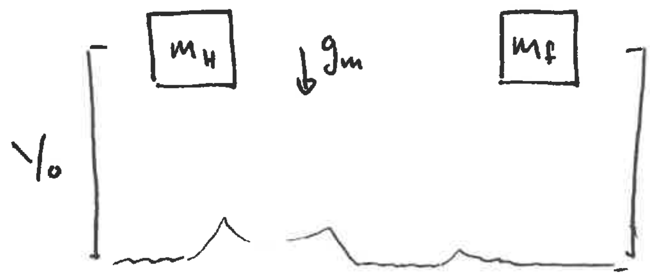
Exam 1 solutions

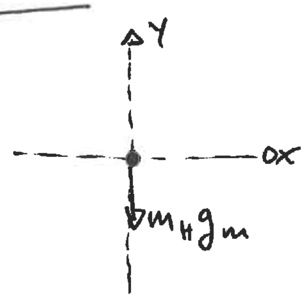
1)



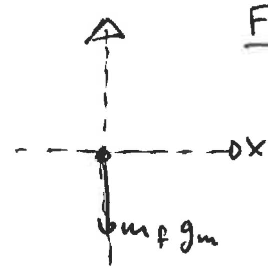
$Y_0 = 1m$

Free body diagrams.

Hammer



Feather



$\Sigma \vec{F} = m\vec{a}$

$\underline{X_H}$ N/A

$\underline{Y_H}$

$-m_H g_m = -m_H a_H$

$-g_m = -a_H$

$g_m = a_H$

$\underline{X_f}$ N/A

$\underline{Y_f}$

$-m_f g_m = -m_f a_f$

$-g_m = -a_f$

$g_m = a_f$

so $a_H = a_f = g_m$

a) $\underline{Y}$ (linear motion)

$a_y = -g_m$ — ①

$v_y = -g_m t + \cancel{v_0}^{\,0}$ — ②

$y = -\frac{1}{2} g_m t^2 + Y_0$ — ③

③, solve for t, $y = 0$

$0 = -\frac{1}{2} g_m t^2 + 1$

$\frac{1}{2} g_m t^2 = 1$

$t^2 = \frac{2}{g_m}$

$t = \sqrt{\frac{2}{g_m}} = \sqrt{\frac{2}{1.6}}$

$\boxed{t = 1.12 \text{ sec.}}$

1) continued

1) b)

Velocities at ground?

② $\leftarrow t$, solve for V.

$V = -g_m t$

$V = -g_m (1.12\,sec)$ — I used more digits stored in my Calculator. Rounding at end.

$V = -1.6\,(1.12)$

$\boxed{V_f = -1.789\,m/s}$

c) Average velocity:

$\bar{V} = \frac{V_f + V_o}{2}$

$\bar{V} = \frac{V_f + 0}{2}$

$\bar{V} = \frac{-1.789}{2}$ ← again, keeping more digits in calc.

$\boxed{\bar{V} = -0.894\,m/s}$

d) Average acceleration:

$\bar{a}$ is constant, so $\bar{a}_f = \bar{a}_o = \bar{a}$

$|\bar{a}| = |g_m| = |1.6\,m/s^2|$

or

$\bar{a} = \frac{a_f + a_o}{2}$

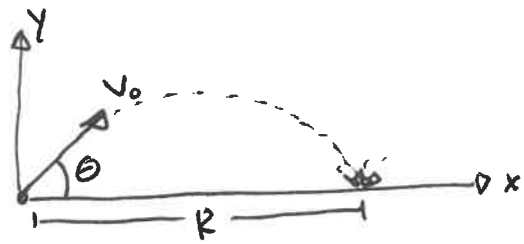
$\bar{a} = \frac{g_m + g_m}{2}$

$\bar{a} = \frac{2g_m}{2}$

$\bar{a} = g_m$

$\boxed{\bar{a} = 1.6\,m/s^2}$

2)



