Chapter 22: Gauss’s Law

- What is “flux”? And What is “electric flux”?
- How to calculate electric flux?
- Electric flux for a closed surface.
- What is the source of the electric flux?
- How to use Gauss’s Law to calculate electric field?

Goal of this chapter

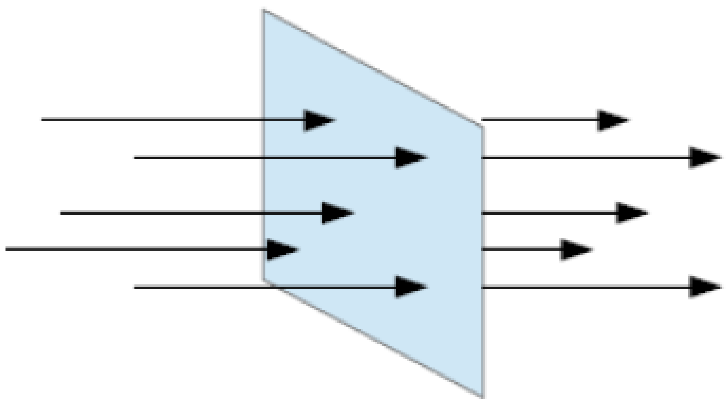
Understand

$$\vec{E} \longrightarrow \Phi_E$$

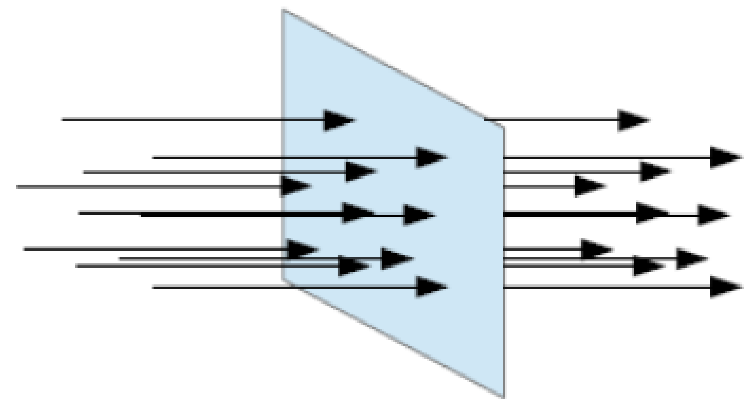
Then, use

$$\Phi_E \longrightarrow \vec{E}$$

Electric field lines



$\vec{E}$



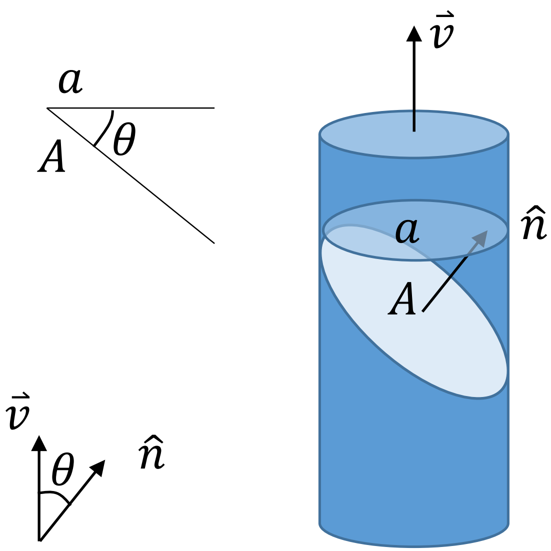
$2\vec{E}$

- Electric field strength = electric field line density. (you can use the electric field line to represent and think about how to calculate the electric field strength)
- However, electric field lines are conceptual lines, there is no rule tells you how many lines should be plotted for a certain strength of electric field.

What is “flux”? And What is “electric flux”?

