PHYS 1220, Engineering Physics, Chapter 31 – Alternating Current

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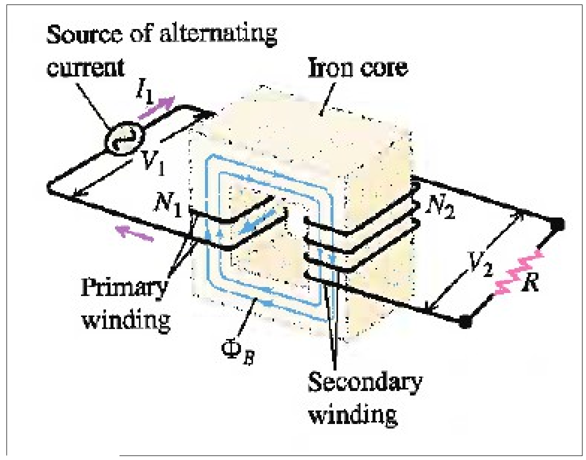
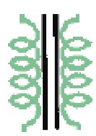
Goal of this chapter is to learn how resistor, capacitor and inductor behaves in a AC circuit

- Transformers are used to convert electric potential from one value to another through the inductance effect with ac current.

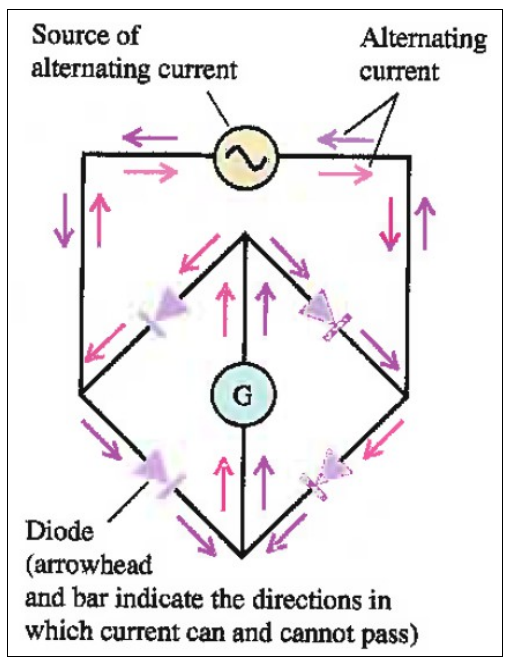
- A transformer is typically formed by two sets of coils in two different circuits with an iron core that keeps magnetic field inside it.
- $\epsilon_1 = -N_1 \frac{d\Phi_B}{dt}$, also, $\epsilon_2 = -N_2 \frac{d\Phi_B}{dt}$
- The magnetic flux, Φ_B , inside the two coils are the same (same iron core).
- $\frac{\epsilon_2}{\epsilon_1} = \frac{N_2}{N_1}$ or $\frac{V_2}{V_1} = \frac{N_2}{N_1}$ (How voltage is converted in transformer)
- $P_1 = V_1 I_1 = V_2 I_2 = P_2$ (energy/power conservation)



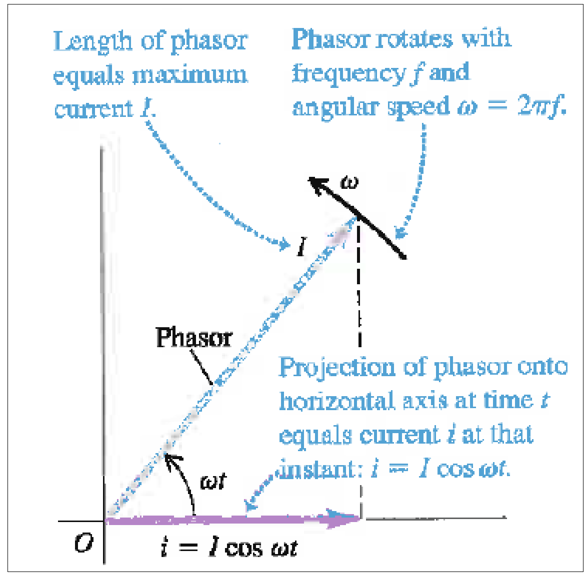


- Rectified alternating current

- Diode is an electronic device that has bipolar resistance: very low resistance along one direction and very large resistance along the other direction. In other words, current can only flow in one direction through the diode, but not the other.
- A set of diodes could be design to rectify an alternating current into a direct current.



