Chapter 4

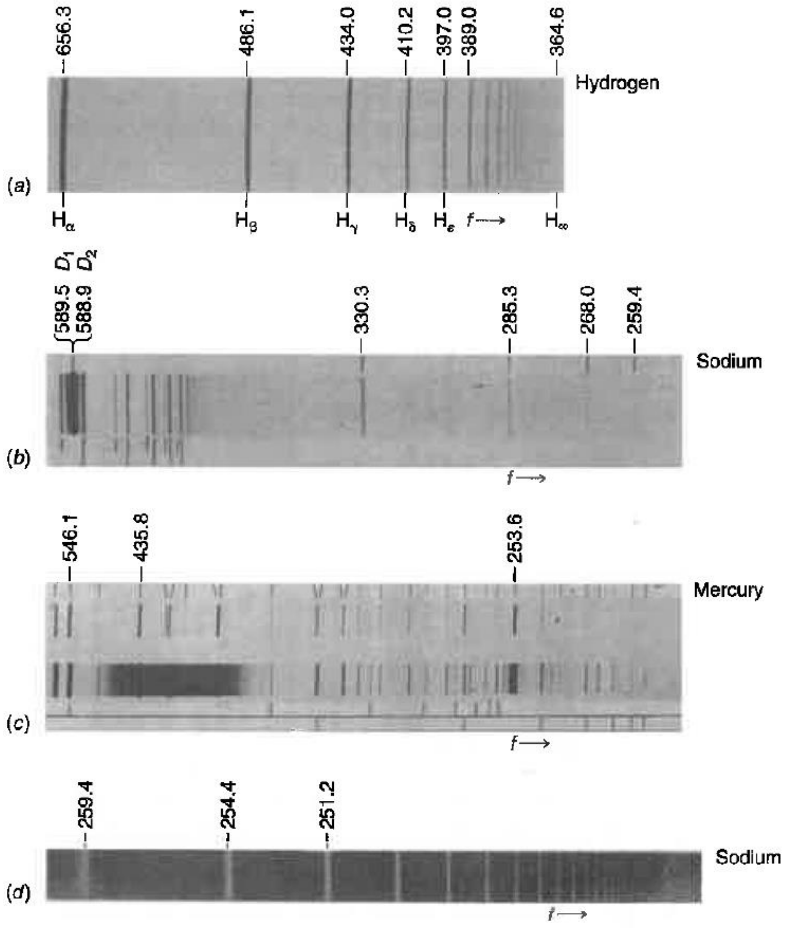
The Nuclear Atom

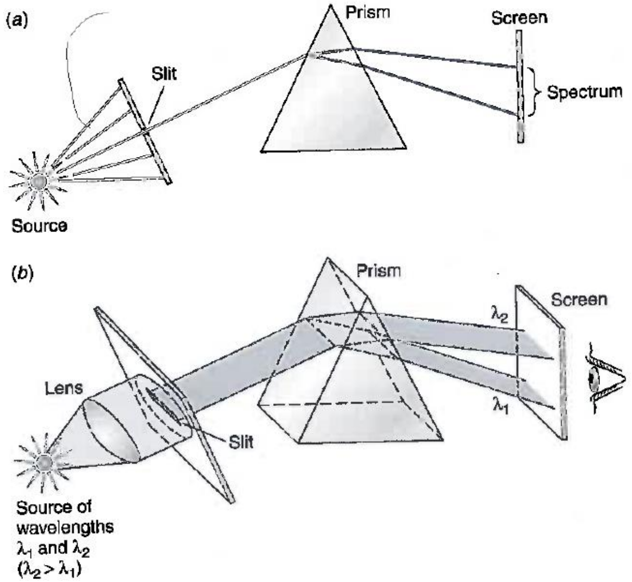
Atomic Spectra: Balmer series (1885)

Balmer series

$$\lambda_n = 364.6 \frac{n^2}{n^2-4} nm \text{ , n = 3, 4, 5, ...}$$

Balmer suggested that his formula might be a special case of a more general expression applicable to the spectra of other elements when ionized to a single electron – hydrogenlike elements.





Atomic Spectra: Rydberg-Ritz formula (1908)

Rydberg-Ritz formula $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$, for n > m. $R_H = 1.096776 \times 10^7 m^{-1}$

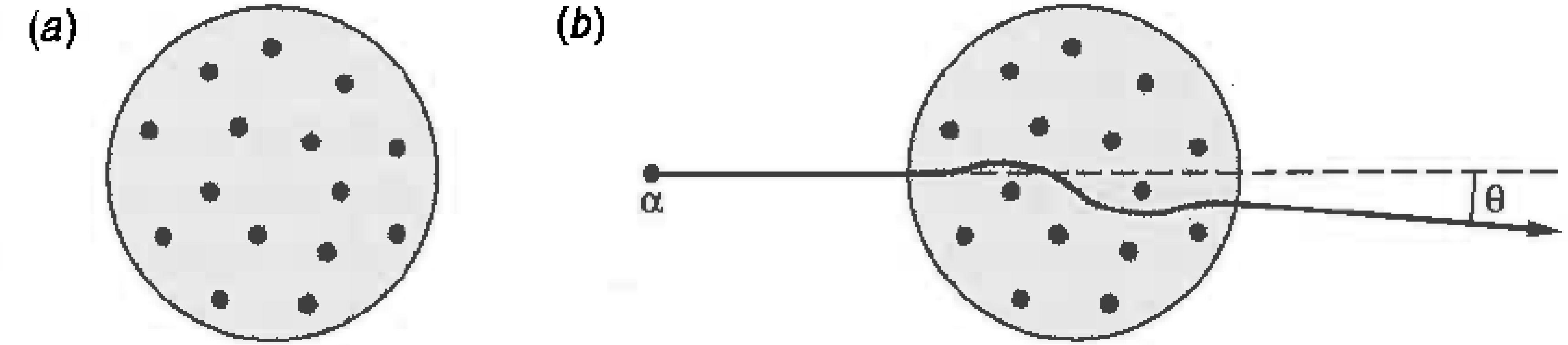
$$R_\infty = 1.097373 \times 10^7 m^{-1}$$

The hydrogen Balmer series reciprocal wavelengths are those given by Eq. 4-2 with m = 2 and n = 3, 4, 5, ….

