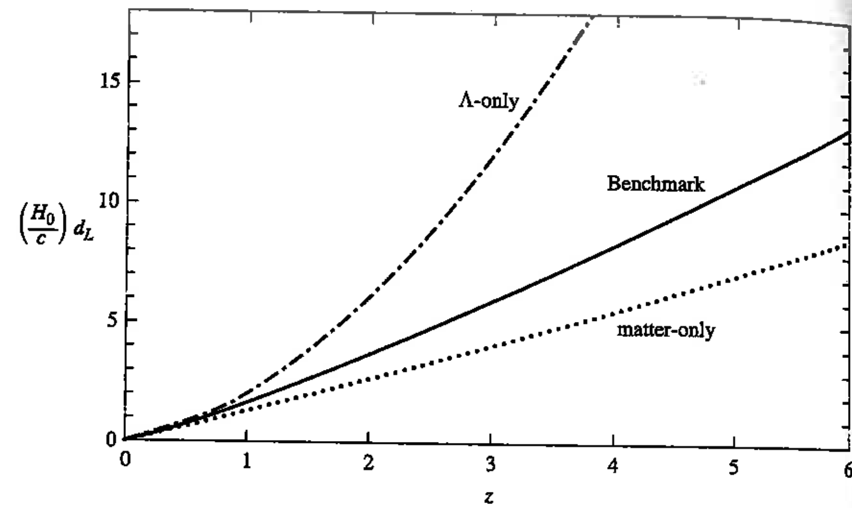

FIGURE 7.2 The luminosity distance of a standard candle with observed redshift z. The bold solid line gives the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.

distance may be approximated as

$$d_P(t_0) \approx \frac{c}{H_0} z \left(1 - \frac{1+q_0}{2} z\right). \tag{7.30}$$

In a nearly flat universe, the luminosity distance may thus be approximated as

$$d_L \approx \frac{c}{H_0} z \left(1 - \frac{1+q_0}{2} z\right)(1+z) \approx \frac{c}{H_0} z \left(1 + \frac{1-q_0}{2} z\right). \tag{7.31}$$

Note that in the limit $z \to 0$,

$$d_p(t_0) \approx d_L \approx \frac{c}{H_0} z. \tag{7.32}$$

In a universe described by the Robertson–Walker metric, the luminosity distance is a good approximation to the current proper distance for objects with small redshifts.

7.3 ■ ANGULAR-DIAMETER DISTANCE

The luminosity distance d_L is not the only distance measure that can be computed using the observable properties of cosmological objects. Suppose that instead of a standard candle, you observed a *standard yardstick*. A standard yardstick is an object whose proper length ℓ is known. In most cases, it is convenient to choose as your yardstick an object that is tightly bound together, by gravity or duct tape

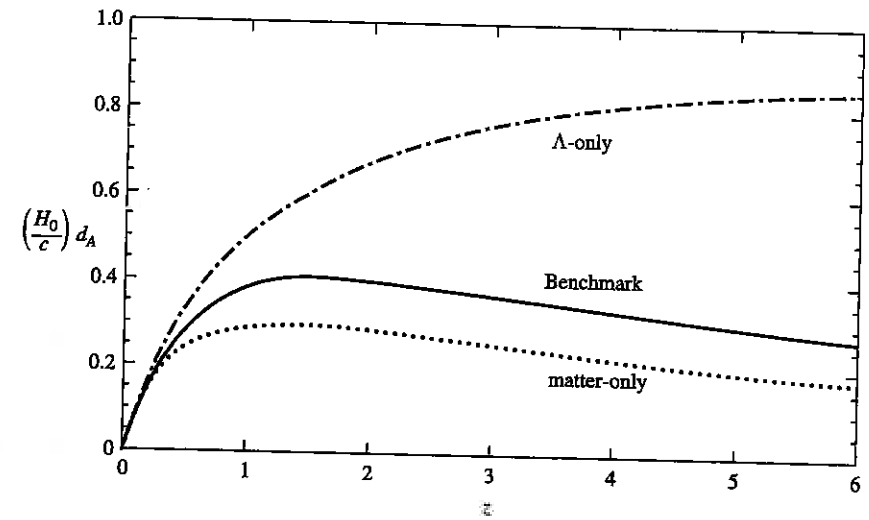

FIGURE 7.4 The angular-diameter distance for a standard yardstick with observed redshift z. The bold solid line gives the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.