Flux: **How much “something” flows through a certain area.**

Electric flux: **How much electric field (lines) flow through a certain area.**



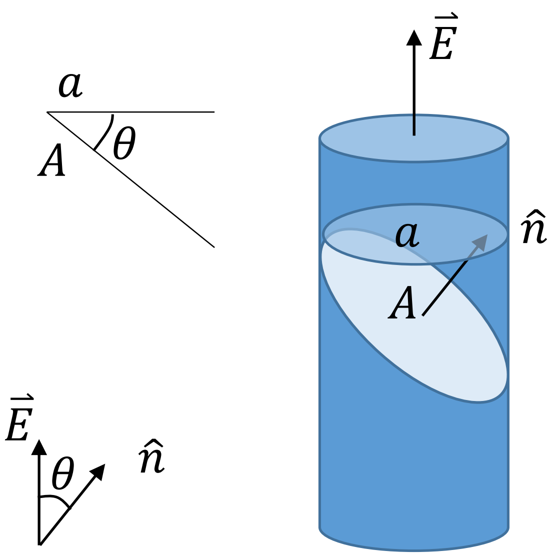
$$\frac{dV}{dt} = a \cdot |\vec{v}| = A \cdot cos\theta \cdot |\vec{v}| = \vec{v} \cdot A\hat{n} = \vec{v} \cdot \vec{A}$$

$$flux = \vec{v} \cdot \vec{A}$$

How to calculate electric flux?

Flux: **How much "something" flows through a certain area.**

Electric flux: **How much electric field (lines) flow through a certain area.**



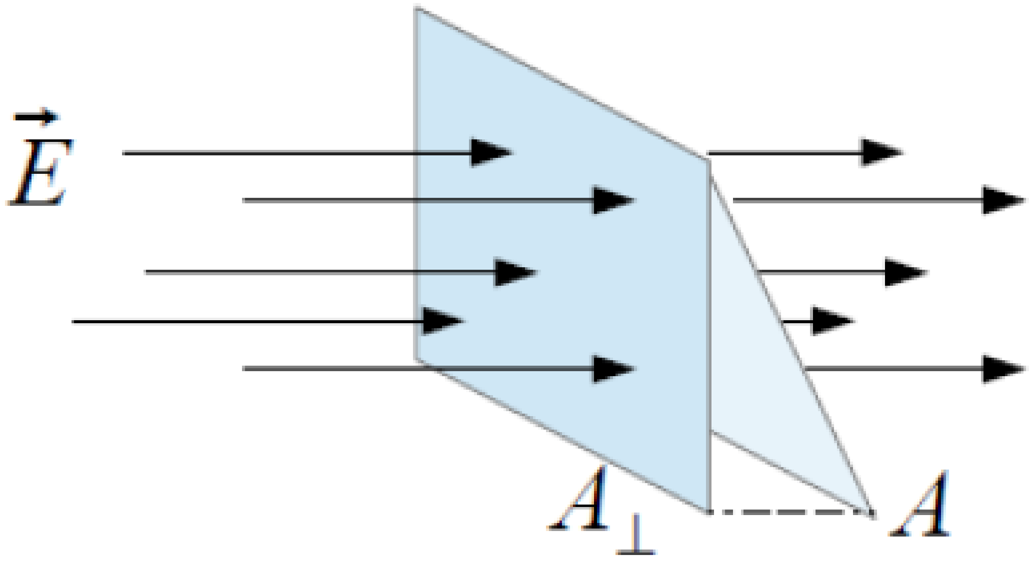
$$electric\ flux = a \cdot |\vec{E}| = A \cdot cos\theta \cdot |\vec{E}| = \vec{E} \cdot A\hat{n} = \vec{E} \cdot \vec{A}$$

$$\Phi_E = \vec{E} \cdot \vec{A}$$

$\vec{A} = A\hat{n}$ is called "area vector"

$\hat{n}$ is the unit vector of the area vector, with the direction normal to the surface (area)