$V_{oH} = 30 m/s$ $\qquad$ $V_{oSH} = 25 m/s$

$\theta_H = 10°$ $\qquad$ $\theta_{SH} = 45°$

Projectile motion for both (same physics / diff initial conditions)

$\boxed{\text{General}}$

X

$a_x = 0$ — ①

$V_x = V_o \cos\theta$ — ②

$x = V_o t\cos\theta + \cancel{x_o}^{0}$ — ③

Y

$a_y = -g$ — ④

$V_y = -gt + V_o \sin\theta$ — ⑤

$y = -\frac{1}{2}gt^2 + V_o t\sin\theta + \cancel{y_o}^{0}$ — ⑥

⑥, solve for t. y=0 (at landing)

$0 = -\frac{1}{2}gt^2 + V_o t\sin\theta$

$0 = t\left(-\frac{1}{2}gt + V_o\sin\theta\right)$

$\cancel{t=0}$ or $0 = -\frac{1}{2}gt + V_o\sin\theta$

$t = \frac{2V_o\sin\theta}{g}$ ✓ — ⑥'

③ ⟵ ⑥', x=R, solve for R.

$R = V_o\left(\frac{2V_o\sin\theta}{g}\right)\cos\theta$

$R = \frac{V_o^2}{g}\cdot 2\sin\theta\cos\theta$ $\qquad$ $2\sin\theta\cos\theta = \sin(2\theta)$

$R = \frac{V_o^2}{g}\sin(2\theta)$ — ③'

Solve ③' for H.

$R_H = \frac{V_{oH}^2}{g}\sin(2\theta_H)$

$R_H = \frac{(30)^2}{10}\sin(2\cdot 10)$

$R_H = 30.78m$

③' solve for SH.

$R_{SH} = \cancel{\frac{V_{osA}^2}{g}}$ $\frac{V_{oSH}^2}{g}\sin(2\theta_{SH})$

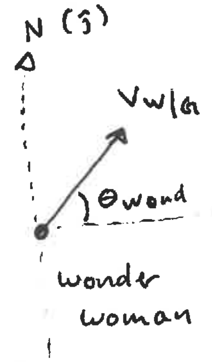
$R_{SH} = \frac{(25)^2}{10}\sin(2\cdot 45)$

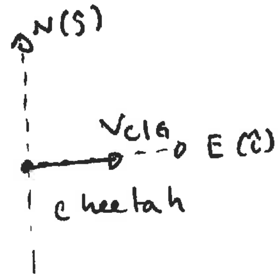
$R_{SH} = 62.5m$

$R_{SH} - R_H = 62.5 - 30.78 = 31.72m$

$\boxed{\text{She Hulk wins by 31m}}$

3)





$|V_{c|G}| = 10\,m/s$

$|V_{w|G}| = 12\,m/s$, $\theta_{wond} = 45°$

$$\vec{V}_{w|c} = \vec{V}_{w|G} + \vec{V}_{G|c} \quad \text{and} \quad \vec{V}_{G|c} = -\vec{V}_{c|G}$$

$$\vec{V}_{w|c} = \vec{V}_{w|G} - \vec{V}_{c|G}$$

$$V_{w|cx}\hat{i} + V_{w|cy}\hat{j} = (V_{w|Gx}\hat{i} + V_{w|Gy}\hat{j}) - (V_{c|Gx}\hat{i} + V_{c|Gy}\hat{j})$$

$$V_{w|cx}\hat{i} + V_{w|cy}\hat{j} = (12\cos(45)\hat{i} + 12\sin(45)\hat{j}) - (10\cos(0)\hat{i} + 10\sin(0)\hat{j})$$

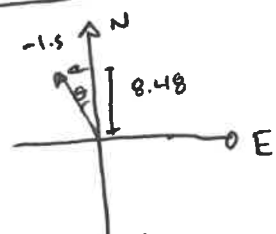
$V_{w|cx} = 12\cos 45 - 10 = -1.515$ ← keeping more digits in calc.

