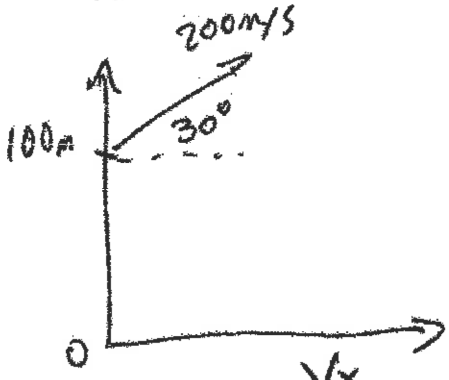
Practice Final Solutions

① Godzilla vs. Hulk.



when $y=0$, $x=?$

$$x = V_0(\cos 30^\circ)t = (173\,\text{m/s})t$$

$V_0 = 200\,\text{m/s}$, $\cos 30^\circ = 0.866$

$$y = y_0 + V_0 \sin 30^\circ\, t + \tfrac{1}{2} a_y t^2$$

just use $g = 10\,\text{m/s}^2$

$$y = 100\,\text{m} + (200\,\text{m/s})(0.5)t + \tfrac{1}{2}(-10\,\text{m/s}^2)t^2$$

$$0 = y = 100\,\text{m} + (100\,\text{m/s})t - 5\,\text{m/s}^2\, t^2$$

$a = -5 \quad b = 100 \quad c = 100$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-100 \pm \sqrt{10000 + 2000}}{-10}$$

$$t = +21\ \text{sec.}$$

$$x = (173\,\text{m/s}) \cdot 21.0 = 3633\,\text{m}$$

~ 3.6 km w/ sig figs.

② Batman Revised

$(m_R g) - T = m_R a$

$m_B g \sin 45^\circ + T + (-)\mu_k g \cos 45^\circ = m_B a$

$\rightarrow a = g - \frac{T}{m_R}$

$(1 - \mu_k)\, m_B g (0.707) + T = (m_B)\left(g - \frac{T}{m_R}\right)$

$\mu_k = 0.3$, $(1 - \mu_k) = 0.7$

$0.49 g + \frac{T}{m_B} = g - \frac{T}{m_R}$

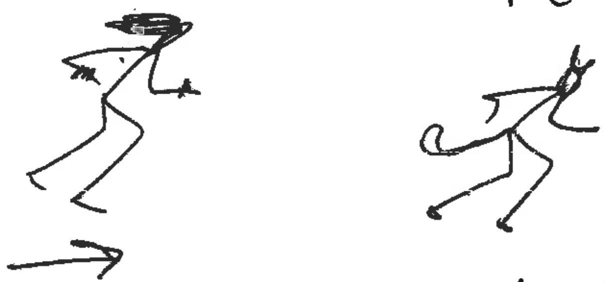
$T\left(\frac{1}{m_B} + \frac{1}{m_R}\right) = 5.1 g$

$\left(\frac{1}{m_B} + \frac{1}{m_R}\right) = 0.03\ kg^{-1}$; g: use $10\ m/s^2$

$$\boxed{T = 170\ N}$$

③ Wonder Woman vs. Cheetah

$m_{WW} = 80\,kg$ $m_c = 60\,kg.$



$V_{WW} = 10\,m/s$ $V_c = 8\,m/s$

Inelastic collision. Momentum conserved. Energy Not.

$$p = m_{WW} \cdot V_{WW} + m_c V_c = (m_{WW} + m_c) V_{final}$$

$$\overset{\shortparallel}{80} \quad \overset{\shortparallel}{10} \quad \overset{\shortparallel}{60} \quad \overset{\shortparallel}{8} \quad \overset{\shortparallel}{140\,kg} \quad \shortparallel\,?$$

$$1280\,kg \cdot m/s = 140 \cdot V_{final}$$

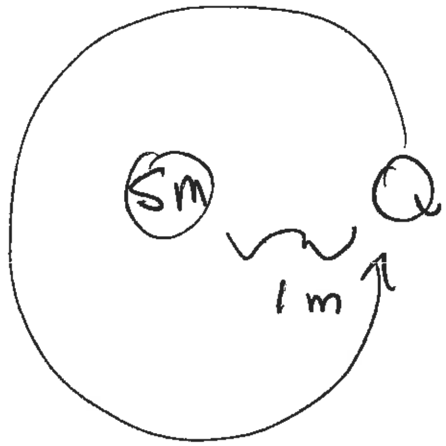
$$\boxed{V_{final} = 9.1\,m/s}$$

$$\frac{K_1}{K_2} = \frac{\frac{1}{2} m_{WW} V_{WW}^2 + \frac{1}{2} m_c V_c^2}{\frac{1}{2} (m_{WW} + m_c) V_{final}^2} = \frac{8000\,J + 3840\,J}{(140)(9.1)^2\,J}$$

$$= \frac{11840\,J}{11593\,J}$$

$$= \boxed{1.02}$$

④ Quicksilver Hits Spider-man



$a = 10\ m/s^2$ for $t = 10s$.

5 hits per circle.