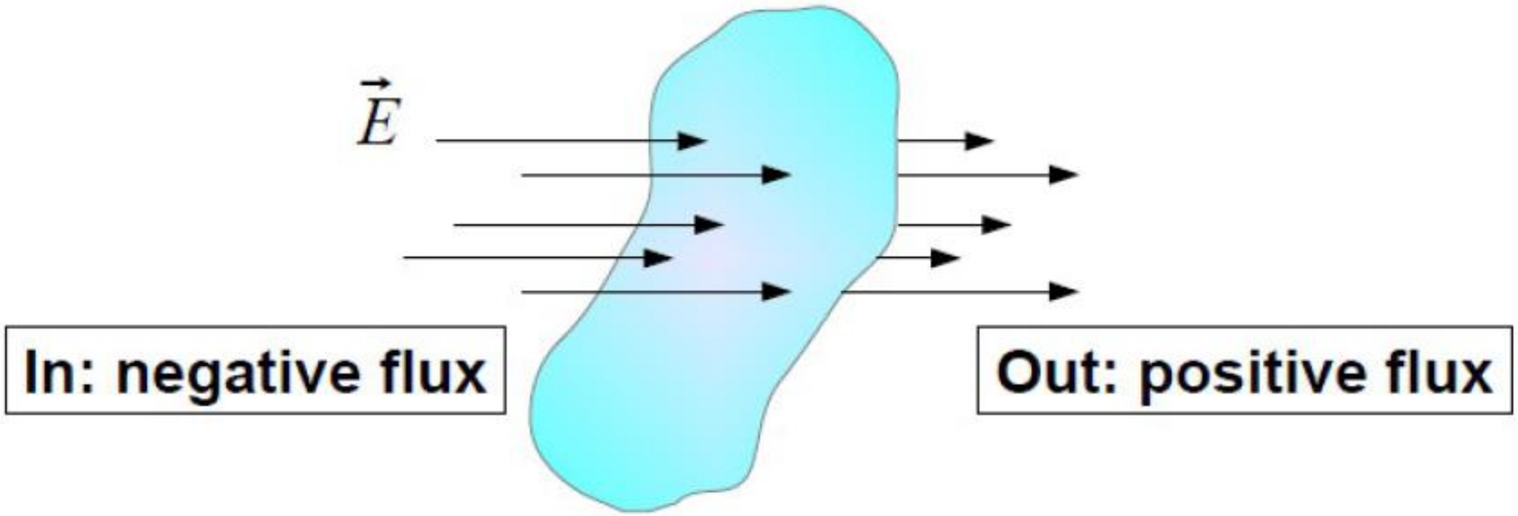
Electric flux



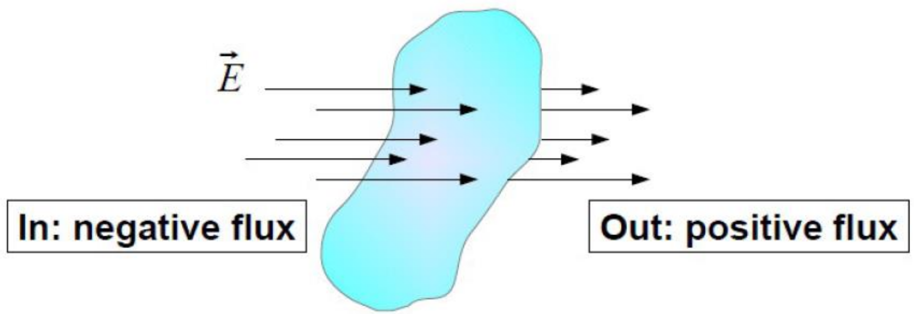
$\Phi_E = \vec{E} \cdot \vec{A} = |\vec{E}| \cdot A_\perp$

Electric flux for a closed surface

- What is interested is how much electric flux flowing ***"out"*** of a closed surface.
- So, define: flux flows ***"into"*** is considered ***"negative flux";*** while flux flows ***"out of"*** is considered as ***"positive flux"***.
- In practice, we do it by defining the direction of $\hat{n}$ to be pointing ***"out"*** *of the closed surface*.



