Name SOLUTIONS Afternoon Section

Physics 1210 Exam 2 – November 8, 2012

Please write directly on the exam and attach other sheets of work if necessary. Calculators are allowed. No notes or books may be used. Multiple-choice problems have only one correct answer. You may choose to circle two answers on a multiple-choice problem and, if one of them is correct, receive half credit. Circle three and if one is correct, 1/3 credit, etc. For worked problems, be complete and show all work, beginning with diagrams and fundamental, general equations used.

Kinematics

$$v_{\text{avg}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad \vec{v} = \frac{d\vec{r}}{dt} \qquad a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

$$x_1 = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad v_1 = v_0 + at \qquad v_1^2 = v_0^2 + 2a(x_1 - x_0) \qquad a_{\text{rad}} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$\sum \vec{F} = m\vec{a} \qquad \vec{w} = m\vec{g} \qquad f_s \le \mu_s N \qquad f_k = \mu_k N \qquad f = kv \qquad f = Dv^2 \qquad f_{\text{spring}} = -kx$$

Momentum/Impulse

$$\vec{p} = m\vec{v} \qquad J = \Delta(mv) = F\Delta t \qquad x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i}$$

Work/Energy

$$W = \vec{F}\cdot\vec{s} = Fs\cos\theta \qquad K_1 + U_1 + W_{\text{other}} = K_2 + U_2 \qquad P = \frac{\Delta W}{\Delta t} = \vec{F}\cdot\vec{v}$$

$$W = \Delta K \qquad K = \tfrac{1}{2}mv^2 \qquad U_{\text{spring}} = \tfrac{1}{2}kx^2 \qquad U_{\text{grav}} = mgy \qquad F = -\frac{dU}{dx}$$

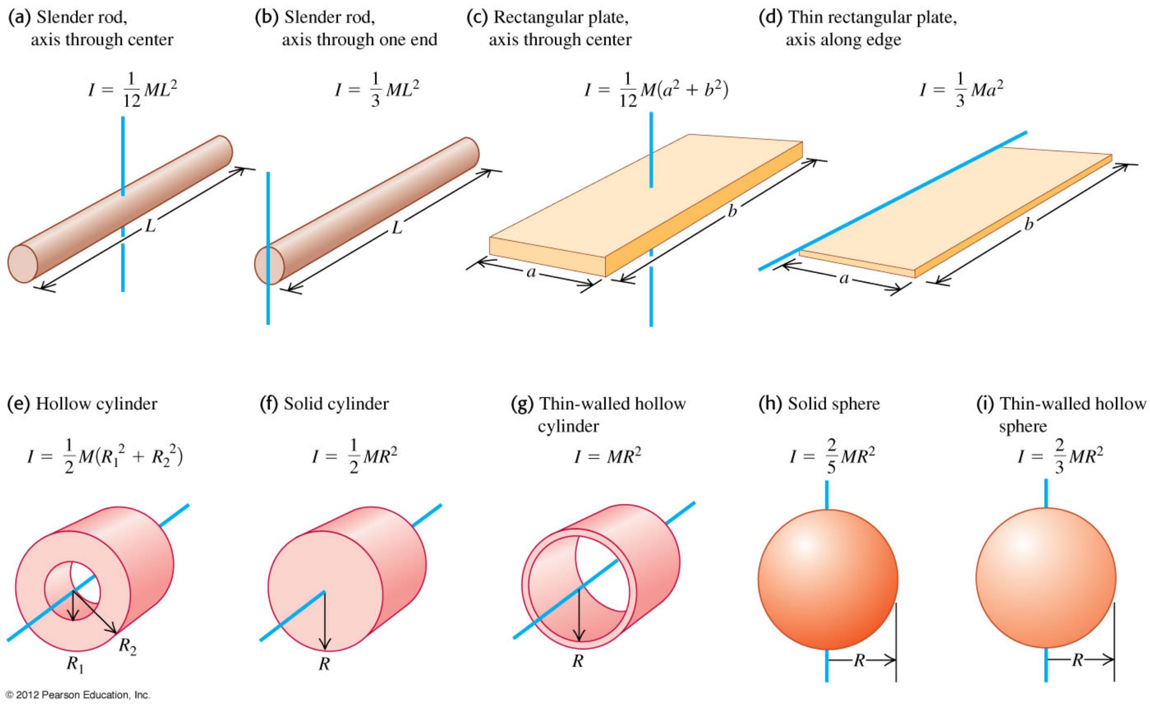
Angular Motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d\omega}{dt} \qquad \theta_1 = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad \omega_1 = \omega_0 + \alpha t \qquad \omega_1^2 = \omega_0^2 + 2\alpha(\theta_1 - \theta_0)$$