Example

- The hydrogen Balmer series reciprocal wavelengths are those given by Eq. 4-2 with $m = 2$ and $n = 3, 4, 5, ...$ Other series of hydrogen spectral lines were found for $m = 1$ (by Lyman) and $m = 3$ (by Paschen). Compute the wavelengths of the first lines of the Lyman and Paschen series.

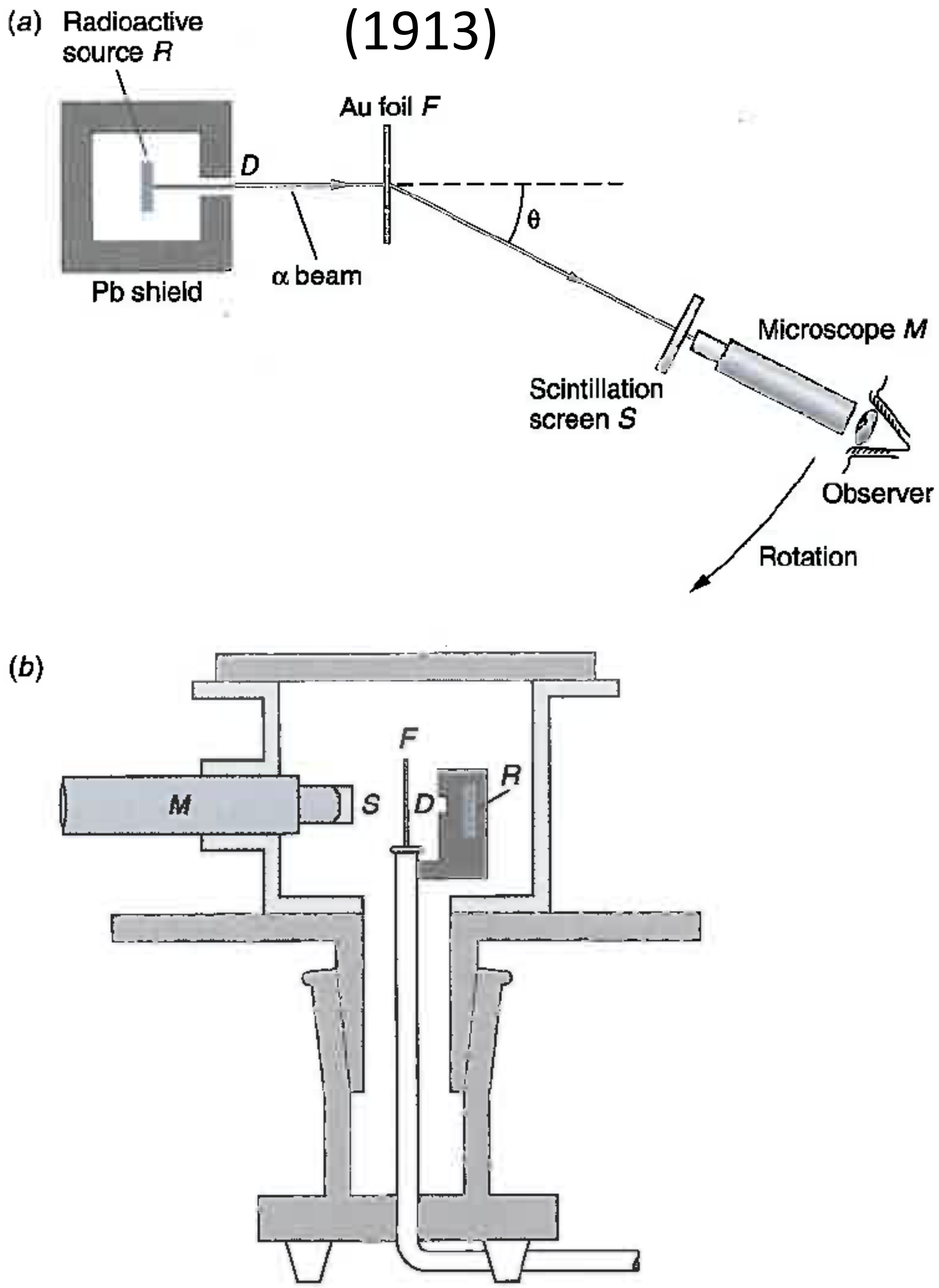
JJ Thomson’s Nuclear Model



Cannot explain the atomic spectra and cannot explain Rutherford’s experiment.