However, the angular-diameter distance to highly redshifted objects approaches zero as $z \to \infty$, with

$$d_A(z \to \infty) \approx \frac{d_{\rm hor}(t_0)}{z}. \qquad (7.41)$$

In model universes other than the lambda-only model, the angular-diameter distance d_A has a maximum for standard yardsticks at some critical redshift z_c. (For the Benchmark Model, $z_c = 1.6$, where $d_A(\max) = 0.41c/H_0 = 1800\,\mathrm{Mpc}$.) This means that if the universe were full of glow-in-the-dark yardsticks, all of the same size ℓ, their angular size $\delta\theta$ would decrease with redshift out to $z = z_c$, but then would increase at larger redshifts. The sky would be full of big, faint, redshifted yardsticks.

In principle, standard yardsticks can be used to determine H_0. To begin with, identify a population of standard yardsticks (objects whose physical size ℓ is known). Then, measure the redshift z and angular size $\delta\theta$ of each standard yardstick. Compute the angular-diameter distance $d_A = \ell/\delta\theta$ for each standard yardstick. Plot cz versus d_A, and the slope of the relation, in the limit $z \to 0$, will give you H_0. In addition, if you have measured the angular size $\delta\theta$ for standard candles at $z \sim z_c$, the shape of the cz versus d_A plot can be used to determine further cosmological parameters. If you simply want a kinematic description, estimate q_0 by fitting equation (7.39) to the data. If you are confident that the universe is dominated by matter and a cosmological constant, you can see which values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ provide the best fit to the observed data.