- Basic math for understanding ac circuits with resistors, capacitors, and inductors
  - For convenience, let's use sinusoidal function for the time dependent current and voltage.
  - $I(t)=I_0\cos(\omega t)$
  - Phase: everything inside the "cos" or "sin" function is called phase, ranging from 0 to 2π.
  - Amplitude: the factor in front of "cos" or "sin" is the amplitude of this function.

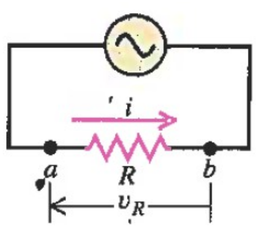


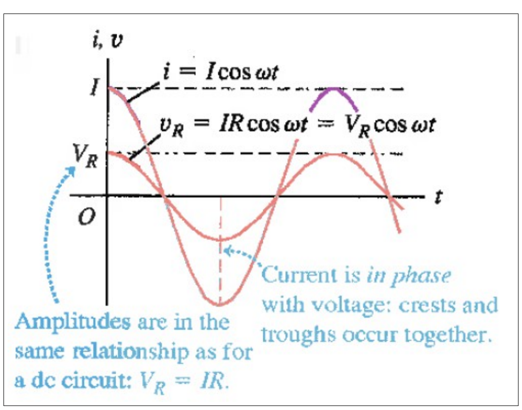
- Root-mean-square (rms) and average numbers
  - "rms" refers to a function is first, (1) **squared**, then, (2) find the **mean** (average), and finally, (3) take the **square root** of the mean: $I_{rms}=\sqrt{\frac{\int_0^T I^2(t)\,dt}{T}}$
  - For example: if $I(t)=I_0\cos(\omega t)$.

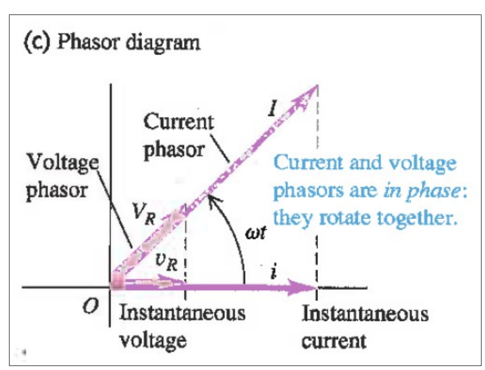
$$I_{rms}=\sqrt{\frac{\int_0^T I_0^2\cos^2(\omega t)\,dt}{T}}=\sqrt{\frac{I_0^2}{T}\int_0^T\frac{1}{2}(1+\cos(2\omega t)\,dt)}=\frac{I_0}{\sqrt{2}}$$

  - Average is just take the average of a function: $I_{ave}=\frac{\int_0^T I(t)\,dt}{T}$

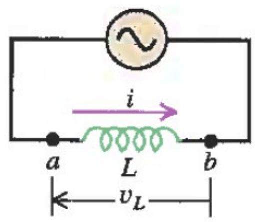
- Resistance and Reactance in AC circuit (with an AC current source: $I(t)=I_0\cos(\omega t)$)
  - For **Resistor**
  - $V_R(t)=I(t)R=RI_0\cos(\omega t)$