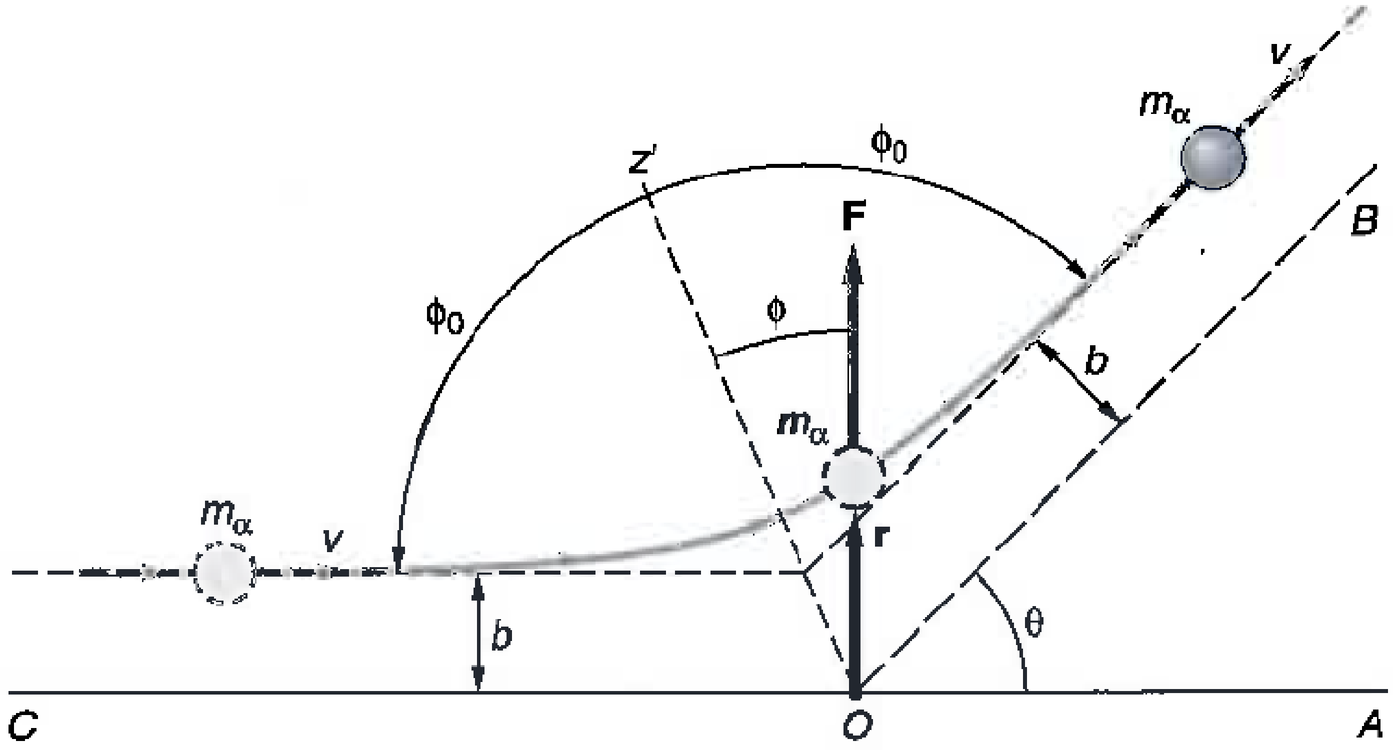
Rutherford’s Nuclear Model

(1913)



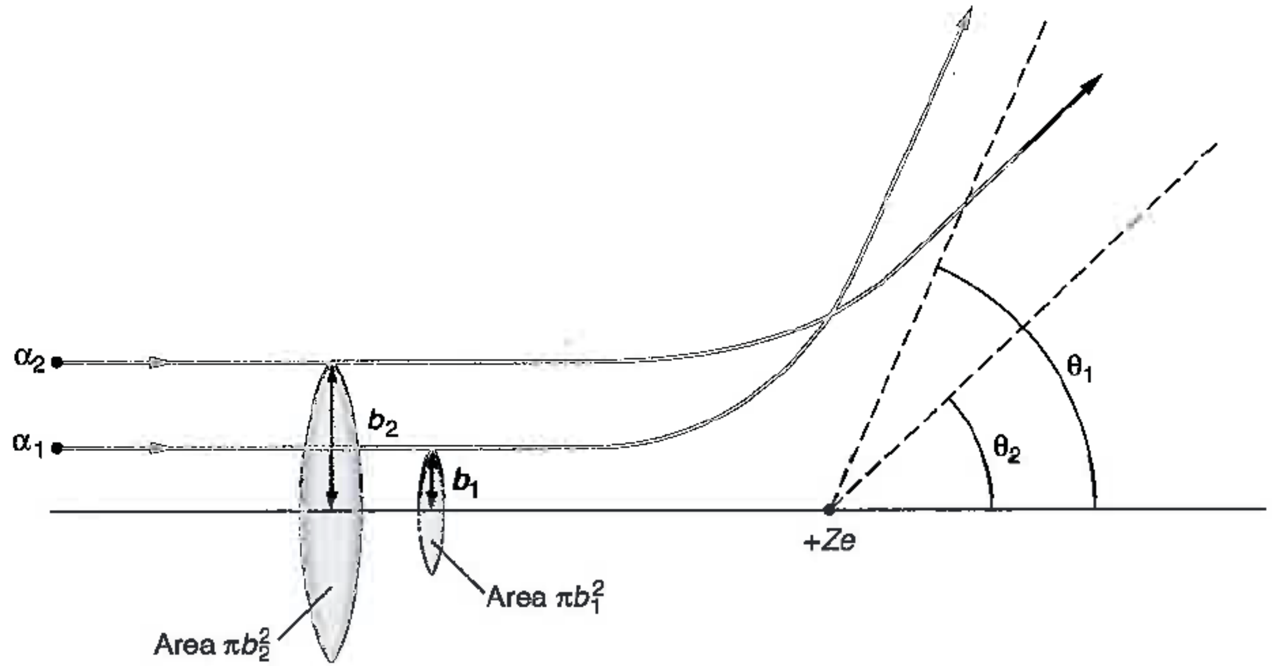
- Rutherford and his students Geiger and Marsden found the α particle’s q/m value is half that of the proton.
- Spectral line of α particle confirmed that it is helium nucleus.
- It is found that some α particles were deflected as large as 90° or more, even 180° was possible.

