Chapter 23: Electric Potential

- Defining Electric Potential Energy
- Defining Electric Potential
- What is the relationship among electric force, field, potential energy, and potential?

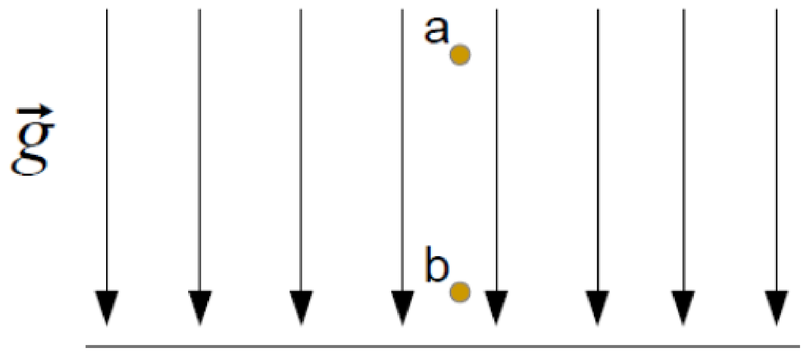
Defining Electric Potential Energy

- Conservative Forces: The forces depend only on the position.
- Potential Energy could only be defined for conservative forces: ***The work done by the conservative force with a "-" sign.***

$$\Delta U = -W$$

Gravitational Force vs Potential Energy

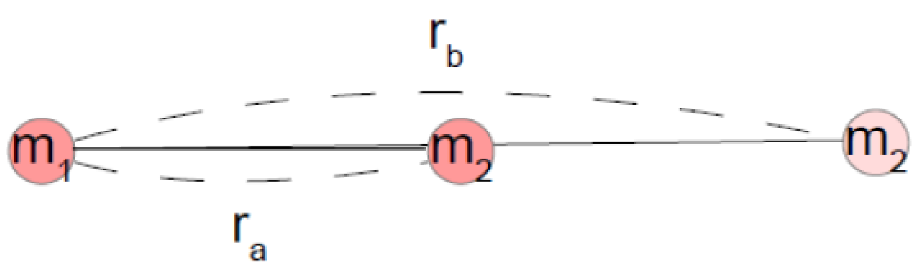
Near earth surface: gravitational force is a constant value for a particular object.



$\Delta U = mg\Delta h$

$U = mgh$

Far from earth surface: gravitational force is a function of distance.