$m_Q = 70\ kg$

a. What is final speed in m/s? **100 m/s**

b. Final Kinetic Energy? **350 kJ**

c. Final angular speed ω? **100 rad/sec**

d. Number of hits total? **398**

$a = \frac{dv}{dt}$ "constant". So $V_f = V_0 + at$ $= \boxed{100\ m/s}$ ($V_0 = 0$, $a = 10\ m/s^2$, $t = 10$)

$K = \frac{1}{2} m_Q V_Q^2 = \frac{1}{2}(70kg)(100\ m/s)^2 = 350{,}000\ J$

Not a rigid rotator, not spinning! Only $\frac{1}{2} m v_Q^2$.

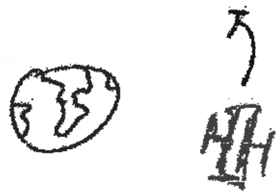
$V = r\omega$ so $\omega = \frac{V_{final}}{r = 1m} = 100\ rad/s$

$\#\ Hits = \left(\frac{\text{total distance}}{2\pi r}\right) \times 5$

$x = \cancel{x_0}^{0} + \cancel{v_0}^{0} t + \frac{1}{2} a t^2 = \boxed{500\ m}$

$= \left(\frac{500\ m}{2\pi\ m}\right) 5 = 398$

⑤. Geosynchronous JLA Watchtower



$T = P = 1 \text{ day} = 86\,400 \text{ sec}$

$M_E = 5.97 \times 10^{24} \text{ kg}$

Kepler $T = 2\pi r^{3/2} / \sqrt{G M_E}$

$G = 6.67 \times 10^{-11} \; N (m/kg)^2$

$$r^{3/2} = \frac{T}{2\pi} \sqrt{G M_E}$$

$$r = \left(\frac{86400 s}{2\pi} \sqrt{6.67 \times 10^{-11} \times 6 \times 10^{24}} \right)^{2/3}$$

2π: $= 13800$

$\sqrt{\ldots} = 1.99 \times 10^{7}$; $6.67 \times 10^{-11} \times 6 \times 10^{24} = 39.8 \times 10^{13}$

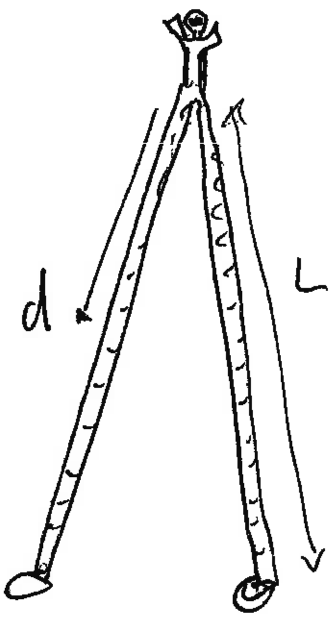
$$r = \left(2.75 \times 10^{12} \right)^{2/3}$$

$$\boxed{r = 4.27 \times 10^{6} \text{ m} = 42{,}700 \text{ km}}$$

(26,500 miles)

Note: often altitude is given above sea level, $R_E \approx 6400$ km.

⑥ Stiltman Walks!



$L = 75\ m$

Mass Leg = 100 kg

$I_{rod} = \frac{1}{3} ML^2$ (pivot at end)

$\omega = \sqrt{\frac{mgd}{I}}$ ← d = c.o.m. distance

$\omega = \sqrt{\frac{\cancel{100\,kg} \cdot 9.8\,m/s^2 \cdot 37.5\,\cancel{m}}{1/3 \cdot \cancel{100\,kg} \cdot (75\,\cancel{m})^2}}$ (10 m/s² ?)

$\omega = \sqrt{0.02}\ s^{-1}$

$\omega = 0.14$ rad/s

Period $T = 2\pi/\omega = 44.5$ sec.

$\text{Velocity} = \frac{\text{Stride}}{\text{Time}} = \frac{25\,m}{44.5\,sec.} = \boxed{0.56\ m/s}$

(7) Black Canary's Cry

$V_{sound} = 344\ m/s$
$f = 50\ kHz$

a) $\lambda = \frac{V_{sound}}{f} = \frac{344\ m/s}{50{,}000/s} = \boxed{0.00688\ m}$

$\approx 7\ mm$

$B_1 = 150\ dB$

b) 150 dB but need to convert to W/m^2

$\frac{d_1}{d_2} = \frac{1}{2}$

$$150\ dB = 10\ dB \log\left(\frac{I_1}{10^{-12}\ W/m^2}\right)$$

$$15 = \log \frac{I_1}{10^{-12}\ W/m^2}$$

$$W/m^2\left(10^{15} \times 10^{-12}\right) = I_1$$

$$I_1 = 1000\ W/m^2$$

I_2 is 2x distance, $\Rightarrow \frac{1}{4}$ the Intensity

$$\boxed{I_2 = 250\ W/m^2}$$

$$10\ dB * \log\left(\frac{250\ W/m^2}{10^{-12}\ W/m^2}\right) = ?\ 144\ dB$$