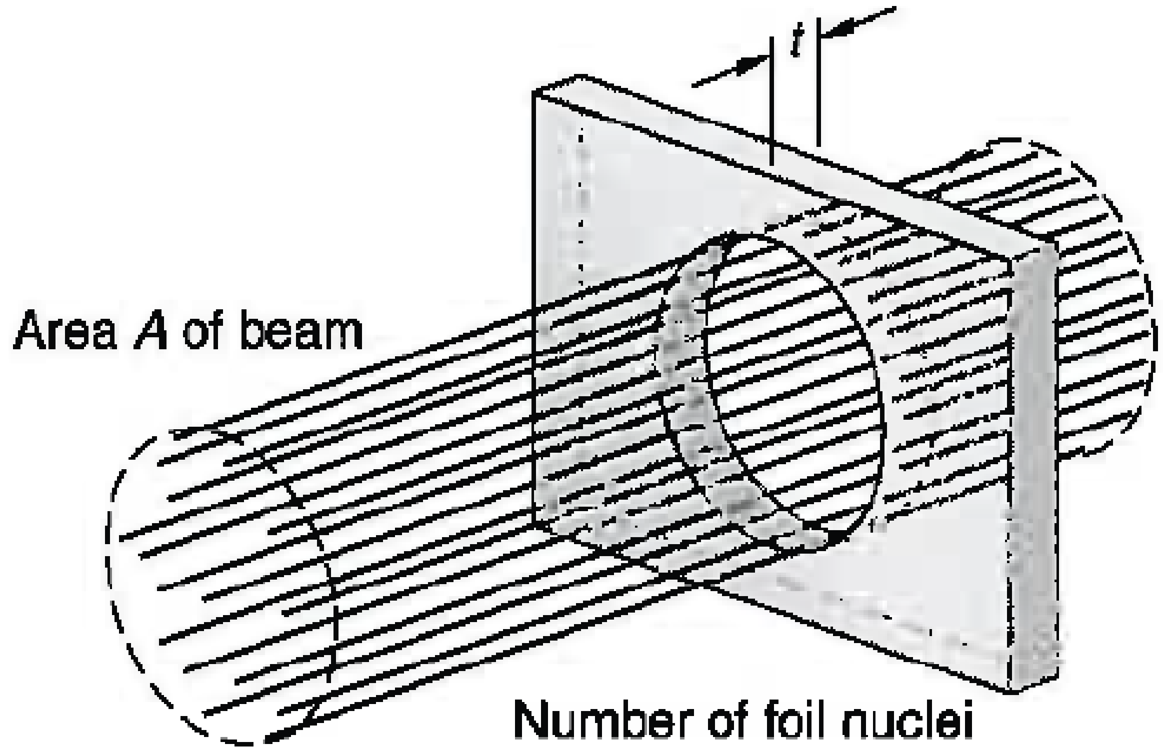
Rutherford's Scattering Theory



$$b = \frac{kq_\alpha Q}{m_\alpha v^2} cot \frac{\theta}{2}$$

Cross section and scattered fraction





Cross section $\sigma = \pi b^2$

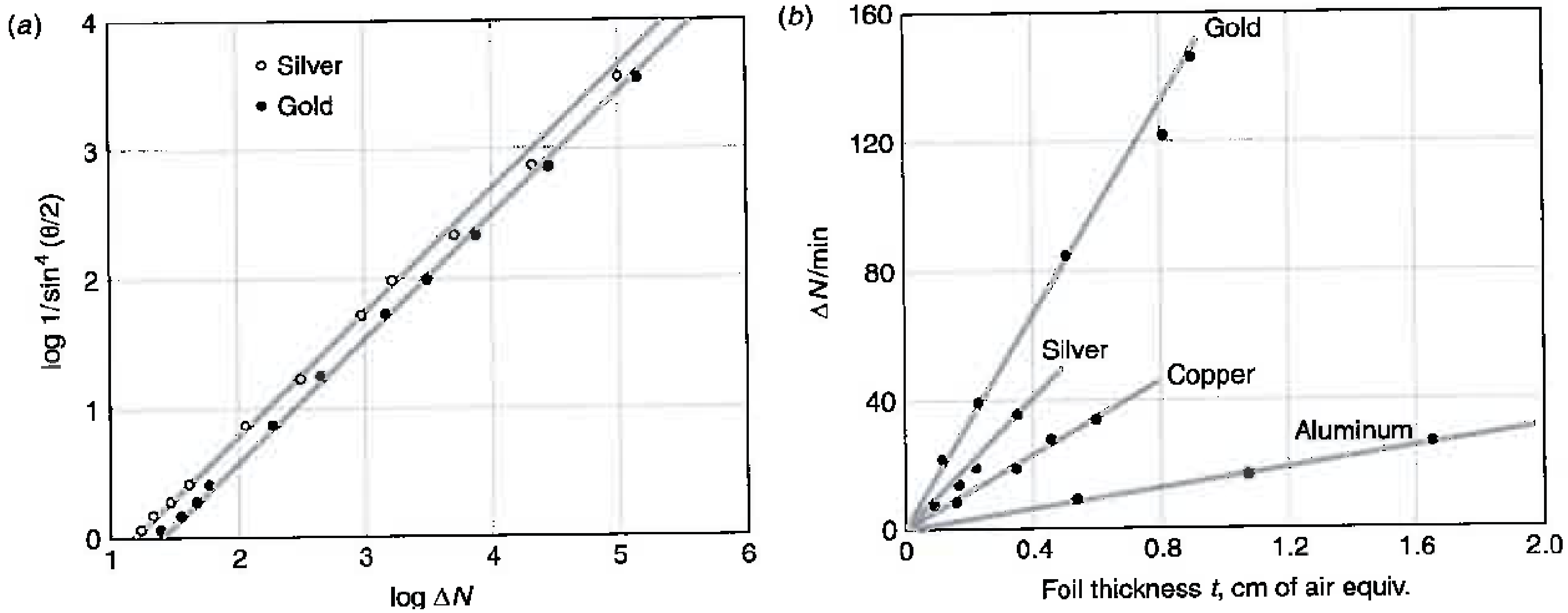
Scattered fraction $f = \pi b^2 nt$

$$n = \frac{\rho N_A}{M}$$

Example

- Calculate the fraction of an incident beam of α particles of kinetic energy 5 MeV that Geiger and Marsden expected to see for θ ≥ 90° from a gold foil (Z = 79) 10^{-6} m thick. The density of gold is 19.3 g/cm^3; M = 197.

More quantitative agreements



$$\Delta N = \left(\frac{I_0 A_{sc} n t}{r^2}\right)\left(\frac{kZe^2}{2E_k}\right)^2 \frac{1}{sin^4 \frac{\theta}{2}}$$

Size of the nucleus

$$r_d = \frac{kq_\alpha Q}{\frac{1}{2}m_\alpha v^2}$$

$$r_d = \frac{(2)(79)(1.44\ \text{eV}\cdot\text{nm})}{7.7 \times 10^6\ \text{eV}} = 3 \times 10^{-5}\ \text{nm} = 3 \times 10^{-14}\ \text{m}$$

Example

- A beam of α particles with E_k = 6.0 MeV impinges on a silver foil 1.0 μm thick. The beam current is 1.0 nA. How many a particles will be counted by a small scintillation detector of area equal to 5 mm^2 located 2.0 cm from the foil at an angle of 75°? (For Silver Z = 47, ρ = 10.5 gm/cm^3, and M = 108).

Nuclear Model of Hydrogen Atom

Classical Mechanics

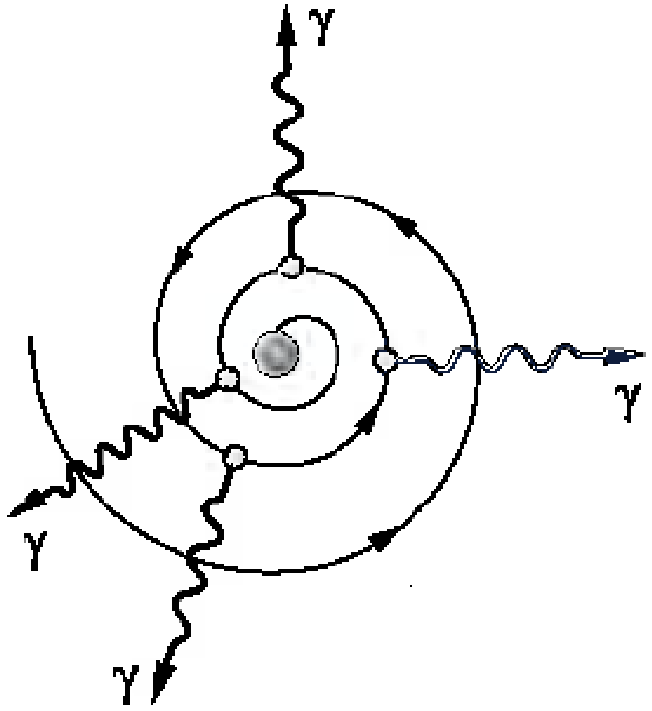
$$F = \frac{kZe^2}{r^2} = \frac{mv^2}{r}$$

Electrons could orbit at any value of *r*.