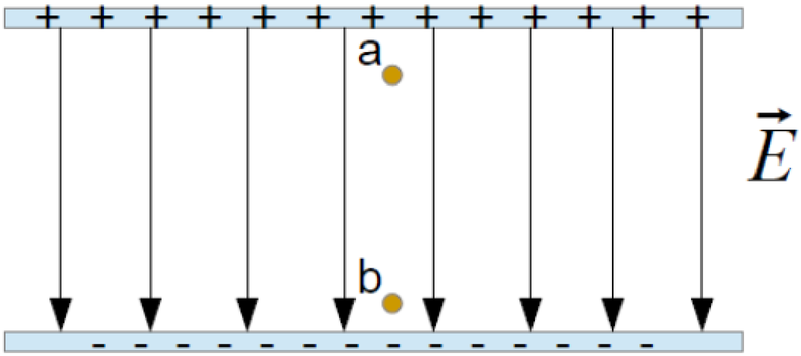


$\Delta U = -Gm_1m_2\Delta\frac{1}{r}$

$U = -G\frac{m_1m_2}{r}$

Electric Force vs Potential Energy

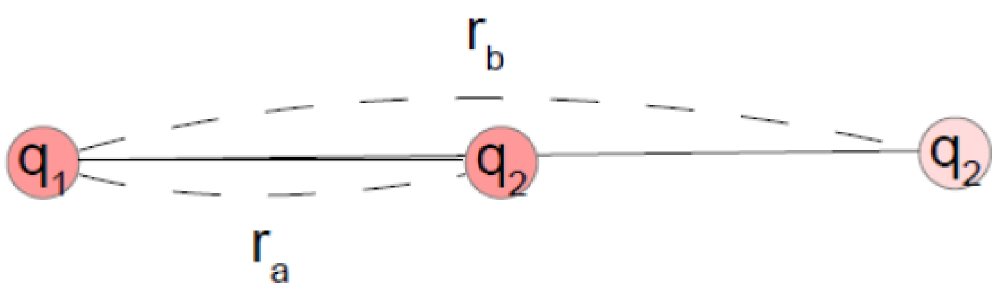
In an uniform electric field



$$\Delta U = q|\vec{E}|\Delta y$$

$$U = q|\vec{E}|y$$

For point charges



$$\Delta U = \frac{1}{4\pi\varepsilon_0} q_1 q_2 \Delta \frac{1}{r}$$

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

Example

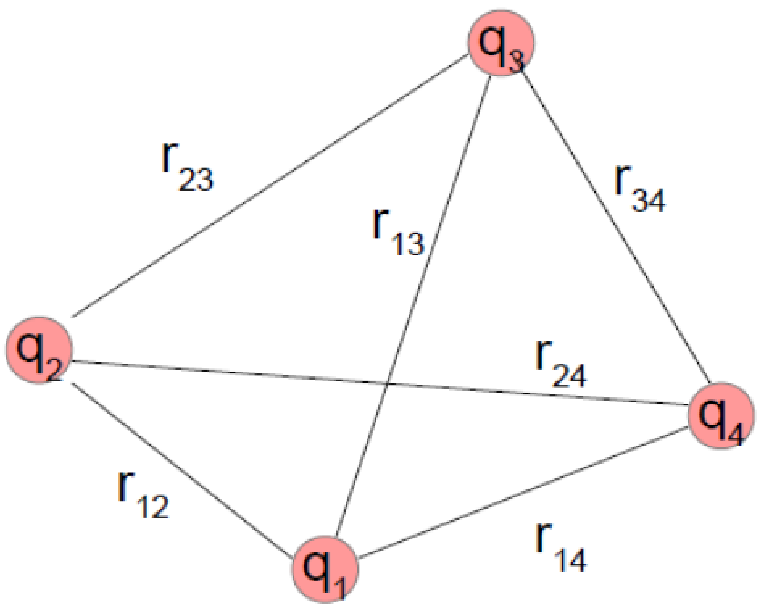
- Two point charges are located on the x-axis, $q1 = -e$ at $x = 0$ and $q2 = +e$ at $x = a$.

(a) Find the work that must be done by an external force to bring a third point charge $q3 = +e$ from infinity to $x = 2a$.

(b) Find the total potential energy of the system of three charges.

Properties of Potential Energy

- Potential Energy is a **scalar**, which mean they can be added ***without considering directions***.



$$U = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right) = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}}$$

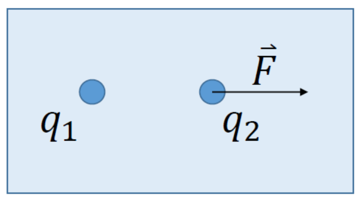
Defining Electric Potential

Recall that in Ch21:

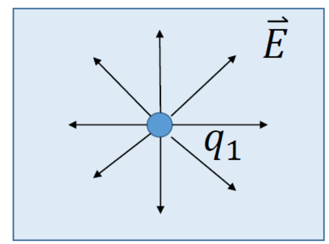
What is electric field?

$$\vec{F} = q_2 \vec{E} \quad \Longleftrightarrow \quad \vec{E} = \frac{\vec{F}}{q_2}$$

$$\vec{E} = k\frac{q_1}{r^2}\hat{r}$$



For one particular situation



Generalized situation

$$U = q_2 V \quad \Longleftrightarrow \quad V = \frac{U}{q_2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

Defining Electric Potential

The unit of electric potential

$$1V = 1\ volt = 1\frac{J}{C} = 1\frac{joule}{coulomb}$$

Electric potential due to a collection of point charges:

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric potential due to a continuous distribution of charges:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

Example:

Find the potential at a distance r from a very long line of charge with linear charge density (charge per unit length) λ.

