Astr 2310 Tues. Feb. 23, 2016

Today's Topics

- Chapter 4: Earth-Moon System
  - Effects of Earth's Rotation
  - Coriolis Effect
  - Aberration of Starlight
  - Roche and Instability Limits
  - Tidal Evolution of Earth-Moon System

Chapter 4: Homework

Chapter 4: #1, #2, #5, #7, #8, #9

- Due Tues. March 1

Chapter 4: Dynamics of the Earth-Moon System

Direct evidence for the Earth's Rotation is Subtle:

Equatorial Bulge (Oblateness)

Foucault's Pendulum: A long pendulum set in motion will oscillate in a plane due to the conservation of momentum. We observe it to precess over the course of one day due to the Earth's rotation.

Doppler Shift: A given star shows a small Doppler shift (~ 0.5 km/sec) between rising and setting due to the Earth's rotational motion.

Direct Evidence for the Earth's Orbital Motion is also Subtle:

Abberation of Starlight: The finite speed of light means that telescopes have to be pointed slightly away from the position of a star in order for the light to be seen or recorded. Result is an apparent shift in the position of the stars (θ ~ tan θ =v/c).

Parallax: Apparent position of nearby stars shift as Earth orbits the Sun.

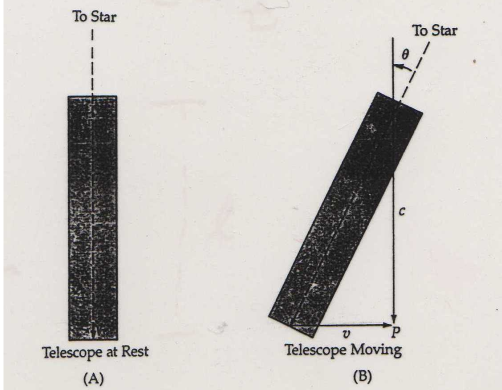


Figure 3–11 The aberration of starlight. (A) A telescope is at rest and so pointed upward to observe the light from a star. (B) When a telescope moves at speed v (as a result of the Earth's revolution), it is tilted through an angle θ, so that the starlight reaches P at the same time as the bottom of the telescope does.

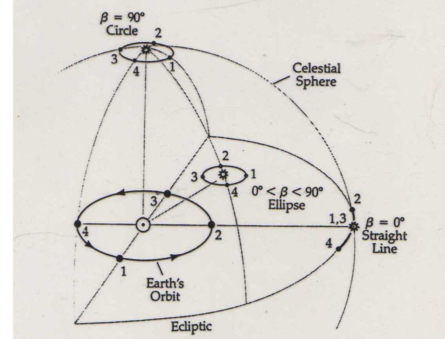
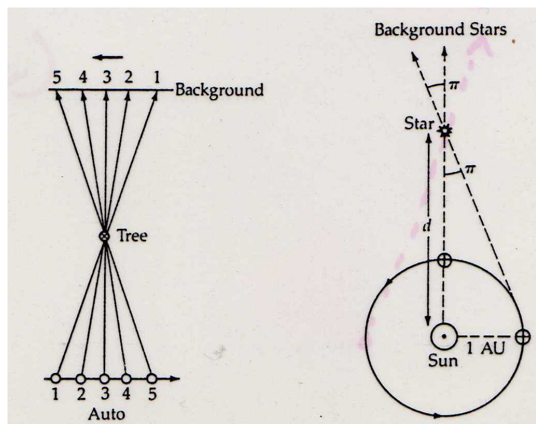
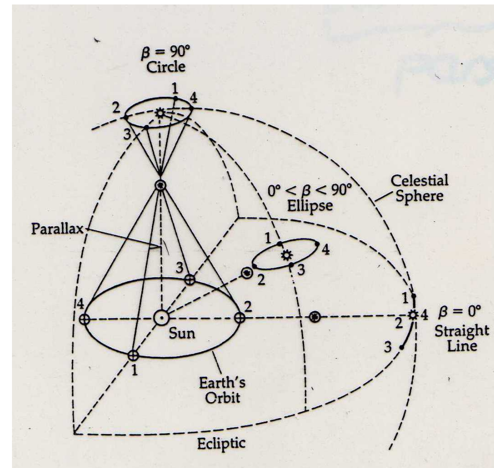