$$s = r\theta \qquad v = r\omega \qquad a_{\tan} = r\alpha \qquad a_{\text{rad}} = \omega^2 r \qquad 2\pi = 360^\circ$$

$$I = \sum_i m_i r_i^2 \qquad I = I_{\text{cm}} + Md^2 \qquad \vec{\tau} = \vec{r}\times\vec{F} = rF\sin\phi \qquad \sum\vec{\tau} = I\vec{\alpha} \qquad W = \Delta K = \tau\Delta\theta$$

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 \qquad K_{\text{tot}} = \tfrac{1}{2}mv_{\text{cm}}^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2 \qquad \vec{L} = \vec{r}\times\vec{p} = rmv = I\omega \qquad \Delta L = \tau\Delta t \qquad \text{power} = \tau\omega$$



1) [5 pts] Box A that has mass 4.00 kg collides with Box B that has mass 1.00 kg. After collision, the two boxes stick together and move on a level, frictionless surface. After the collision, the kinetic energy of box A is

A) 1/16 the kinetic energy of block B.
B) 1/4 the kinetic energy of block B.
C) 4 times the kinetic energy of block B.
D) 16 times the kinetic energy of block B.
E) the same as the kinetic energy of block B.

2) [5 pts] A heavy truck and a small car are traveling at speed *v* on the same road. They both slam on their brakes and skid to a halt. They both have the same tires, so the coefficient of friction is the same. Which of the following is true?

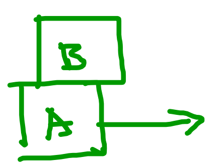
A) They both come to a stop in the same distance and same time.
B) The car stops in the least distance and in the least time.
C) The truck stops in the least distance and in the least time.
D) The truck stops in less distance, but they both stop in the same time.
E) The car stops in less distance, but they both stop in the same time.

3) [5 pts] When a rigid body rotates about a fixed axis, all the points in the body have the same

A) tangential acceleration.
B) angular acceleration.
C) linear displacement.
D) centripetal acceleration.
E) tangential speed.

4) [6pts] Use diagrams and words to give one example for each of the following situations.

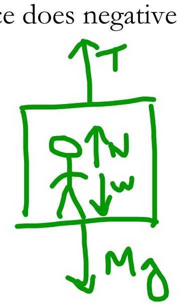
A) A static frictional force does positive work.



When A is pulled, static frictional force does positive work on B

B) A normal force does negative work.

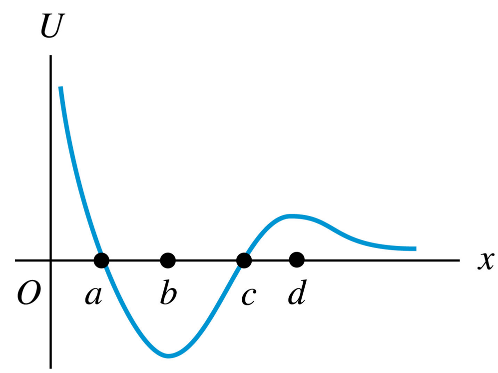
Elevator:



↓ accelerating down

As the elevator accelerates downward, the normal force does negative work.

The graph at right shows the potential energy U for a particle that moves along the x-axis. Use this graph to answer questions 5 & 6.