Classical Electromagnetism

$$f = \frac{v}{2\pi r} = \left(\frac{kZe^2}{4\pi^2 m}\right)^{1/2} \frac{1}{r^{3/2}}$$

Electrons with acceleration would emit radiation as E&M wave.



Bohr Model of Hydrogen Atom (1913)

Bohr's postulates

- Electrons could move in certain orbits without radiating – stationary state.
- The atom radiates when the electron makes a transition from one stationary state to another and that the frequency f of the emitted radiation is related to the energy difference between them.

$$hf = E_i - E_f$$

- In the limit of large orbits and large energies, quantum calculations must agree with classical calculations.

Stationary State/Orbital in Hydrogen Atom

Based on measurements and assumptions made by Bohr, angular momentum is quantized as $L = \frac{nh}{2\pi}$

$$L = mvr = \frac{nh}{2\pi} = n\hbar$$

$$r = r_n = \frac{n^2 a_0}{Z} \quad \text{where} \quad a_0 = \frac{\hbar^2}{mke^2} = 0.529\text{Å} \quad \text{(Bohr radius)}$$

$$E = E_n = -E_0 \frac{Z^2}{n^2} \quad \text{where} \quad E_0 = \frac{mk^2e^4}{2\hbar^2} = 13.6\ eV$$

Bohr's Radiation Energy

$$hf = E_i - E_f$$

$$E_n = -E_0 \frac{Z^2}{n^2}$$

$$\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

where

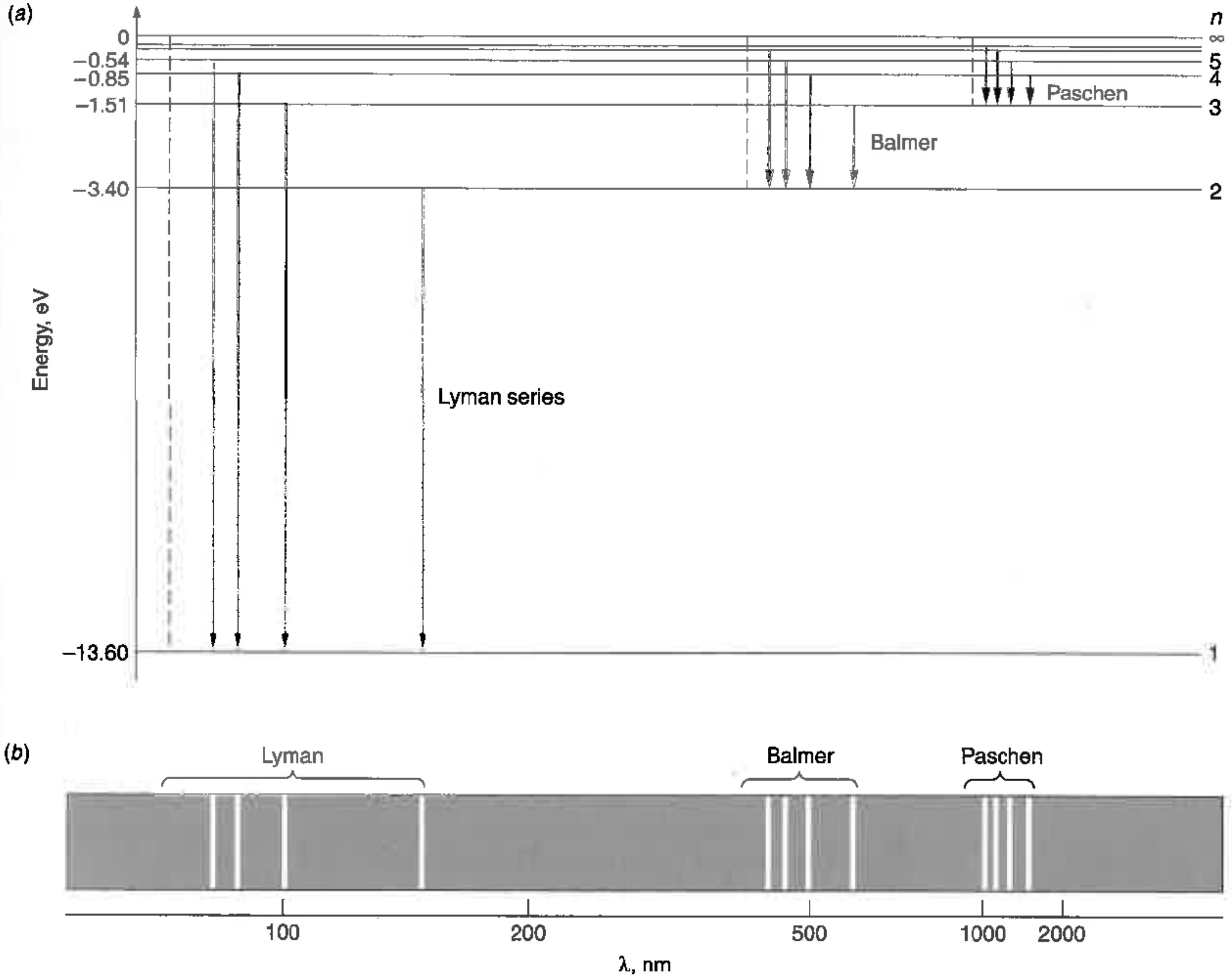
$$R = \frac{mk^2e^4}{4\pi c\hbar^3} = 1.097 \times 10^7 m^{-1}$$

Recall, from measurements

$$R_H = 1.096776 \times 10^7 m^{-1}$$
$$R_\infty = 1.097373 \times 10^7 m^{-1}$$

Bohr’s Radiation Energy in Hydrogen Atom

$$E_n = -\frac{E_0}{n^2} = -\frac{13.6}{n^2}\ eV$$



Example

- Compute the wavelength of the H_β spectral line, that is, the second line of the Balmer series predicted by Bohr's model. The H_β line is emitted in the transition from $n_i = 4$ to $n_f = 2$.