Electric flux for a closed surface

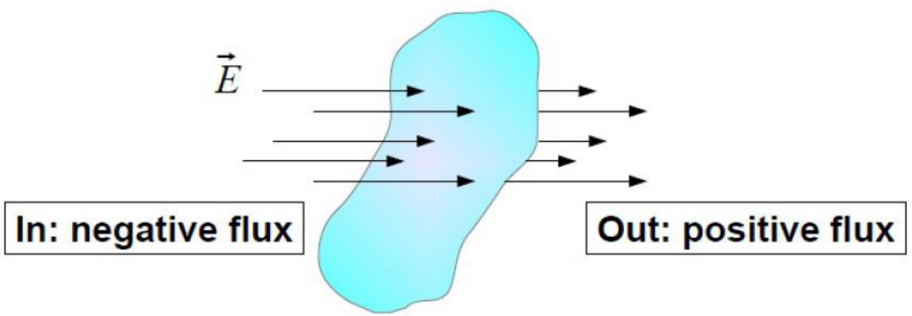


$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A}$$

- *"S"* means "integrate over a closed surface *S*". It is a conceptual symbol. Real integral depends on the geometry of the closed surface
- The "circle" in the integral symbol means "a closed surface".

What is the source of the electric flux?

Remember: Electric field lines start at positive charges and end at negative charges.



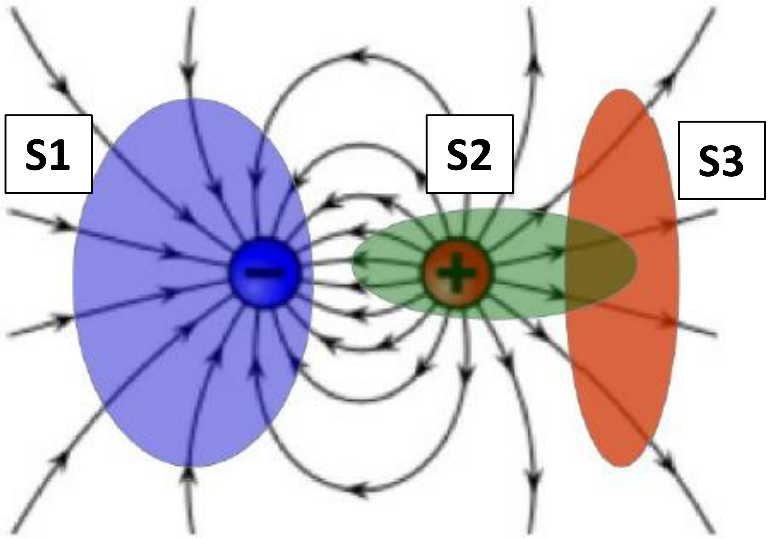
- If there is no charge in the closed surface: electric lines,

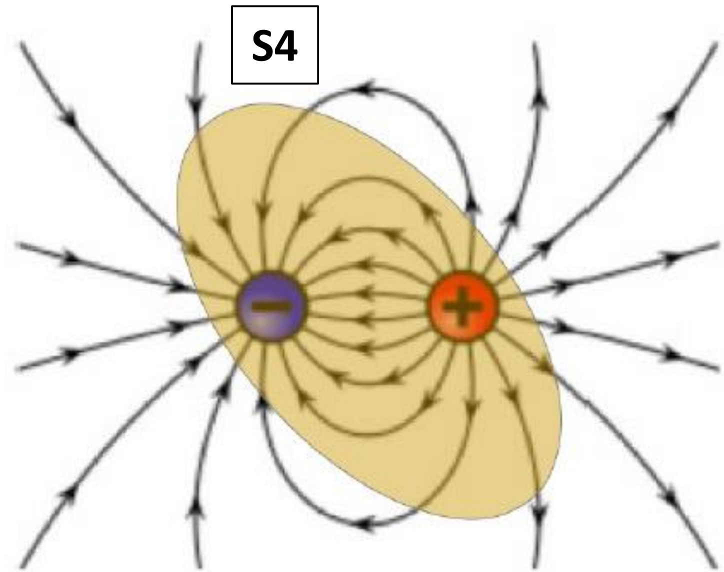
  "in = out" ➔ $\Phi_E = 0$

- If the total charge in the closed surface is positive (negative): electric lines,