$V_{w|cy} = 12\sin 45 - 0 = 8.485$

$$|\vec{V}_{w|c}| = \sqrt{V_{w|cx}^2 + V_{w|cy}^2}$$

$$= \sqrt{(-1.515)^2 + (8.485)^2}$$

$$\boxed{|\vec{V}_{w|c}| = 8.619\,m/s}$$

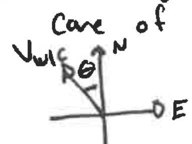


$\tan\theta = \dfrac{-1.5}{8.48}$

$\theta = \tan^{-1}\left(\dfrac{-1.515}{8.485}\right)$

$\theta = -10.12°$ ← but negative is already taken care of with quadrant

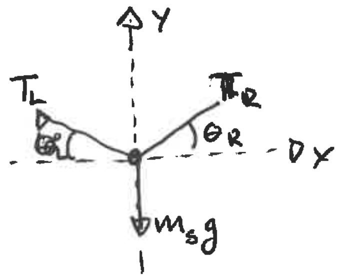
$\boxed{\theta = 10.12°}$



or $\boxed{10.12° \text{ West of North}}$

4)

Free body diagram



$\theta_L = 10°$
$\theta_R = ?$
$T_L = 10{,}000\ N$
$T_R = ?$
$m_s = 50\ kg$

$\sum \vec{F} = 0$ (static problem)

<u>X</u>

$-T_L \cos(\theta_L) + T_R \cos\theta_R = 0$ — ①

<u>Y</u>

$T_L \sin\theta_L + T_R \sin\theta_R - m_s g = 0$ — ②

①, solve for T_R.

$T_R \cos\theta_R = T_L \cos\theta_L$

$T_R = \dfrac{T_L \cos\theta_L}{\cos\theta_R}$ — ①'

② ⟵ ①', solve for θ_R

$T_L \sin\theta_L + \left(\dfrac{T_L \cos\theta_L}{\cos\theta_R}\right) \sin\theta_R - m_s g = 0$

$T_L \sin\theta_L + T_L \cos\theta_L \tan\theta_R = m_s g$

$\sin\theta_L + \cos\theta_L \tan\theta_R = \dfrac{m_s g}{T_L}$

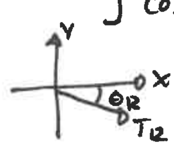
$\cos\theta_L \tan\theta_R = \dfrac{m_s g}{T_L} - \sin\theta_L$

$\tan\theta_R = \left[\dfrac{m_s g}{T_L} - \sin\theta_L\right] \cdot \dfrac{1}{\cos\theta_L}$

$\theta_R = \tan^{-1}\left(\left[\dfrac{m_s g}{T_L} - \sin\theta_L\right] \cdot \dfrac{1}{\cos\theta_L}\right)$

$\theta_R = \tan^{-1}\left(\left[\dfrac{50 \cdot 10}{10{,}000} - \sin(10)\right] \cdot \dfrac{1}{\cos(10)}\right)$

Continued. ↓ $\theta_R = -7.156°$



4) Continued

$\theta_R = -7.156°$ (I'm keeping more digits in my calc.)

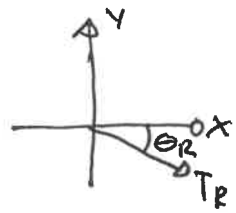
① ← θ_R, solve for T_R

$$T_R = \frac{T_L \cos\theta_L}{\cos\theta_R}$$

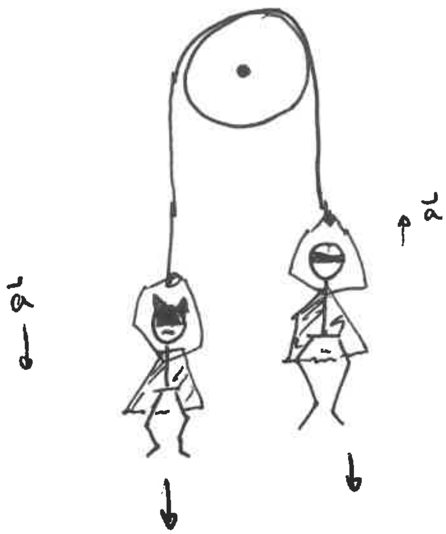
$$T_R = \frac{10{,}000 \cdot \cos(10)}{\cos(-7.156)}$$