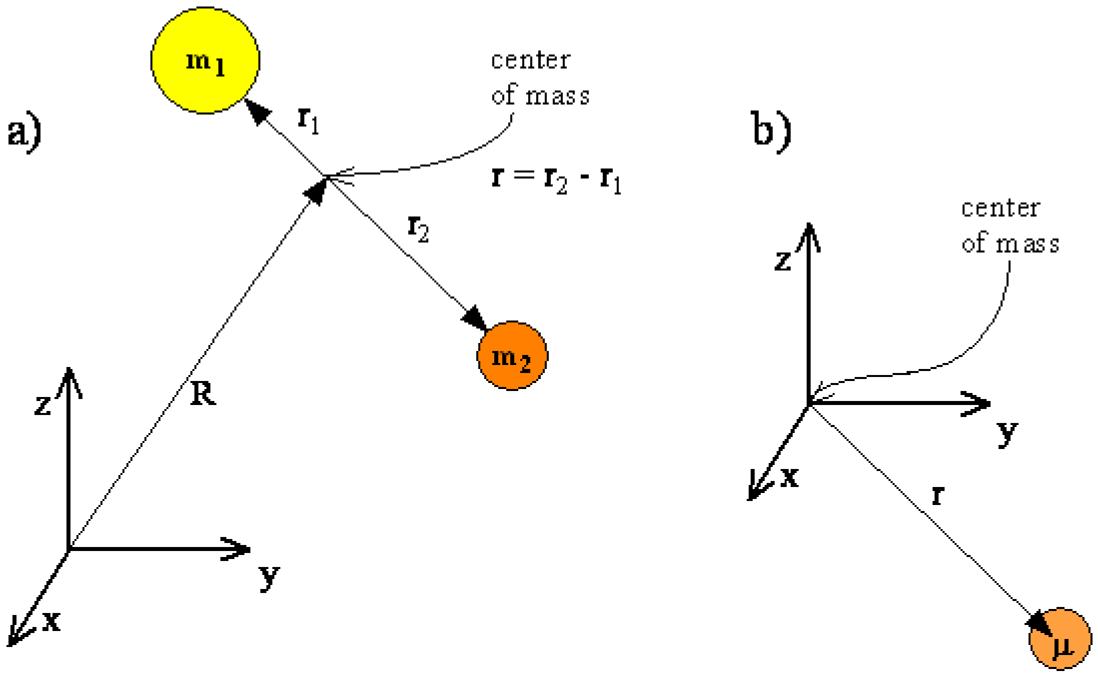
Reduced Mass Correction

$$R = \frac{mk^2e^4}{4\pi c\hbar^3}$$

$$E_0 = \frac{mk^2e^4}{2\hbar^2}$$

$$R = \frac{\mu k^2e^4}{4\pi c\hbar^3}$$

$$E_0 = \frac{\mu k^2e^4}{2\hbar^2}$$



$$R = \frac{k^2e^4}{4\pi c\hbar^3}\frac{m}{1+\frac{m}{M}} = \frac{mk^2e^4}{4\pi c\hbar^3}\frac{1}{1+\frac{m}{M}} = R_\infty\frac{1}{1+\frac{m}{M}}$$

$$\mu = \frac{m}{1+\frac{m}{M}}$$

Example

- Compute the Rydberg constants for H and He^{+} applying the reduced mass correction (m = $9.1094 \times 10^{-31} kg; m_p = 1.6726 \times 10^{-27} kg;$ $m_{\alpha} = 6.6447 \times 10^{-27} kg$).

Correspondence Principle

$$f = \frac{Z^2 m k^2 e^4}{4\pi\hbar^3}\frac{2n-1}{n^2(n-1)^2}$$

For large n

$$f \approx \frac{Z^2 m k^2 e^4}{4\pi\hbar^3}\frac{2n}{n^4}$$

$$f_{rev} = \frac{v}{2\pi r}$$

$$r = \frac{n^2\hbar^2}{Zmke^2}$$

$$v = \frac{n\hbar}{mr}$$

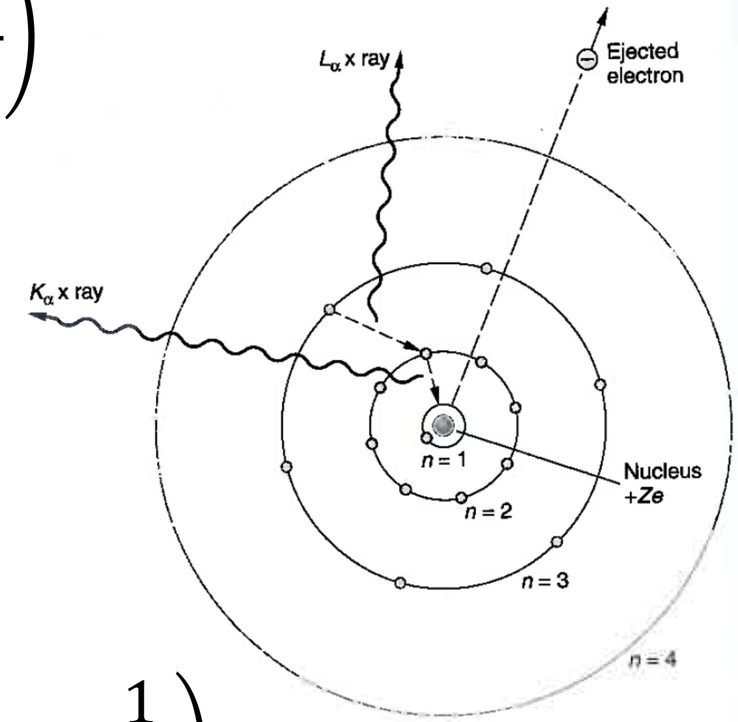
X-ray spectra

Bohr's model

$$f = \frac{mk^2e^4Z^2}{4\pi\hbar^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