  "in < out" ("in > out") ➔ $\Phi_E > 0$ $(\Phi_E < 0)$

Quiz

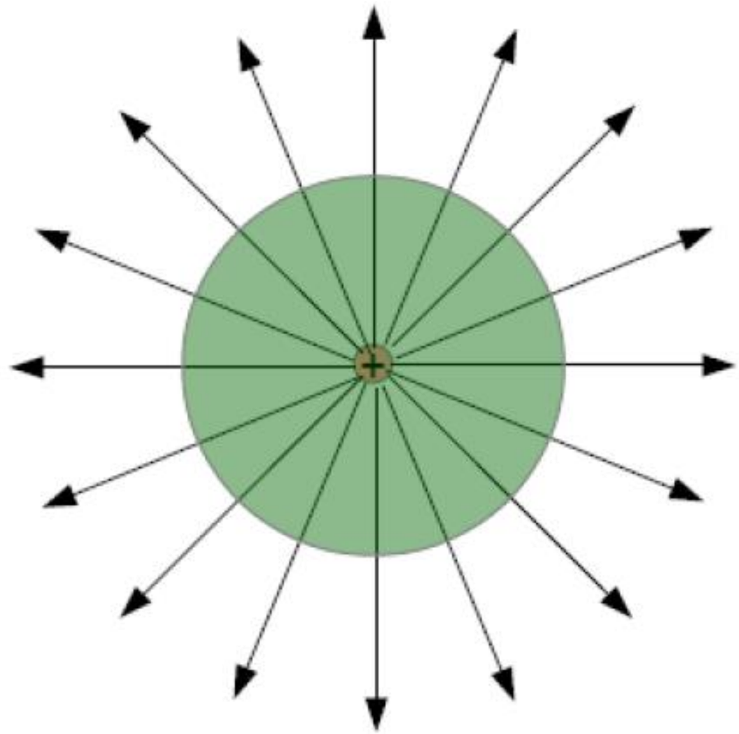




Which statement about the electric flux for each closed surface is correct:

A. S1 = 0; S2 > 0; S3 = 0; S4 < 0
B. S1 < 0; S2 > 0; S3 > 0; S4 = 0
C. S1 > 0; S2 = 0; S3 < 0; S4 < 0
D. S1 < 0; S2 > 0; S3 = 0; S4 = 0
E. S1 < 0; S2 < 0; S3 > 0; S4 > 0