New unit for **ENERGY**: electron volt (eV)

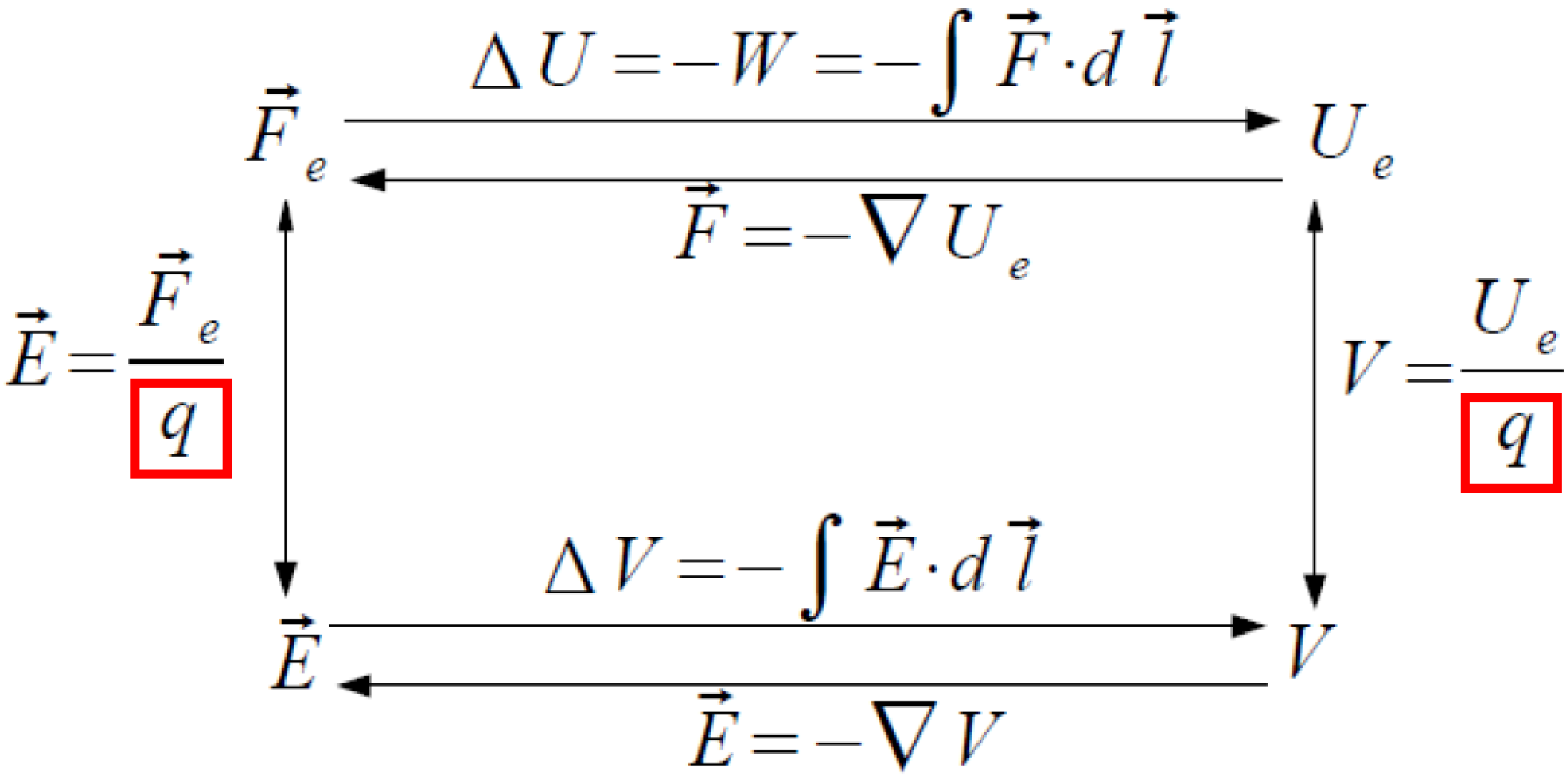
1 eV = the electric potential energy gained by one electron with change of 1 V electric potential

