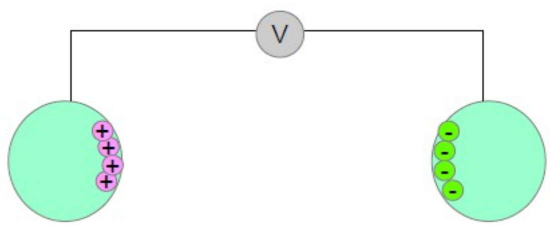
PHYS 1220, Engineering Physics, Chapter 24 – Capacitance and Dielectrics

Instructor: TeYu Chien
Department of Physics and Astronomy
University of Wyoming

Goal of this chapter is to learn what is Capacitance, its role in electronic circuit, and the role of dielectrics.

- How to use electric potential to move opposite signed charges to two different conductors?

- Using an external electric potential sources (batteries, power supply etc)



- In general, any two separated conductors could be used in above schematics.
- A capacitor is composed of a pair of conductors insulated from each other.
- A capacitor could be used to store opposite signed of electric charges on the two different conductors.
- Capacitor is represented as the following symbols when draw in circuit

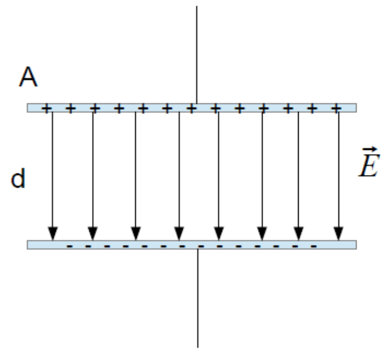
- How much charge could be stored? (related to the definition of Capacitance)

- Capacitance is defined as *how much charge* could be stored in the capacitor *per unit volt*:

$$C = \frac{Q}{V_{ab}}$$

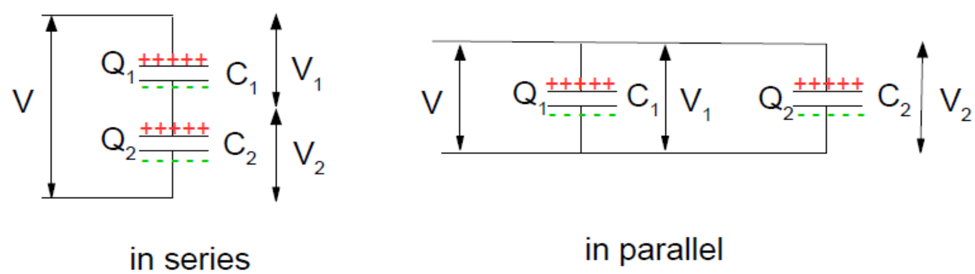
- NOTE: The unit used for capacitance is *Farad* (F) and is defined as $1\,F = 1\,\frac{C}{V}$
- NOTE #2: Please pay attention that don't be confused with to the notation used for capacitance (C) and the unit for charge (Q): coulomb (C).

- Capacitance calculation for few different types of capacitors
  - Parallel-plate capacitor



    - $C=\frac{Q}{V_{ab}}=\epsilon_0\frac{A}{d}$
  - Spherical capacitor (**Do Example 24.3 (page 792)**).
  - Cylindrical capacitor (**Do Example 24.4 (page 792)**).

- Multiple capacitors in circuit – equivalent capacitance



  - When two capacitors are in series, the amount of charge is the same.
    - $\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}$
  - When two capacitors are in parallel, the electric potential is the same.
    - $C_{eq}=C_1+C_2$
  - Equivalent capacitance is the capacitance that you measured from the two terminals regardless how complex the real capacitor collection is. In other words, when analyzing circuit, you can simplify multiple capacitors into one equivalent capacitor.

Do example 24.6 (page 796)

- How much energy is stored in capacitors?
  - $dW=V\,dq=\frac{q}{C}dq$ (Remember: $C=\frac{Q}{V}$)
  - $W=\int dW=\int_0^Q \frac{q}{C}dq=\frac{1}{2}\frac{Q^2}{C}=\frac{1}{2}CV^2=\frac{1}{2}QV$
- Electric-field energy
  - The energy stored in capacitors could be considered as the form of electric field.

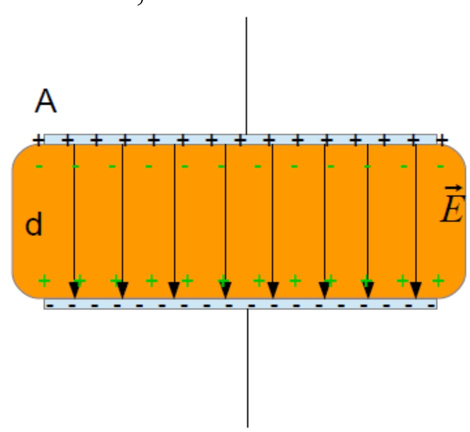
We know that electric field is distributed all over the vacuum space between the two conductors (of the capacitor). Thus, we could find out the energy density that is stored in the form of electric field as (take parallel-plate capacitor as example):

$$u = energy\ per\ unit\ volumn = \frac{total\ energy}{total\ volumn} = \frac{\frac{1}{2}CV^2}{Ad}$$

- Note that $C = \epsilon_0 \frac{A}{d}$ for parallel-plate capacitors.
- Electric-field energy density in vacuum: $u = \frac{1}{2}\epsilon_0 E^2$ (This will be used again in Chapter 32)

Do Example 24.7 (page 798)

- How to increase the capacitance without changing the dimension of the capacitor?
  - The answer is to add insulating materials between the two conductors to replace vacuum.
  - This insulating material is called "dielectric" material.
  - How does that work? (1) charges in the dielectric material near the conductors will be induced; (2) the induced charges produce electric field with OPPOSITE direction to the original one; (3) to maintain the same electric potential (V), the conductors need more charge Q; thus the capacitance increase; (3') or if charge (Q) does not change, the electric field is lowered (as mentioned in (2)), thus the electric potential (V) decreases, which increases the capacitance C.