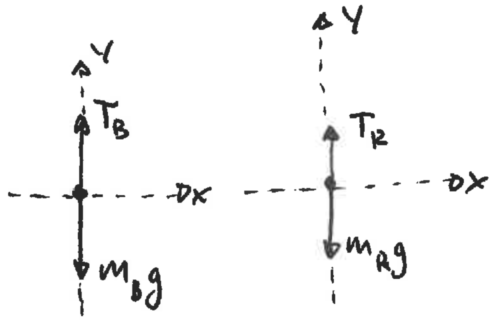
$$\boxed{T_R = 9925.4\,N}$$

$$\boxed{\theta_R = -7.156°}$$



5)





$\sum \vec{F} = m\vec{a}$

$\boxed{\text{Batman}}$

X	Y
N/A	$T_B - m_B g = \cancel{m_B a} = -m_B a$ — ①

$\boxed{\text{Robin}}$

X	Y
N/A	$T_R - m_R g = m_R a$ — ②

By Pulley rule, $T_B = T_R$:

① , solve for T_B

$T_B = -m_B a + m_B g$ — ①'

② ← ①', solve for $\vec{a}$. ($T_B = T_R$)

$(-m_B a + m_B g) - m_R g = m_R a$

$m_B g - m_R g = m_R a + m_B a$

$(m_B - m_R) g = (m_R + m_B) a$

$a = \dfrac{m_B - m_R}{m_B + m_R} g$

$a = \dfrac{80 - 50}{80 + 50}(10) = \boxed{2.307\ m/s^2}$

Note on #5

$T_R - m_R g = m_R a$

$T_R = m_R a + m_R g$

$T_R = (50)(2.307) + (50)(10)$

$\boxed{T_R = 615.38\,N} \boxed{= T_B}$

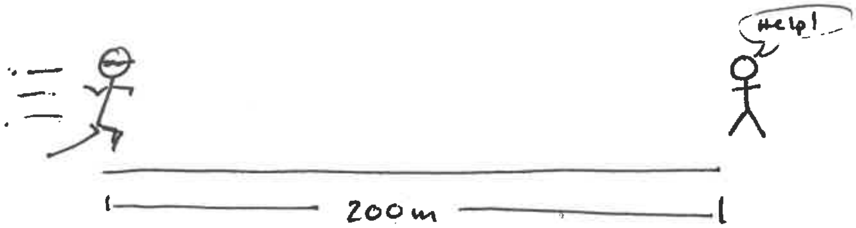
Note that $T_R = T_B$

but $T_B \neq m_B g$ $\quad \cancel{T_B}\; m_B g = 800$

$T_R \neq m_R g$ $\quad m_R g = 500$

This wasn't asked for, but this shows that The tension on Batman is <u>not</u> Batman's weight, and is <u>not</u> Robin's weight.

6)



$\cancel{a_Q}$ $a_Q = 100\ m/s^3\ t$

$$V_Q = \int a_Q\,dt = \int 100\,t\,dt = \frac{100}{2} \cdot t^2 + \cancel{v_0}^{\,0}$$

$$V_Q = 50t^2$$

$$X_Q = \int V_Q\,dt = \int 50t^2\,dt = \frac{50}{3}t^3 + \cancel{x_0}^{\,0}$$

$$X_Q = \frac{50}{3}t^3$$

looking for t, so set $x = 200m$

$$200 = \frac{50}{3}t^3$$

$$t^3 = \frac{600}{50}$$

$$t^3 = 12$$

$$t = \sqrt[3]{12} = (12)^{1/3}$$

$$t = 2.289s$$

$$\boxed{t = 2.3s}$$