Intro to Stellar Atmospheres

ASTR 5420 – Stellar Evolution & Interiors

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1 Definition of Stellar Atmosphere

The stellar atmosphere consists of the outer regions of the star. These are the regions that are actually observed, so the properties of the radiated flux determined here tell us what we actually observe about the star. We need to understand radiative transfer to study the properties of the stellar atmosphere, and how that translates into the observed properties.

2 Radiation Pressure

Radiation pressure:

$$P_{\text{rad}} = \frac{aT^4}{3}$$

Gas pressure:

$$P_{\text{gas}} = \frac{\rho N_A kT}{\mu}$$

Where does radiation pressure become important?
Exercise: calculate and compare pressures within the sun, and in its atmosphere.

3 Eddington Limit

Radiation pressure becomes important at the surface, where density is low. Recall equation for flux:

$$F_\nu = \frac{4\pi}{3}\frac{\partial B_\nu}{\partial \tau_\nu} = -\frac{4\pi}{3}\frac{1}{\chi_\nu \rho}\frac{\partial B_\nu}{\partial r} \tag{1}$$

because

$$\tau_\nu = -\int_\infty^r \chi_\nu \rho \, dr'$$

Recall

$$\begin{aligned} P_{\text{rad}} &= aT^4/3 && (2)\\ B(T) &= \frac{\sigma}{\pi}T^4 = \frac{ac}{4\pi}T^4 && (3) \end{aligned}$$

$$P_{\rm rad} = \frac{4\pi}{3c}B(T) \tag{4}$$
$$\tag{5}$$

From equation (1),

$$F_\nu = \frac{L_\nu}{4\pi r^2} = -\frac{4\pi}{3}\frac{1}{\chi_\nu \rho}\frac{\partial B_\nu}{\partial r} \tag{6}$$
$$\frac{\chi_\nu L_\nu}{4\pi r^2} = -\frac{4\pi}{3}\frac{1}{\rho}\frac{\partial B_\nu}{\partial r} \tag{7}$$
$$\frac{1}{4\pi r^2}\int_0^\infty \chi_\nu L_\nu \, d\nu = -\frac{4\pi}{3}\frac{1}{\rho}\frac{\partial B}{\partial r} = -\frac{c}{\rho}\frac{\partial P_{\rm rad}}{\partial r} \tag{8}$$

Define a new mean opacity,

$$\chi_P = \frac{\int_0^\infty \chi_\nu L_\nu d\nu}{L}$$

(We'll call it the Planck opacity.) Then

$$\frac{\partial P_{\rm rad}}{\partial r} = -\frac{\chi_P \rho}{4\pi r^2 c}L \tag{9}$$

Let's revisit the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -g\rho$$

If radiation pressure is sufficiently high, it will overwhelm gravity, and we no longer have a hydrostatic model. This is how winds can be launched, mass loss occurs.

$$\frac{dP_{\rm rad}}{dr} = -g\rho \tag{10}$$
$$-\frac{\chi_P \rho}{4\pi R^2 c}L = -\frac{GM}{R^2}\rho \tag{11}$$
$$L = \frac{4\pi c G M}{\chi_P} \tag{12}$$
$$\tag{13}$$

This is the Eddington luminosity. With greater luminosity, mass loss due to radiation pressure can not be ignored. Some typical values: $\chi_P = 0.34$ cm^2 g^{-1} for the photosphere, then

$$\frac{L_{\rm Edd}}{L_\odot} \approx 3.5 \times 10^4 \left(\frac{M}{M_\odot}\right)$$

What is the photosphere?

Let's consider conditions at or near the surface.

4 Eddington Approximation

The Eddington approximation means that we assume that the radiation field is nearly isotropic. Make it up to linear in μ

$$I_\nu(\tau,\mu) = a_\nu(\tau) + b_\nu(\tau)\mu \tag{14}$$

where $\tau_\nu = -\int \chi_\nu \rho \, dr$.

Define

$$J_\nu \equiv \frac{1}{2}\int_{-1}^{+1} I_\nu \, d\mu = a_\nu \tag{15}$$

$$H_\nu \equiv \frac{1}{2}\int_{-1}^{+1} \mu I_\nu \, d\mu = \frac{b_\nu}{3} \tag{16}$$

$$K_\nu \equiv \frac{1}{2}\int_{-1}^{+1} \mu^2 I_\nu \, d\mu = \frac{a_\nu}{3} \tag{17}$$

$$\tag{18}$$

the 0th, 1st and 2nd moments of I_ν. Then

$$K_\nu = J_\nu/3.$$

Recall the equation of radiative transfer:

$$\mu\frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - S_\nu \tag{19}$$

Integrating this by

$$\frac{1}{2}\int_{-1}^{1} d\mu \quad \Rightarrow \quad \frac{dH_\nu}{d\tau_\nu} = J_\nu - S_\nu \tag{20}$$

$$\frac{1}{2}\int_{-1}^{1} \mu \, d\mu \quad \Rightarrow \quad \frac{dK_\nu}{d\tau_\nu} = H_\nu \rightarrow \frac{dJ_\nu}{d\tau_\nu} = 3H_\nu \tag{21}$$