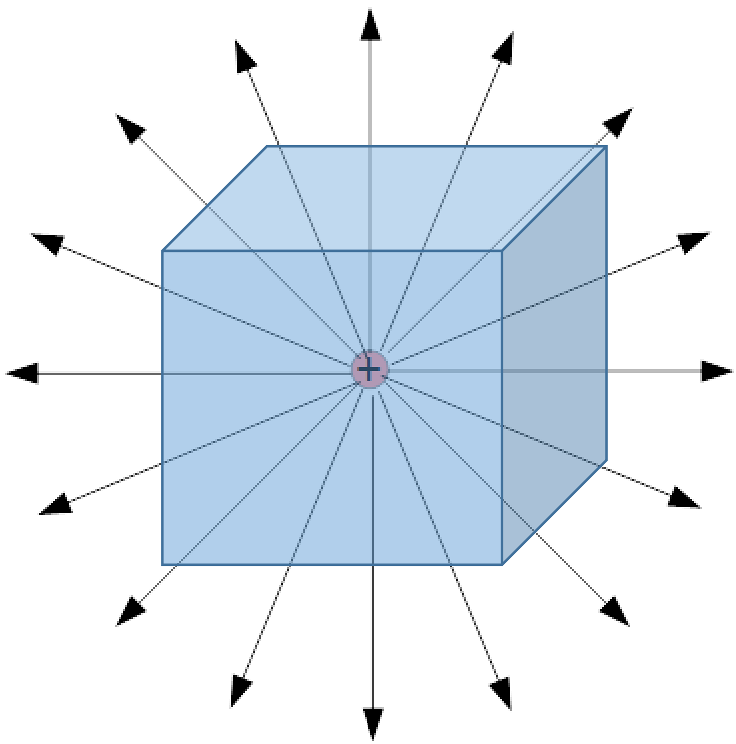
Try it



Charge: *q*

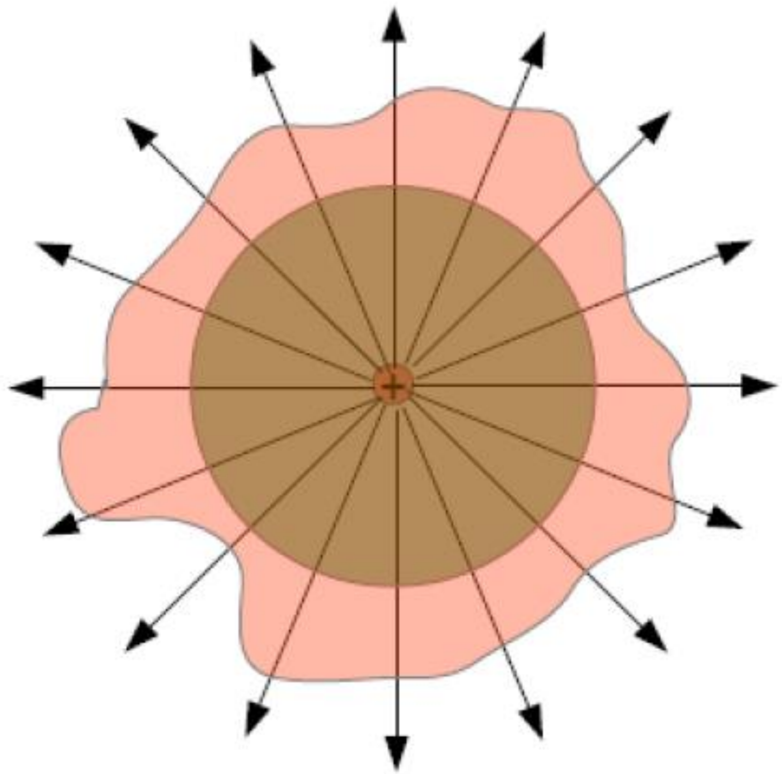
Flux through a spherical closed surface with radius *r*

Try it



Charge: *q*
Flux through a cubical closed surface with side *L*

Try it

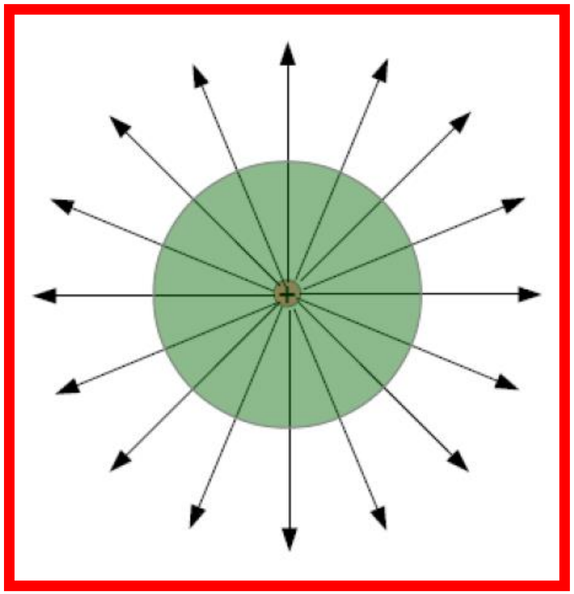


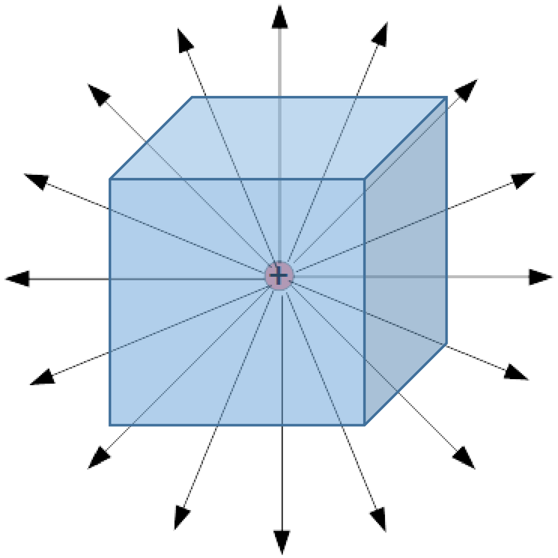
$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