Figure 3–12 Apparent stellar orbits resulting from the aberration of starlight. The Earth is shown at four positions in orbit around the Sun (1 to 4). The apparent paths traced out by stars because of the change in the direction of aberration during a year are shown for stars directly above the orbital plane (ecliptic pole), in the plane (the ecliptic), and at an intermediate position on the celestial sphere. The sizes of the apparent orbits are independent of the distances to the stars.





Tidal Field Between Earth and Moon

The Differential Gravitational Field Produced by One Body on Another is Known as the Tidal Field. Consider the Gravitational Force:

$F_g = Gm_1m_2/r^2$

Differentiation wrt r:

$dFg = - 2\ Gm_1m_2/r^3\ dr$

This is known as the "tidal force". Note that it falls rapidly with distance.

Ocean bulge leads Moon due to Earth's rotation. The tilt of the Earth's axis and centripital force on the water means bulge is not along the Earth-Moon direction. The torque on the Earth and the resulting precession is evident.

Earth's tidal field forces Moon into synchronus rotation (same side faces the Earth)

Earth looses rotational energy and the Moon gains orbital energy. Earth's day was about 20 hours long a few Million years ago and the Moon was much closer to Earth.

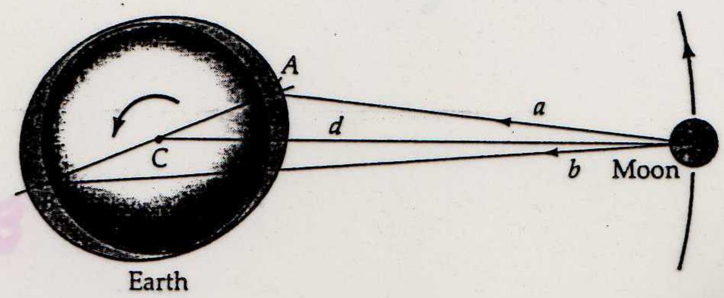


Figure 3–17 Tidal evolution. The Earth's tidal bulges (*A* and *B*) are driven by frictional coupling to the Earth's rotation to be ahead of the Moon's orbital position. The friction slows the Earth's rotation, and the bulges accelerate the Moon in its orbit.

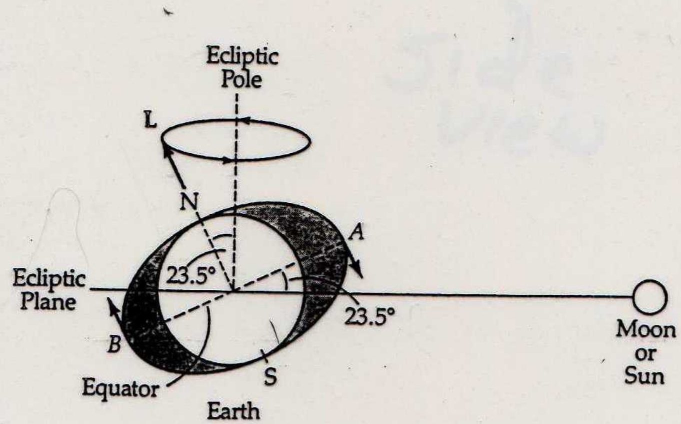


Figure 3–18 Precession of the Earth's axis. The differential tidal forces on the Earth's equatorial bulge (*A* and *B*) result in a torque (pointing into the page) that causes the Earth's angular momentum vector **L** to precess westward around the ecliptic pole.

Roche Instability Limit

- Consider a small moon in orbit about a larger mass.
  - Gravitational tidal and centripital force can be sufficient to tear a moon apart (overcome its self gravity)

The differential gravitational acceleration (tidal field):

$A = 2GMr/d^3$ where r is the satellite's radius and d is its distance.