$$1eV = \Delta U = q\Delta V = (1.602 \times 10^{-19} C)(1V) = 1.602 \times 10^{-19} J$$

Note: CV = J

Note #2: eV is an unit of ENERGY; while V is an unit of electric potential

What is the relationship among electric force, field, potential energy, and potential?

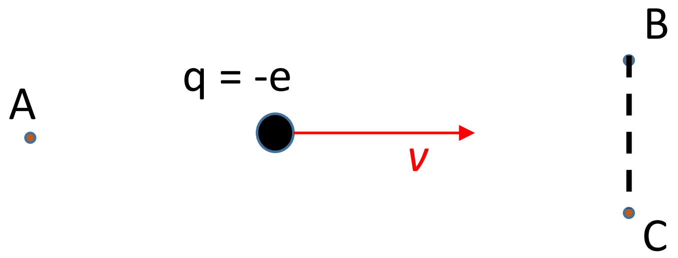


Note: the “q” will flip sign between $\vec{F}$ and $\vec{E}$, or U and V.

Note #2: The absolution value of V (or U) is not important, instead, the change of them (ΔU and ΔV) is the key.

Quiz

Suppose a region of space has a uniform electric field, unknown field direction. An electron sit in this electric field was observed to moving toward the right direction from rest, as shown in figure. Which statement about the electric potential is true?

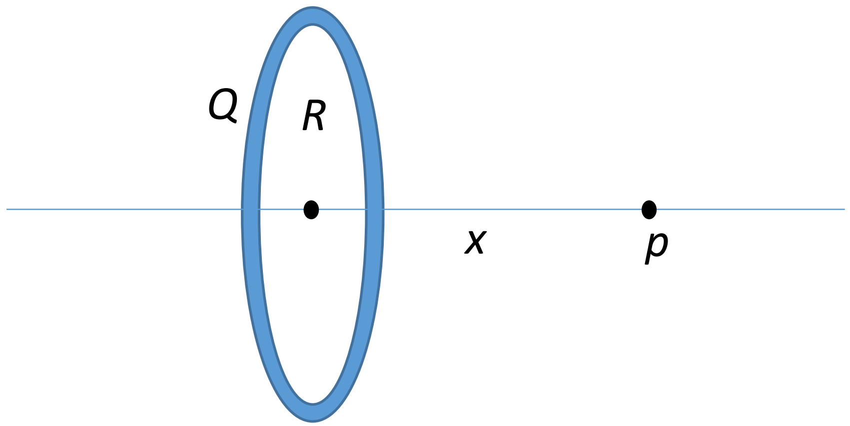


A. The potential at points *A* and *B* are equal, and the potential at point *C* is lower than the potential at point *A*.
B. The potential at point *A* is the highest, the potential at point *B* is the second highest, and the potential at point *C* is the lowest.
C. The potential at points B and C are equal, and the potential at point A is higher than the potential at point B.
D. The potential at all three locations (*A*, *B*, *C*) is the same because the field is uniform.
E. The potential at points *B* and *C* are equal, and the potential at point *A* is lower than the potential at point *B*.

Potential Gradient

$$\vec{E} = -\vec{\nabla}V = -(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z})$$

Example 1: Find the electric field and electric potential at point p.



Example 2:

Find the electric field at a distance r from a very long line of charge with linear charge density (charge per unit length) λ through the electric potential.