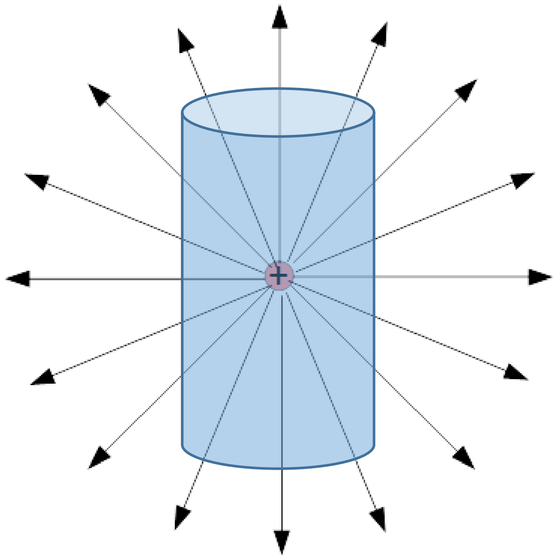
Charge: q
Flux through a irregular closed surface

The electric flux of a closed surface only depends on how much charge is enclosed in it.

How to use Gauss's Law to calculate electric field?

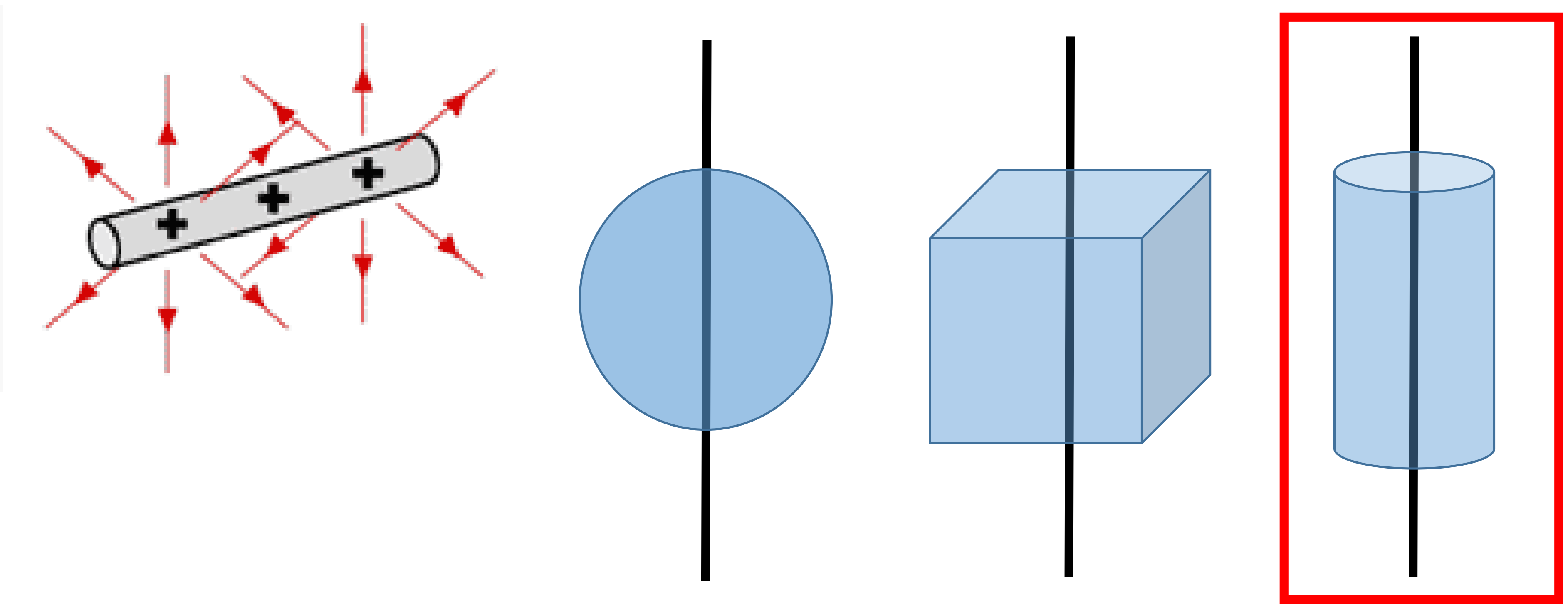






If you need to calculate the electric flux, which shape you would like to do?

How to use Gauss's Law to calculate electric field?



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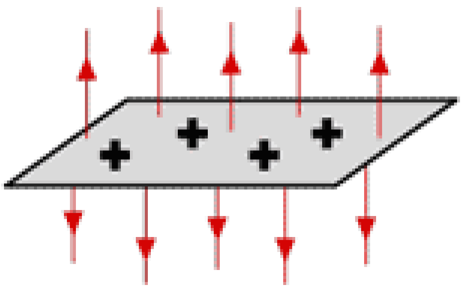
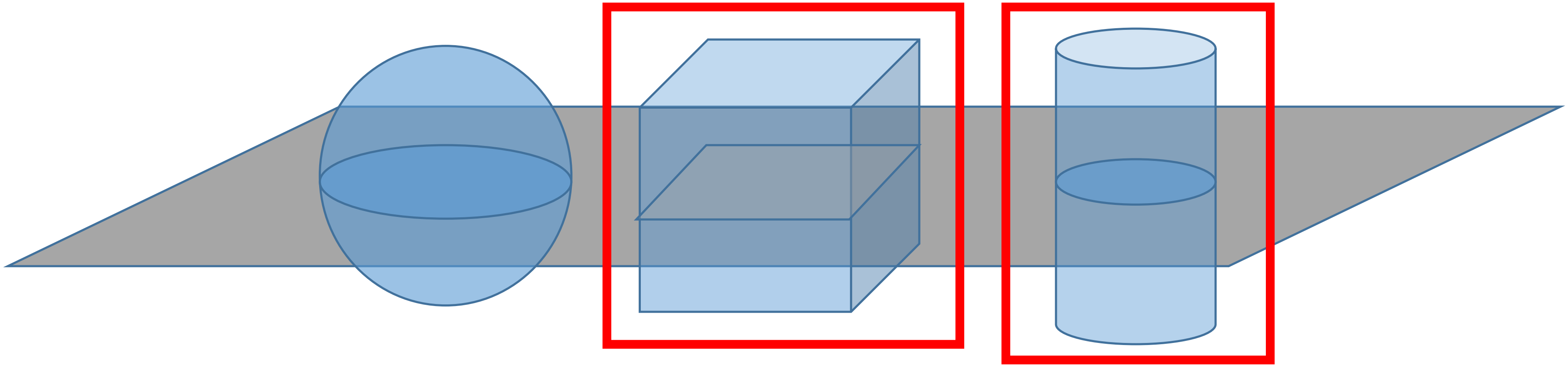
How to use Gauss's Law to calculate electric field?



Important trick: Find the symmetry of the electric field lines (patterns) before you choose what closed surface to calculate.

- Make the area vector either perpendicular or parallel to the local electric field.
- Make the magnitude of the electric field to be the same all over the surface for integration.



If you need to calculate the electric flux, which shape you would like to do?

Example 1

We place a total positive charge q on a solid conducting sphere with radius R.
Find the electric field at any point inside or outside the sphere.

Example 2

Electric charge is distributed uniformly along an infinitely long, thin wire. The charge per unit length is λ (assumed positive).
Find the electric field using Gauss's Law.

Example 3

Use Gauss's Law to find the electric field caused by a thin, flat, infinite sheet with a uniform positive surface charge density σ.

Example 4

Positive electric charge Q is distributed uniformly throughout the volume of an insulating sphere with radius R.
Find the electric field at any point inside or outside the sphere.