The differential centripetal acceleration (tidally locked):

$B = GMr/d^3$ The combination must be balanced by self gravity:

$A+B = 3GMr/d^3 = Gm/r^2$ Solving for d the Roche limit:

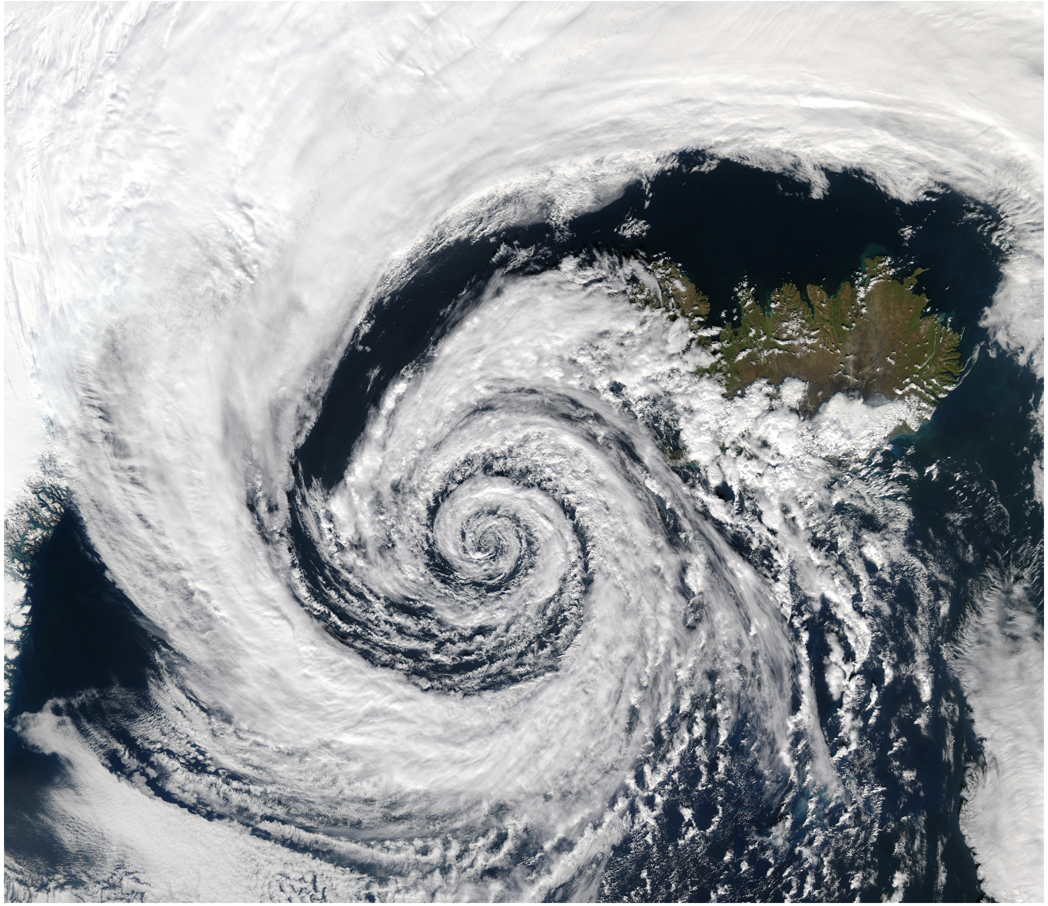
$d = r(3M/m)^{1/3}$ or in terms of the densities ($\rho_M = 3M/4\pi R^3$ and $\rho_m = 3m/4\pi r^3$) we have:

$d = R(3\rho_M/\rho_m)^{1/3} \sim 1.44(3\rho_M/\rho_m)^{1/3}R$ (really ~ 2.44, see Eq. 4.36)

Coriolis Force

- **Rotational velocity at equator is ~ 0.5 km/sec**
  - **Air moving south lags behind (west)**
  - **Air moving north leads (east)**
  - **Cyclonic storms (Lows) in the northern hemisphere rotate counter-clockwise**
  - **Cyclonic storms (Lows) in the southern hemisphere rotate clockwise.**

- **Corresponding acceleration:**
  $a_{cor} = 2(v \times \omega)$



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