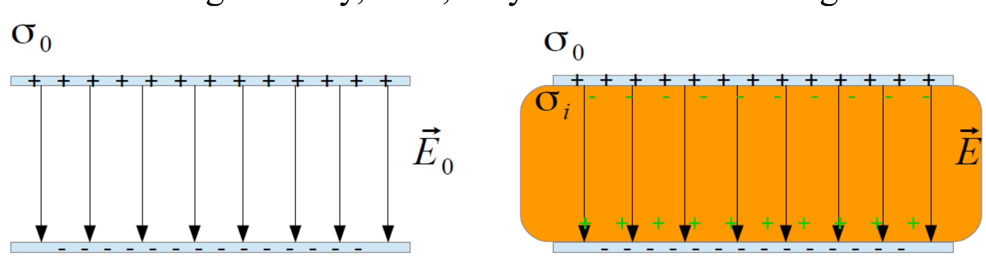
  - The change of the capacitance depends on the selection of the dielectric materials. The key property of the dielectric material here is the Dielectric Constant, K (see Table 24.1 on page 801 for various types of dielectric materials)
  - Quantitatively, the dielectric constant could be defined as: $K = \frac{C}{C_0}$.
  - Remember, when without the dielectric material, $C_0 = \epsilon_0 \frac{A}{d}$. Thus,

    $C = K C_0 = K \epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$. Here, we define $K\epsilon_0 = \epsilon$, where ϵ is called the **permittivity** of the dielectric material.

- NOTE: Engineers use K more; while scientists use ϵ more.
- NOTE #2: In many cases, K is also denoted as ϵ_r , means relative permittivity, due to the relationship: $K=\frac{\epsilon}{\epsilon_0}=\epsilon_r$.
- Electric-field energy density in dielectric material: $u=\frac{1}{2}K\epsilon_0 E^2=\frac{1}{2}\epsilon E^2$.

- Is there a maximum energy storage (or maximum voltage) in capacitor?
  - Yes, there is a maximum value. The threshold comes from the **dielectric breakdown** when the electric field is stronger than the **dielectric strength**.
  - Dielectric strength refers to the maximum electric field the dielectric material could withstand without breakdown.

- Free charge vs Bound charge (Let's assume the charge on the conducting plates do not change):
  - Free charge (σ_0): charges that are free to move.
  - Bound charge (σ_i): charges that are bounded to molecules and cannot move freely. They typically will form electric dipole in electric field. (See Fig. 24.19 on page 806).
  - Total charge (σ_{total}): the combination of free and bound charges: $\sigma_{total}=\sigma_0-\sigma_i$. Note here that charge density, σ , only take care of the magnitude.



  - From Gauss's Law we learned: $\vec{E}_0=\frac{\sigma_0}{\epsilon_0}$; and $\vec{E}=\frac{\sigma_0-\sigma_i}{\epsilon_0}$. Note here that charge density, σ , only take care of the magnitude.
  - Remember, capacitance is defined: $C=\frac{Q}{V}$, and the charge Q is actually the free charge: $C=\frac{Q_{free}}{V}$.
  - In other words, for the two situations above. We know $C=KC_0=K\frac{Q_{free}}{V_1}=\frac{Q_{free}}{V_2}$. where V_1 is the voltage difference in the left condition (without dielectric material), while V_2 is the voltage difference in the right condition (with dielectric material). This leads to: $\frac{V_1}{K}=V_2$.
  - And we know in this parallel-plate condition, $V=|\vec{E}|d$. That leads us to the

conclusion of $\frac{\vec{E}_0}{K} = \vec{E}$, and thus $\sigma_i = \sigma_0 (1 - \frac{1}{K})$.

- Modification of Gauss's Law with the presence of dielectric materials

- Originally, $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} = \frac{Q_{free} + Q_{bound}}{\epsilon_0} = \frac{Q_{free}}{K \epsilon_0}$. Note here, the sign of charge is in Q.
- $\oint K \vec{E} \cdot d\vec{A} = \frac{Q_{encl-free}}{\epsilon_0}$ or $\oint \epsilon \vec{E} \cdot d\vec{A} = Q_{encl-free}$. Note that when doing the integration, the ϵ is depending on the materials where the Gauss's surfaces located. For a closed Gauss's surface, different part of the closed surfaces might immersed in different materials.
- Typically, we only care about free charge, that's why in the modified Gauss's Law, we only relate it to the free charge.

Math Preview for Chapter 25:

- Derivative

Question to think:

- Let's focus on a conducting wire that connects to the capacitors. During the charging/discharging process (moving and removing charges in the capacitors, respectively), there is a voltage difference. How to describe the process of the charge movement?