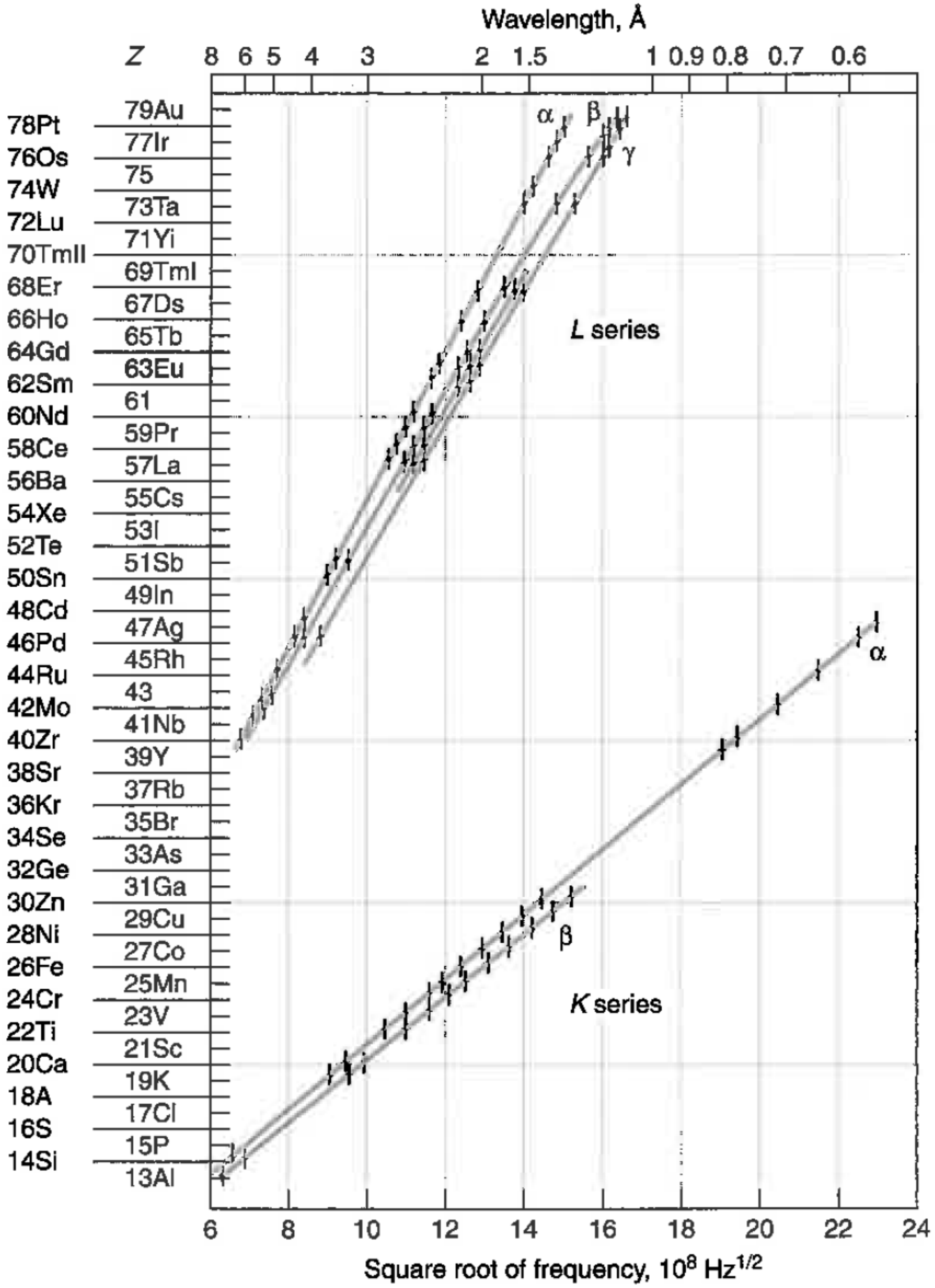
K series

$$f = \frac{mk^2e^4(Z-1)^2}{4\pi\hbar^3}\left(1 - \frac{1}{n_i^2}\right)$$



Moseley plot

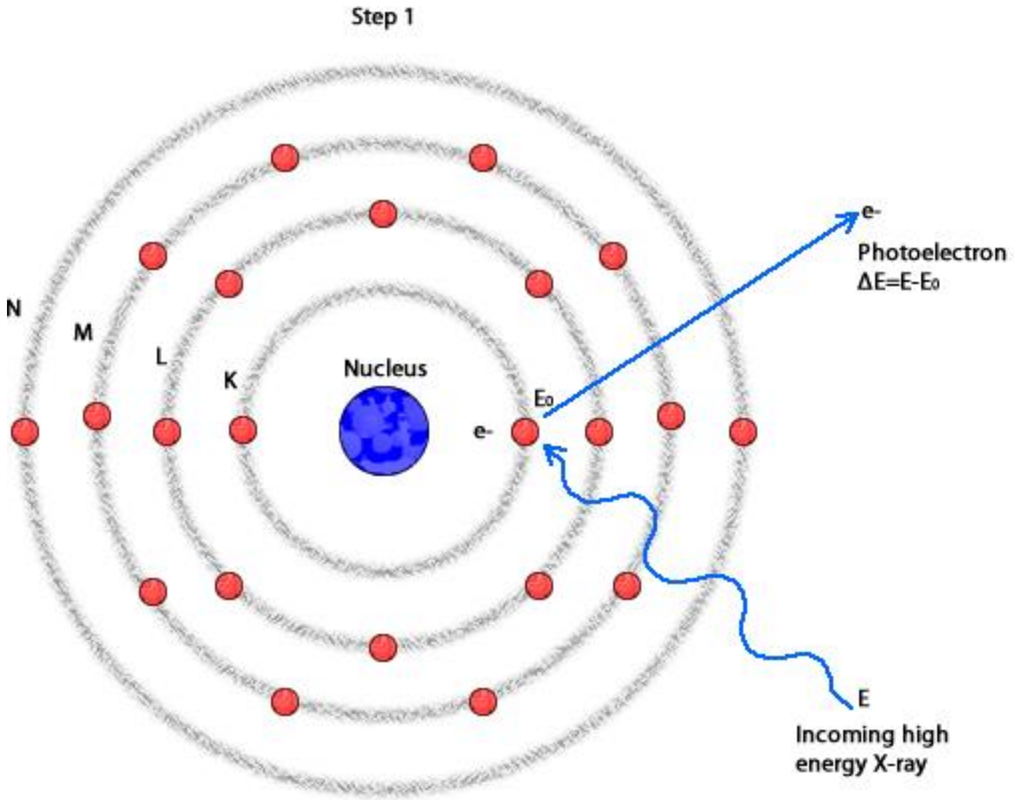
$$f^{1/2} = A_n(Z-b)^2$$



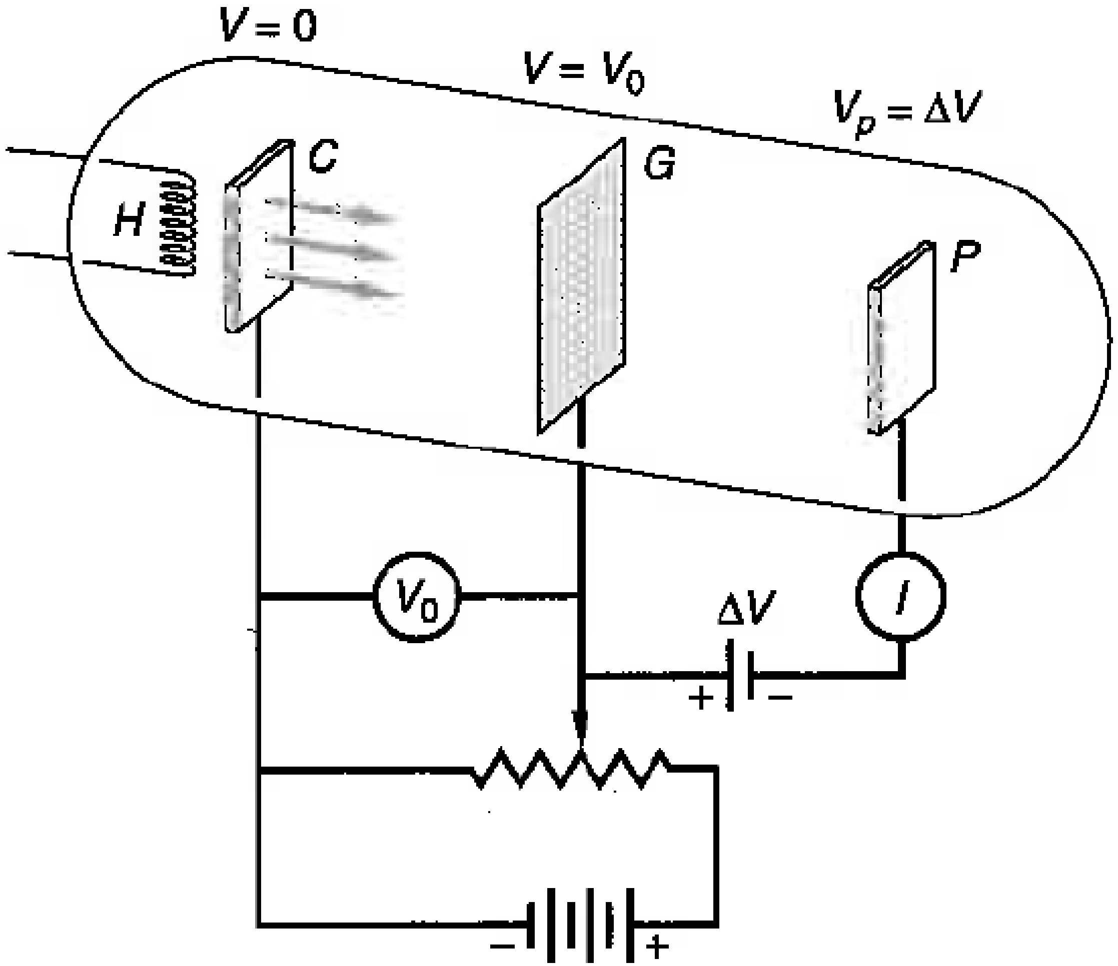
Example

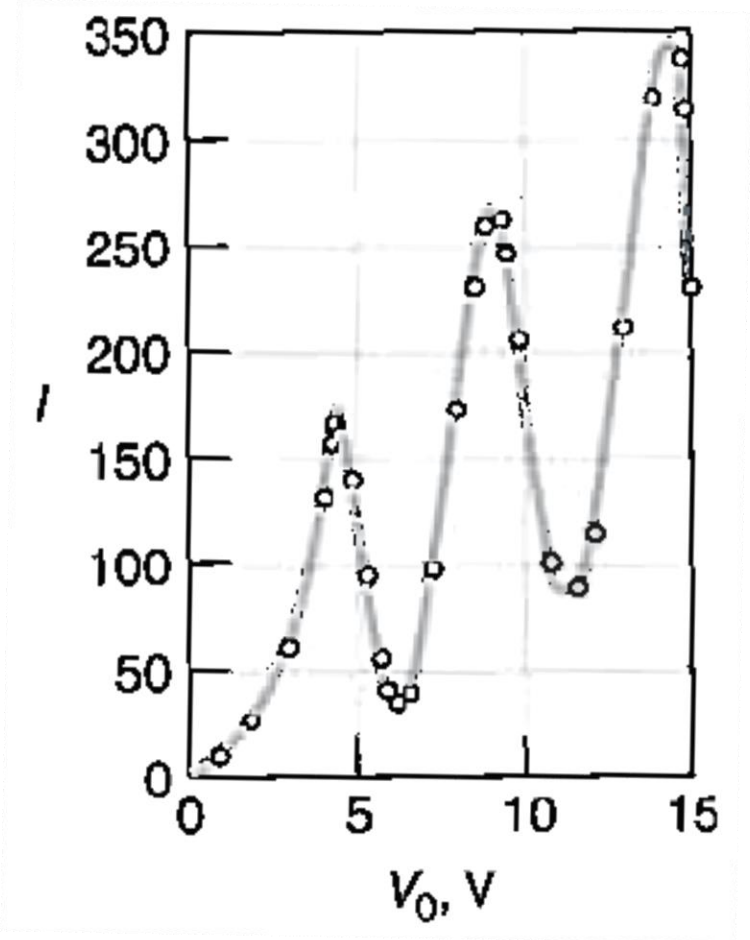
- Calculate the wavelength of the K_α line of molybdenum (Z = 42), and compare the result with the value $\lambda = 0.0721\ nm$ measured by Moseley and with the spectrum in Figure 3-15b (page 141)

Auger Electrons



Franck-Hertz Experiment





Electron Energy Loss Spectroscopy (EELS)