1 Intro: Tinbergen Wollaston Technique

Consider a star intrinsically polarized where

$$q = \frac{Q}{I} = \frac{I_0 - I_{90}}{I_0 + I_{90}} = \frac{1-\beta}{1+\beta} \text{ so } I_{90} = \beta I_0 \text{ and } \beta = \frac{1-q}{1+q}$$

2 Photometric

Now, introduce a throughput coefficient to account for the different response between the two sides of the Wollaston. Let the left side be A and the right side be B and let $B = \alpha A$ when $q = 0$. Now let SET 1 be HWP $= 0°$ and SET 2 be HWP $= 45°$, swapping I_0 and I_{90}. We are assuming α is not dependent on the HWP position!

Following Jaap Tinbergen, define the measured R_q as:

$$R_q^2 = \frac{A_1/B_1}{A_2/B_2} = \frac{1/\alpha\beta}{\beta/\alpha} = \frac{1}{\beta^2}$$

Substituting in for β in terms of q and some algebra we have:

$$q = \frac{R_q - 1}{R_q + 1}$$

Thus the measured q is the same value as the intrinsic q, independent of α.

3 Non-Photometric

In the preceding, we assumed that SET 1 and SET 2 had the same response conditions. That is, if $q = 0$, then $A_1 = A_2$ and $B_1 = B_2$. What if the response changed between the two positions of the HWP due to, say, clouds? Here we introduce yet another factor, γ, to represent the change in throughput between SET 1 and SET 2. The definition of R_q from Section 2 is now:

$$R_q^2 = \frac{A_1/B_1}{A_2/B_2} = \frac{1/\alpha\beta}{\gamma\beta/\gamma\alpha} = \frac{1}{\beta^2}$$

So, even in non-photometric weather, the method still works exactly.