5) [4 pts] The particle is initially at $x = d$ and moves in the negative x-direction. At which of the labeled x-coordinates does the particle have the greatest *speed*?

A) at $x=a$
B) at $x=b$
C) at $x=c$
D) at $x=d$
E) More than one of the above.

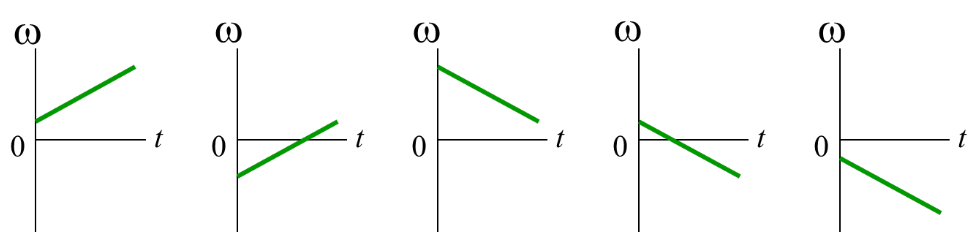
6) [4 pts] At which of the labeled x-coordinates is there zero force on the particle?

A) at $x=a$ and $x=c$
B) at $x=b$ only
C) at $x=d$ only
D) at $x=b$ and d
E) misleading question – there is a force at all values of x.

7) [5 pts] A car speeds up while the engine delivers constant power. Which of the following statements is correct?

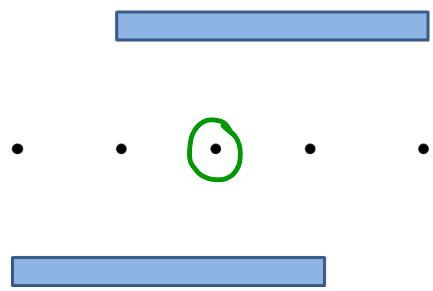
A) The car's acceleration is greater at the beginning of this process.
B) The car's acceleration is greater at the end of this process.
C) The car's acceleration is the same during this process.
D) None of the above is correct.

8) [5 pts] An object rotates with constant acceleration about an axis. The initial angular position is positive, the initial angular velocity is negative, and the angular acceleration is positive. Which of the following ω–t graphs best describes this rotational motion?

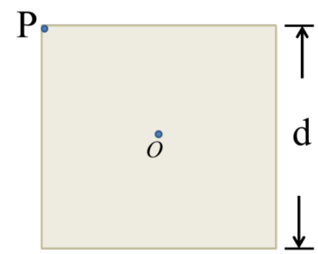


A. B. C. D. E.

9) [5 pts] Two geometrically identical bars with the same mass are put together according to the following figure. Circle the correct location of the center of mass of this two-bar system.



10) [5 pts] A square metal plate with length *d* on each side is pivoted about an axis through a corner (point P as indicated in the figure) and perpendicular to the plate. The mass of the plate is M, and the moment of inertia of the plate about an axis through its center of mass O and perpendicular to the plate is I_{CM}. What is the moment of inertia about the first axis through point P?



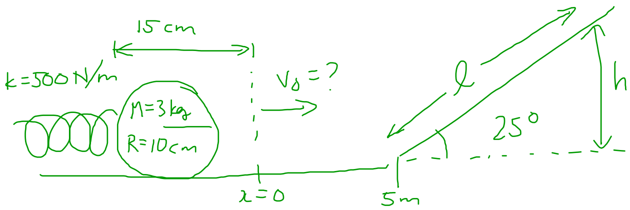
A) $I_P = I_{CM} + Md^2$
B) $I_P = I_{CM} + M(d/2)^2$
C) $I_P = I_{CM} + \frac{1}{2} Md^2$
D) $I_P = 1/6\ Md^2$
E) None of the above

11) [6 pts] A girl moves quickly to the center of a spinning merry-go-round, traveling along the radius of the merry-go-round. Which of the following statements are true? **Circle all that apply.**

A) The angular speed of the system decreases.
B) The angular speed of the system increases.
C) The angular speed of the system remains constant.
D) The moment of inertia of the system decreases.
E) The moment of inertia of the system increases.
F) The moment of inertia of the system remains constant.
G) The kinetic energy of the system decreases.
H) The kinetic energy of the system increases.
I) The kinetic energy of the system remains constant.