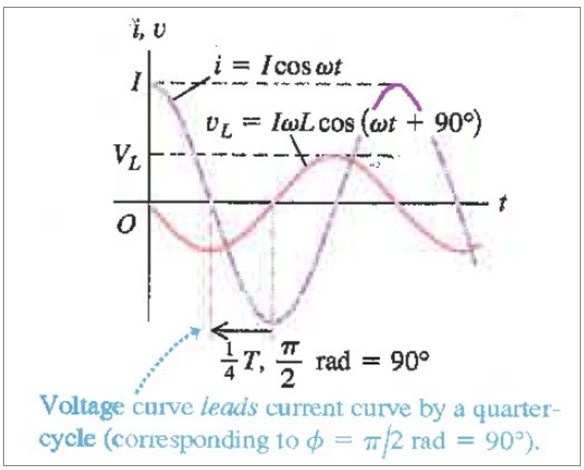


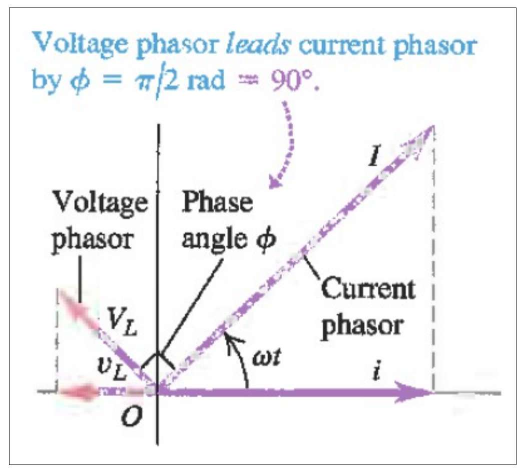




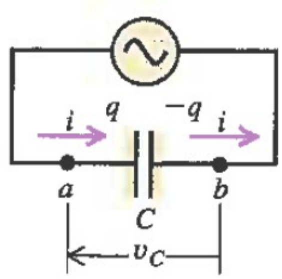
- For **Inductor**
- $V_L(t)=L\frac{dI(t)}{dt}=-LI_0\omega\sin(\omega t)=L\omega I_0\cos(\omega t+\frac{\pi}{2})$ (remember: $\cos(A+\frac{\pi}{2})=-\sin(A)$)
- Amplitude of the $V_L(t)$: $V_L=L\omega I_0=I_0X_L$; where $X_L=L\omega$
- Phase of the $V_L(t)$: $\phi=\omega t+\frac{\pi}{2}$ (90 degrees advanced than current)