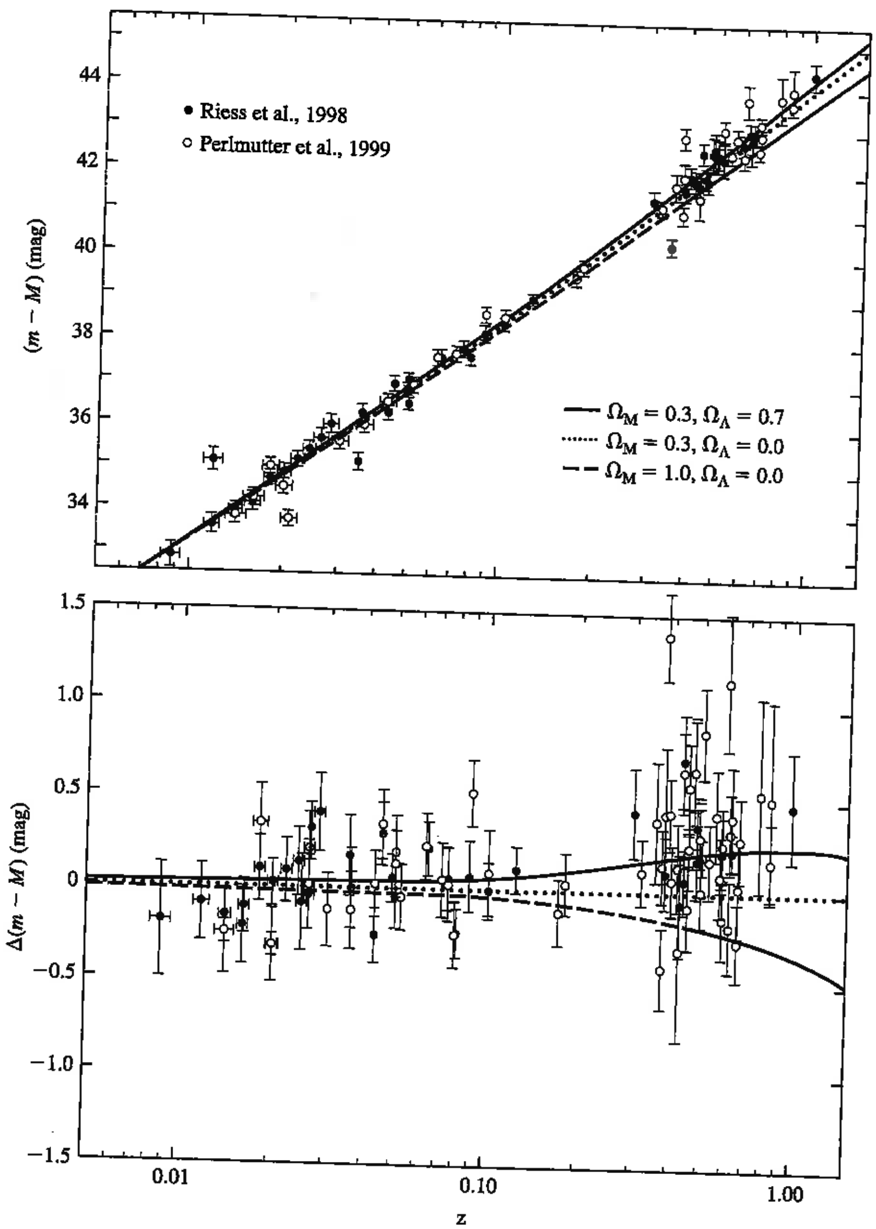

FIGURE 7.5 Distance modulus versus redshift for type Ia supernovae from the Supernova Cosmology Project (Perlmutter et al., 1999, ApJ, 517, 565) and the High-z Supernova Search Team (Riess et al., 1998, AJ, 116, 1009). The bottom panel shows the difference between the data and the predictions of a negatively curved $\Omega_{m,0} = 0.3$ model.

The supernova data extend out to $z \sim 1$; this is beyond the range where an expansion in terms of H_0 and q_0 is adequate to describe the scale factor $a(t)$. Thus, the two supernova teams customarily describe their results in terms of a model universe that contains both matter and a cosmological constant. After choosing values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$, they compute the expected relation between $m - M$ and z, and compare it to the observed data. The results of fitting these model universes are given in Figure 7.6. The ovals drawn on Figure 7.6 enclose those values of $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ that give the best fit to the supernova data. The results of the two teams (the solid ovals and dotted ovals) give very similar results. Three concentric ovals are shown for each team's result; they correspond to 1σ, 2σ, and 3σ confidence intervals, with the inner oval representing the highest probability.

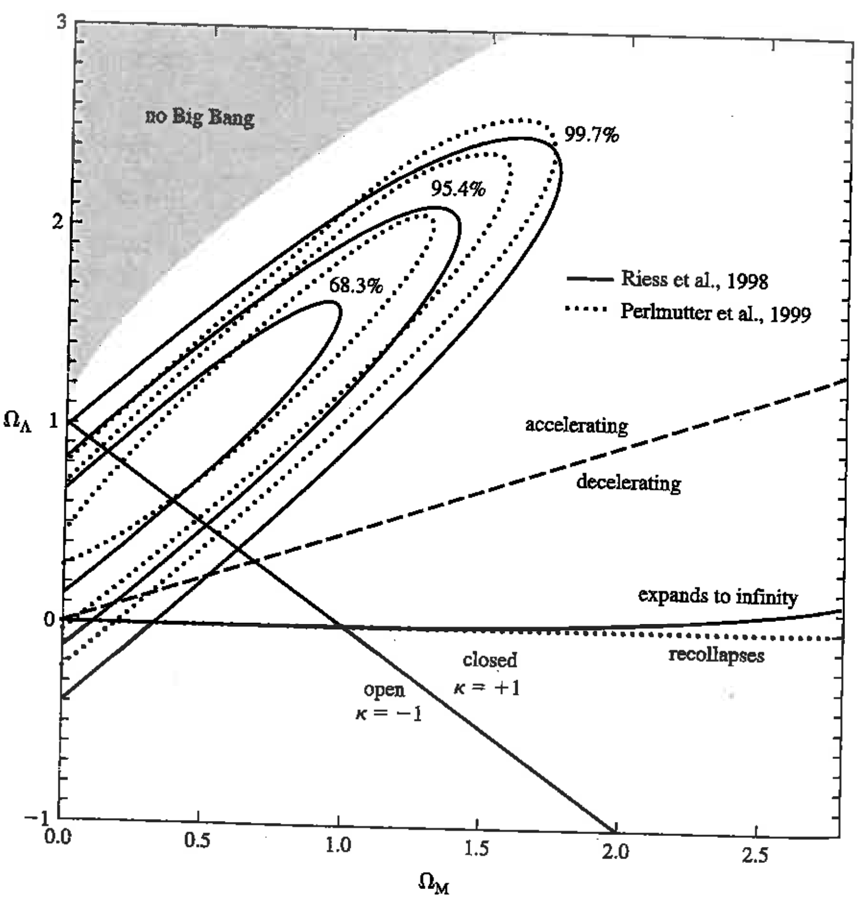

FIGURE 7.6 The values of $\Omega_{m,0}$ (horizontal axis) and $\Omega_{\Lambda,0}$ (vertical axis) that best fit the data shown in Figure 7.5. The solid ovals show the best-fitting values for the High-z Supernova Search Team data; the dotted ovals show the best-fitting values for the Supernova Cosmology Project data.