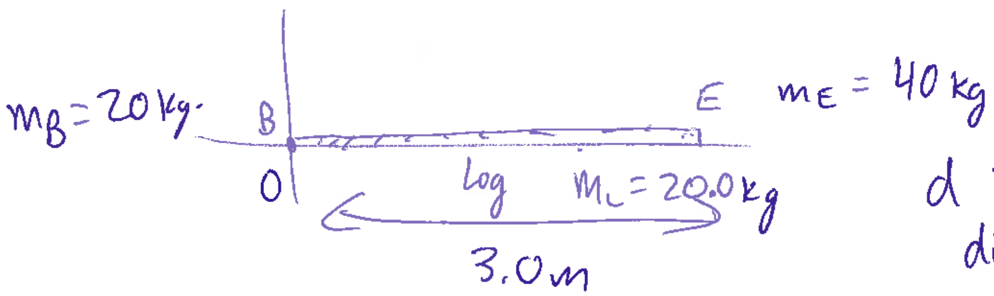
Problem 8.105

No Net External Force on Bert, Ernie, log System
$\Rightarrow$ Center of mass doesn't move.

$$X_{com} = \frac{m_E X_E + m_B X_B + m_L X_{Log}}{m_E + m_B + m_L}$$



d is log displacement

Initially: $X_{c.o.m.} = \dfrac{(20\,kg)(1.5\,m) + (40\,kg)(3\,m)}{90\,kg}$

Final: $X_{c.om.} = \dfrac{(30\,kg)d + (1.5\,m + d)(20\,kg) + (40\,kg)d}{90\,kg}$

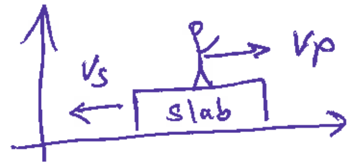
$(X_{com})_i = (X_{com})_f$ so...

$$\cancel{(20\,kg)(1.5\,m)} + (40\,\cancel{kg})(3\,m) = (30\,\cancel{kg})d + (\cancel{1.5} + d)(20\,\cancel{kg}) + 40\,\cancel{kg}\,d$$

$$120\,m = \underbrace{(30 + 20 + 40)}_{90}d$$

$$d = \frac{120}{90}\,m = 1.33\,m$$

Problem 8.107



$w_s = 5w_p$
$m_s = 5m_p$

Frozen lake, ignore friction Between slab and ice

$V_p = 2.00$ m/s relative to ice

$V_s = ?$

Use conservation of momentum, center of mass

$$\vec{V}_{c.o.m} = \frac{P_{TOT}}{m_p + m_s} = 0$$

$$P_{TOT} \text{ moving} = m_p\vec{V}_p + m_s\vec{V}_s = 0$$

$$x: \quad V_s = -\frac{m_p}{m_s} V_p = -\left(\frac{\cancel{m_p}}{5\cancel{m_p}}\right)(2.00\,m/s)$$

$$= \boxed{-0.400\ m/s}$$

Problem 8.112

Assume $\vec{g}$ const. (Not true if Altitude gets too high)
we'll see in a few weeks

How do we modify previous analysis?

Gravity: External force $mg \Rightarrow$ Impulse to change $\vec{p}$ of Rocket.

Imp.–p theorem:

a) $$-mg\,dt = \underbrace{(m+dm)(v+dv) + (-dm)(v-V_{ex})}_{p_2} - \underbrace{mv}_{p_1 \text{ initial}}$$

$$\text{Imp} = \Delta p$$

$$-mg\,dt = m\,dv + V_{ex}\,dm$$

$$-\cancel{m}g = \cancel{m}\frac{dv}{dt} + V_{ex}\frac{dm}{dt}$$

$$m\frac{dv}{dt} = -V_{ex}\frac{dm}{dt} - mg$$ New add.

b) $$\Rightarrow \quad a = \frac{dv}{dt} = -\frac{V_{ex}}{m}\frac{dm}{dt} - g$$

c) What is a for $g = 9.8\ m/s^2$? from Ex. 8.15

$V_{ex} = (2400)\ m/s \qquad \frac{dm}{dt} = -\frac{m_0}{120s}$

Plug & chug $\Rightarrow t = 0, \quad a = 10.2\ m/s^2$

d) $t = 90s$:

$t_0 = 0$
$V_0 = 0$
$V_{ex} = 2400\ m/s$
$g = 9.8\ m/s^2$
$m = m_0/4$ — out in 90s.

$$dv = -V_{ex}\frac{dm}{m} - g\,dt$$

$$\int_{V_0}^{V} dv' = -V_{ex}\int_{m_0}^{m}\frac{dm'}{m} - \int_{t_0}^{t} g\,dt$$

$$V = -V_{ex}\ln\frac{m}{m_0} - gt$$

$$V = 2445\ m/s$$