12) [15 pts] You launch a spherical ball of mass 3.00 kg and radius 10.0 cm horizontally across a level surface, using a massless spring with spring constant k=500 N/m compressed by 15.0 cm. There is no friction between the spring and the ball, so that it immediately begins rolling without slipping on the level surface. After a distance of 5 m, it begins to go up an incline with slope angle 25°, continuing to roll without slipping.

A) What is the ball's center of mass velocity immediately after launch?
B) How far does the ball travel up the incline before rolling back down?
C) What is the angular acceleration of the ball as it rolls up the incline?



A) Conservation of energy

$$U_1 + K_1 = U_2 + K_2$$

$$U_1 = \tfrac{1}{2}kx^2 \qquad U_2 = 0$$

$$K_1 = 0 \qquad K_2 = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega^2$$

$$\Rightarrow \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega^2$$

$$I = \tfrac{2}{5}MR^2, \quad \omega = \frac{v_0}{R} \Rightarrow \tfrac{1}{2}I\omega^2 = \tfrac{1}{5}mv_0^2$$

$$\Rightarrow \tfrac{1}{2}kx^2 = \tfrac{1}{2}mv_0^2 + \tfrac{1}{5}mv_0^2 = \tfrac{7}{10}mv_0^2$$

$$v_0^2 = \frac{5}{7}\frac{k}{m}x^2$$

$$v_0 = x\sqrt{\frac{5}{7}\frac{k}{m}} = (0.15\,m)\sqrt{\frac{5}{7}\frac{500\ N/m}{3\ kg}} = \boxed{1.64\ m/s}$$

B) Conservation of energy:

After launch of ball, U_{el} converted to K_{tot}.

At highest point, K_{tot} converted to U_g.

$$\Rightarrow U_{el} = U_g \Rightarrow \tfrac{1}{2}kx^2 = mgh$$

$$h = \frac{kx^2}{2mg} = \frac{(500\ N/m)(0.15\,m)^2}{2(3\ kg)(9.8\ m/s^2)}$$

$$= 0.191\ m = 19.1\ cm$$

$$\cos 25^\circ = \frac{h}{l} \Rightarrow l = h\cos 25^\circ = \boxed{0.452\ m}$$

12 c) Angular acceleration up the ramp:

$$\omega_f^2 = \omega_0^2 + 2\alpha(\theta_2 - \theta_1)$$

$$\Delta\theta = \frac{l}{r} = \frac{0.452\text{ m}}{0.1\text{ m}} = 4.52\text{ rad}$$

$$\omega_0 = \frac{v}{r} = \frac{1.64\text{ m/s}}{0.1\text{ m}} = 16.4\text{ rad/s}$$

$$\omega_f = 0\text{ rad/s}$$

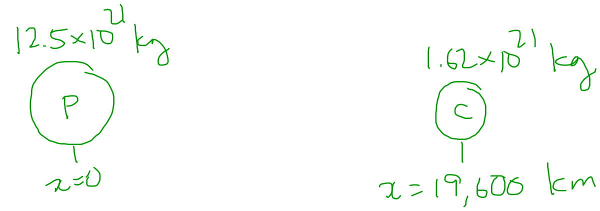
$$\alpha = \frac{\omega_f^2 - \omega_0^2}{2\Delta\theta} = \frac{-(16.4\text{ rad/s})^2}{2(4.52\text{ rad})}$$

$$= \boxed{-29.6\text{ rad/s}^2}$$

negative, because it is slowing down.

13) [15 pts] The Kuiper Belt is a population of rocky and icy bodies at Neptune's orbit and beyond. Pluto is one of the largest Kuiper Belt objects. Pluto and its largest moon, Charon, orbit each other about their center of mass. Their orbit is circular, which means that they always maintain a distance of 19,600 km from each other. Pluto has a mass of 12.5×10^{21} kg and Charon has a mass of 1.62×10^{21} kg. It takes 6.34 days for the Pluto-Charon system to complete an orbit.