5 Grey Atmosphere

Assume a "grey" opacity, independent of wavelength:

$$\chi_\nu = \chi_P = \chi$$

Then we can replace

$$J = \int_0^\infty J \, d\nu \qquad H = \int_0^\infty H \, d\nu \qquad S = \int_0^\infty S \, d\nu$$

$$\frac{dH}{d\tau} = \int_0^\infty \frac{dH_\nu}{d\tau_\nu} \, d\nu \qquad \frac{dJ}{d\tau} = \int_0^\infty \frac{dJ_\nu}{d\tau_\nu} \, d\nu$$

In radiative equilibrium,

$$J = S = B = \sigma T^4/\pi$$

Then Eq. (20) gives us

$$\begin{aligned} \frac{dH}{d\tau} &= 0 && (22) \\ H &= \text{constant} && (23) \end{aligned}$$

The constant is determined by realizing that

$$H = \frac{F}{4\pi} = \frac{\sigma T_{\rm eff}^4}{4\pi}$$

at the surface of a star.

Eq. (21) then gives

$$J = 3H\tau + C = \frac{3\sigma T_{\rm eff}^4}{4\pi}(\tau + C)$$

Get C from surface boundary condition. One definition for the surface is where there is no incoming radiation. Let $I(\theta)$ at the surface be isotropic for outgoing angles and zero for incoming angles.

$$I(\mu) = \begin{cases} I, & \mu > 0 \\ 0, & \mu \leq 0 \end{cases} \quad (24)$$

Then

$$\begin{aligned} J &= \frac{1}{2}\int_{-1}^{+1} I\, d\mu = \frac{I}{2} && (25) \\ H &= \frac{1}{2}\int_{-1}^{+1} \mu I\, d\mu = \frac{1}{4} I\mu^2 \Big|_0^1 = \frac{I}{4} && (26) \\ & && (27) \end{aligned}$$

so at the surface,

$$J(\tau = 0) = 2H(\tau = 0).$$

So,

$$J = 3H(\tau + 2/3) \quad (28)$$

But recalling $J = B$, we now have a relation for temperature versus optical depth:

$$\begin{aligned} \frac{\sigma T^4}{\pi} &= \frac{3\sigma T_{\rm eff}^4}{4\pi}(\tau + 2/3) && (29) \\ T^4 &= \frac{3}{4} T_{\rm eff}^4 (\tau + 2/3) && (30) \\ & && (31) \end{aligned}$$

The photosphere is where $T = T_{\rm eff}$ or $\tau_p = 2/3$.

Another way to think about this is that it makes sense for the photosphere to be where $\tau \approx 1$ because it is from this region that photons become free to escape from the star – the mean free path is large enough that photons can escape.

6 Pressure in the photosphere

Hydrostatic equilibrium:

$$\frac{dP}{dr} = -g\rho \tag{32}$$

Since we are considering the surface, $g_s = GM/R^2$.

$$P(\tau) = g_s \int_0^\tau d\tau'/\chi = \frac{g_s \tau}{\chi} + P(\tau = 0)$$

No gas pressure at surface. If below Eddington luminosity, then $P_{\rm rad}$ is small, too. So,

$$\begin{aligned} P(\tau_p) &= \frac{2g_s}{3\chi_p} && (33) \\ \frac{aT_{\rm eff}^4}{3} + \frac{N_A k}{\mu}\rho_p T_{\rm eff} &= \frac{2}{3}\frac{g_s}{\chi_0 \rho_p^n T_{\rm eff}^{-s}} && (34) \end{aligned}$$

Can solve for ρ_p. Equation simplifies if radiation pressure is small.

7 Limb Darkening

The angle at which the viewer sees the surface of a star is θ, and $\mu = \cos\theta$. In general, a star does not appear uniformly bright because of this. This is called **limb darkening**. We can calculate the brightness as a function of μ to show this effect.

Start with the equation for the temperature in the atmosphere,

$$T^4 = \frac{3}{4} T_{\rm eff}^4 (\tau + 2/3)$$

which shows that temperature decreases toward the stellar surface. And, the equation of radiative transfer is

$$\mu \frac{\partial I_\nu}{\partial \tau_\nu} = I_\nu - S_\nu \tag{35}$$

We can integrate this along the line of sight, keeping μ fixed, to find the intensity emitted at the surface:

$$I_\nu(\tau = 0, \mu) = \int_0^\infty e^{-t/\mu} \frac{S_\nu(t)}{\mu}\, dt \tag{36}$$

Assume LTE and a grey atmosphere. Then we can integrate over all wavelengths and get

$$S(t) = B[T(\tau = t)] = \frac{\sigma}{\pi} T^4(\tau = t) = \frac{3\sigma}{4\pi} T_{\rm eff}^4 (\tau + 2/3)$$

$$\begin{aligned} I(\tau = 0, \mu) &= \int_0^\infty e^{-t/\mu} \frac{S(t)}{\mu}\, dt \\ &= \int_0^\infty e^{-t/\mu} \frac{3\sigma}{4\pi} T_{\rm eff}^4 (\tau + 2/3) \frac{1}{\mu}\, dt \\ &= \frac{3\sigma}{4\pi} T_{\rm eff}^4 \int_0^\infty e^{-t/\mu} (\tau + 2/3) \frac{1}{\mu}\, dt \\ &= \frac{3\sigma}{4\pi} T_{\rm eff}^4 (\mu + 2/3) \end{aligned}$$

Then

$$\frac{I(0,\mu)}{I(0,1)} = \frac{3}{5}(\mu + 2/3) \tag{37}$$