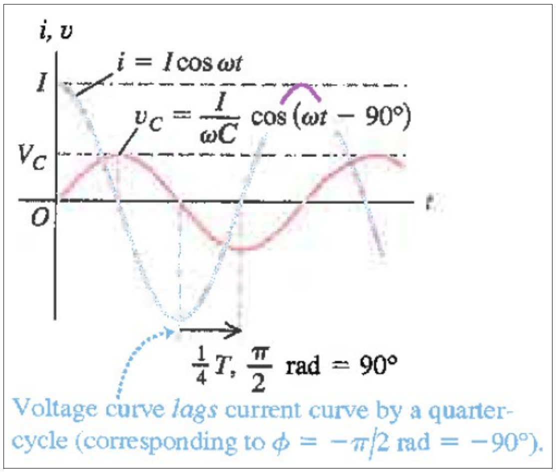


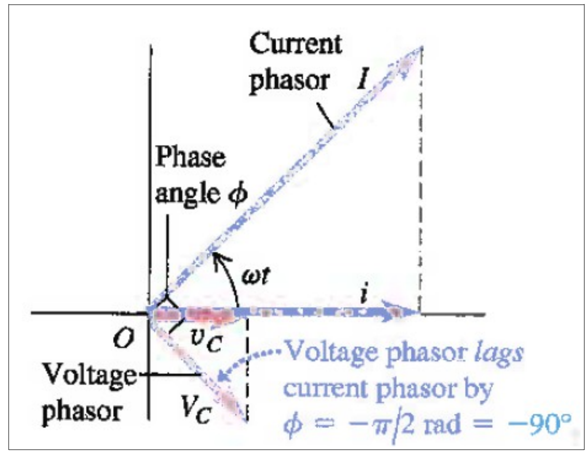




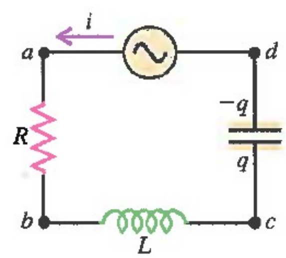
- For **Capacitor**
- $I(t)=\frac{dq}{dt}=I_0\cos(\omega t)$, so $q(t)=\frac{I_0}{\omega}\sin(\omega t)$
- $V_C(t)=q\frac{(t)}{C}=\frac{I_0}{\omega C}\sin(\omega t)=\frac{I_0}{\omega C}\cos(\omega t-\frac{\pi}{2})$ (remember: $\cos(A-\frac{\pi}{2})=\sin(A)$)
- Amplitude of the $V_C(t)$: $V_C=\frac{I_0}{\omega C}=I_0X_C$; where $X_C=\frac{1}{\omega C}$
- Phase of the $V_C(t)$: $\phi=\omega t-\frac{\pi}{2}$ (90 degrees behind the current)

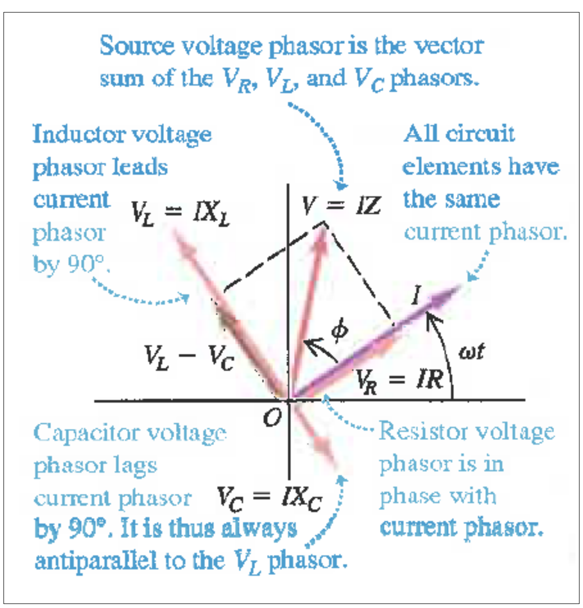


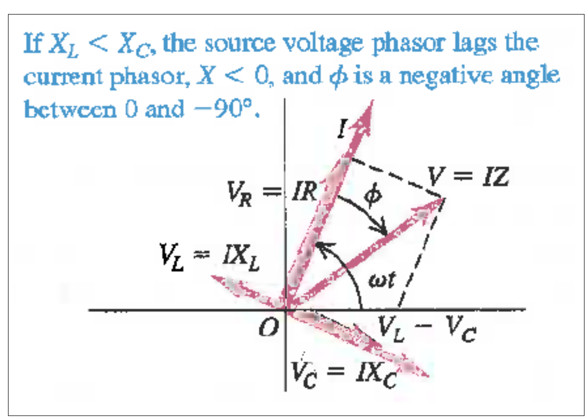




- L-R-C in series in ac circuit (Use phasor diagram to analyze)
  - Amplitude of *V(t)*:
    $V=\sqrt{V_R^2+(V_L-V_C)^2}=\sqrt{(IR)^2+(IX_L-IX_C)^2}=I\sqrt{R^2+(X_L-X_C)^2}=IZ$ where Z is defined as **impedance** of this circuit as: $Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}$
  - Phase of *V(t)*: $\tan\phi=\frac{V_L-V_C}{V_R}=\frac{I(X_L-X_C)}{IR}=\frac{X_L-X_C}{R}$
  - In short: $\tan\phi=\frac{\omega L-\frac{1}{\omega C}}{R}$; or $\phi=\tan^{-1}(\frac{\omega L-\frac{1}{\omega C}}{R})$
  - Then $V(t)=V\cos(\omega t+\phi)$ (if $I(t)=I_0\cos(\omega t)$)







Math Preview for Chapter 32:

- closed surface and closed line integral
- derivative
- differential equation (wave function)
- vector cross product

Questions to think about for Chapter 32:

- We know that time-dependent electric field could produce magnetic field (Ampere's law); and time-dependent magnetic field could produce electric field (Faraday's law). We always call light as electromagnetic wave, which is a wave that is essentially the wave of electric field and magnetic field. With the knowledge you learned about the induced electric (or magnetic) field by the other counterpart, how the light propagate in terms of electric and magnetic fields?