A) What is the distance between Pluto and the center of mass? Between Charon and the center of mass?
B) What is the moment of inertia of the Pluto-Charon system?
C) What is the total angular momentum of the Pluto-Charon system?



A) $$x_{cm} = \frac{\sum x_i m_i}{\sum m_i} = \frac{(0)(12.5\times10^{21}\text{ kg}) + (19{,}600\text{ km})(1.62\times10^{21}\text{ kg})}{12.5\times10^{21}\text{ kg} + 1.62\times10^{21}\text{ kg}}$$

$$= 2{,}250\text{ km}$$

Pluto to CM $= \boxed{2{,}250\text{ km}}$

Charon to CM $= (19{,}600 - 2{,}250)\text{ km} = \boxed{17{,}350\text{ km}}$

B) $$I_{tot} = \sum m_i r_i^2$$

$$= (12.5\times10^{21}\text{ kg})(2.250\times10^6\text{ m})^2 + (1.62\times10^{21}\text{ kg})(17.35\times10^6\text{ km})^2$$

$$= \boxed{5.51\times10^{35}\text{ kg m}^2}$$

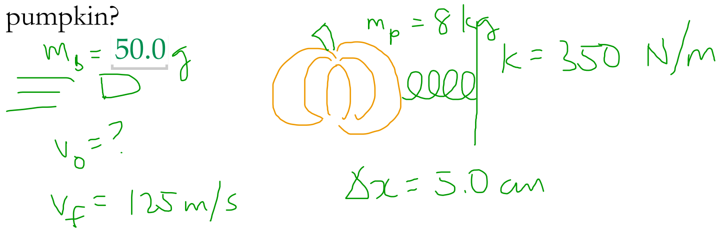
C) $$L_{tot} = I_{tot}\,\omega$$

$$\omega = \frac{2\pi}{6.34\text{ d}}\left(\frac{1\text{ day}}{24\text{ hr}}\right)\left(\frac{1\text{ hr}}{3600\text{ s}}\right) = 1.15\times10^{-5}\text{ s}^{-1}$$

$$L_{tot} = (5.51\times10^{35}\text{ kg m}^2)(1.15\times10^{-5}\text{ s}^{-1})$$

$$= \boxed{6.32\times10^{30}\text{ kg m}^2\text{ s}^{-1}}$$

14) [15 pts] Someone fires a rifle and shoots a bullet of mass ~~5.0~~ 50.0 g through an 8.00 kg pumpkin. The pumpkin is attached to a coil spring and initially rests on a frictionless, horizontal surface. The spring has a force constant of 350 N/m, and the impact compresses the spring 5.0 cm. If the speed of the bullet as it emerges from the pumpkin is 125 m/s, what is the speed of the bullet before it enters the pumpkin?



Impact: conservation of momentum

$$m_B v_0 + m_p(0) = m_B v_f + m_p v_p$$

$$v_0 = v_f + \frac{m_p v_p}{m_B}$$

To get v_p: energy conservation: $K \rightarrow U_{el}$

$$K_1 + U_1 = K_2 + U_2$$

$K_1 = \frac{1}{2} m_p v_p^2$ $\qquad K_2 = 0$

$U_1 = 0$ $\qquad U_2 = \frac{1}{2} k x^2$

$$\Rightarrow \frac{1}{2} m_p v_p^2 = \frac{1}{2} k x^2$$

$$v_p = x\sqrt{\frac{k}{m_p}}$$

$$= (0.05\text{ m})\sqrt{\frac{350\text{ N/m}}{8\text{ kg}}}$$

$$= 0.331\text{ m/s}$$

$$v_0 = (125\text{ m/s}) + \frac{(8\text{ kg})(0.331\text{ m/s})}{0.05\text{ kg}}$$

$$= \boxed{178\text{ m/s}}$$