

Error propagation for Tinbergen technique.

$$R = \sqrt{\frac{A_1 / B_1}{A_2 / B_2}} \quad (1.1)$$

$$q = \frac{R-1}{R+1}$$

We will assume $A_1 \sim A_2 \sim B_1 \sim B_2$ and $\varepsilon_{A_1} \sim \varepsilon_{A_2} \sim \varepsilon_{B_1} \sim \varepsilon_{B_2} = \varepsilon$ (small polarization, small changes in gain or transmission between HWP positioning).

$$\varepsilon_R = \sqrt{\left(\frac{\partial R}{\partial A_1} \varepsilon_{A_1}\right)^2 + \left(\frac{\partial R}{\partial A_2} \varepsilon_{A_2}\right)^2 + \left(\frac{\partial R}{\partial B_1} \varepsilon_{B_1}\right)^2 + \left(\frac{\partial R}{\partial B_2} \varepsilon_{B_2}\right)^2}$$

$$\frac{\partial R}{\partial A_1} = \frac{1}{2} \frac{1}{R} \frac{B_2}{A_2 B_1} \sim \frac{1}{2RA_1} \quad \frac{\partial R}{\partial B_2} = \frac{1}{2} \frac{1}{R} \frac{A_1}{A_2 B_1} \sim \frac{1}{2RA_1} \quad (1.2)$$

$$\frac{\partial R}{\partial B_1} = -\frac{1}{2} \frac{1}{R} \frac{A_1 B_2}{A_2 B_1^2} \sim -\frac{1}{2RA_1} \quad \frac{\partial R}{\partial A_2} = -\frac{1}{2} \frac{1}{R} \frac{A_1 B_2}{A_2^2 B_1} \sim -\frac{1}{2RA_1}$$

After some fussing around:

$$\varepsilon_R \sim \frac{\varepsilon}{RA} \sim \frac{\varepsilon}{RB} \quad (1.3)$$

So....

$$\varepsilon_q = \sqrt{\left(\frac{\partial q}{\partial R} \varepsilon_R\right)^2} = \varepsilon_R \sqrt{\left(\frac{1}{R+1} - \frac{R-1}{(R+1)^2}\right)^2} = \varepsilon_R \sqrt{\frac{4}{(R+1)^4}} = \frac{2\varepsilon_R}{(R+1)^2} \quad (1.4)$$

Since $R \sim 1$

$$\varepsilon_q \sim \frac{\varepsilon_R}{2} \sim \frac{\varepsilon_A}{2A} \sim \frac{\varepsilon_B}{2B} \quad (1.5)$$

This is the same result as for the simple Wollaston technique!