Astr 2310 Thurs. March 25, 2016

This week's Topics

- Chapter 5: Interaction of Radiation and Matter
  - Electromatic Radiation
    - Wave Nature of Light
    - Simple (Classical) Optics
    - Doppler Effect
    - Inverse Square Law
    - Kirchoff's Laws
  - Quantum Nature of Light
    - Bohr Atom
    - Atomic Spectra
    - Molecular Spectra
    - Blackbody Emission
  - Radiative Transfer
    - Intensity vs. Flux
    - Absorption of Light
    - Transfer Equation
    - Simple Solutions & the Source Function
    - Spectral Line Formation
  - Boltzman & Saha Equations
    - Thermal Equilibrium
    - Boltzman Equation
    - Saha Equation

Chapter 5: Homework

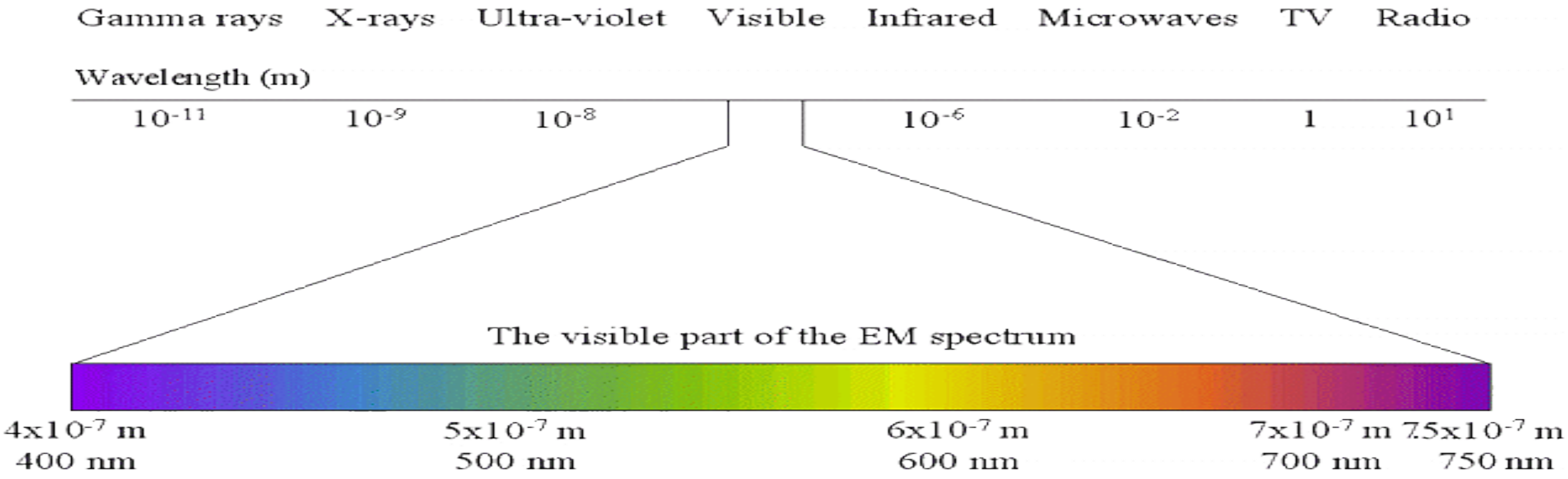
Chapter 5: #1, 2, 3, 4, 5, 6, 7, 8

- Due Tuesday March 8

Electromagnetic Radiation - I

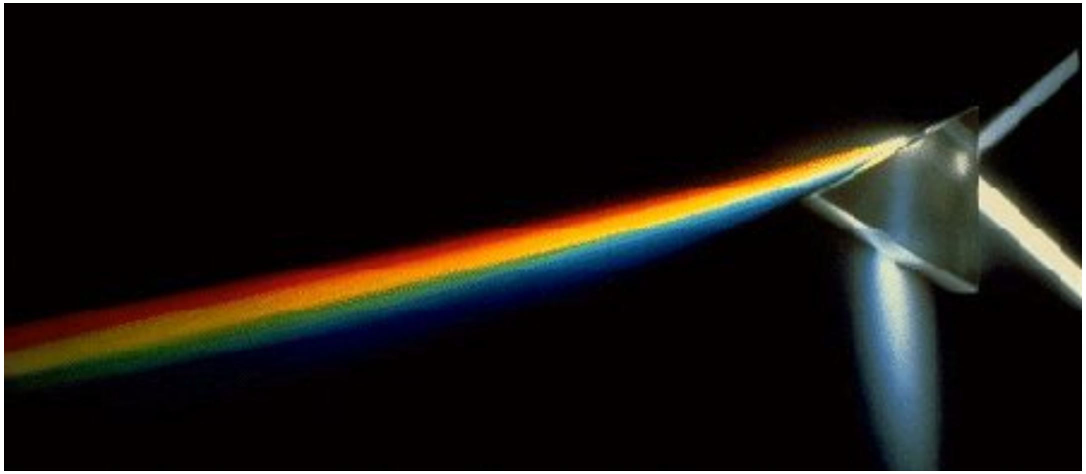
Almost everything we know about the distant universe comes from the information contained in light. We need to understand light in order to understand its information content.

- Electromagnetic Spectrum
  - Visible Spectrum
    - Recall that the visible spectrum is just a small portion of the electromagnetic spectrum
    - Light can be ordered by its energy
      - Light can be produced in the lab that is invisible
      - Common units of measurement are the micron (10^{-6} meter) or nanometer (10^{-9} meter)
        - » Shorter Wavelength = Higher Energy
        - » Longer Wavelength = Lower Energy
        - » Micro-waves and Radio waves
        - » Ultra-violet and X-rays
    - Visible spectrum is defined by the colors (wavelengths) our eyes see. Why?
      - Sun's maximum is in the visible
      - Atmospheric Window



Electromagnetic Radiation - II

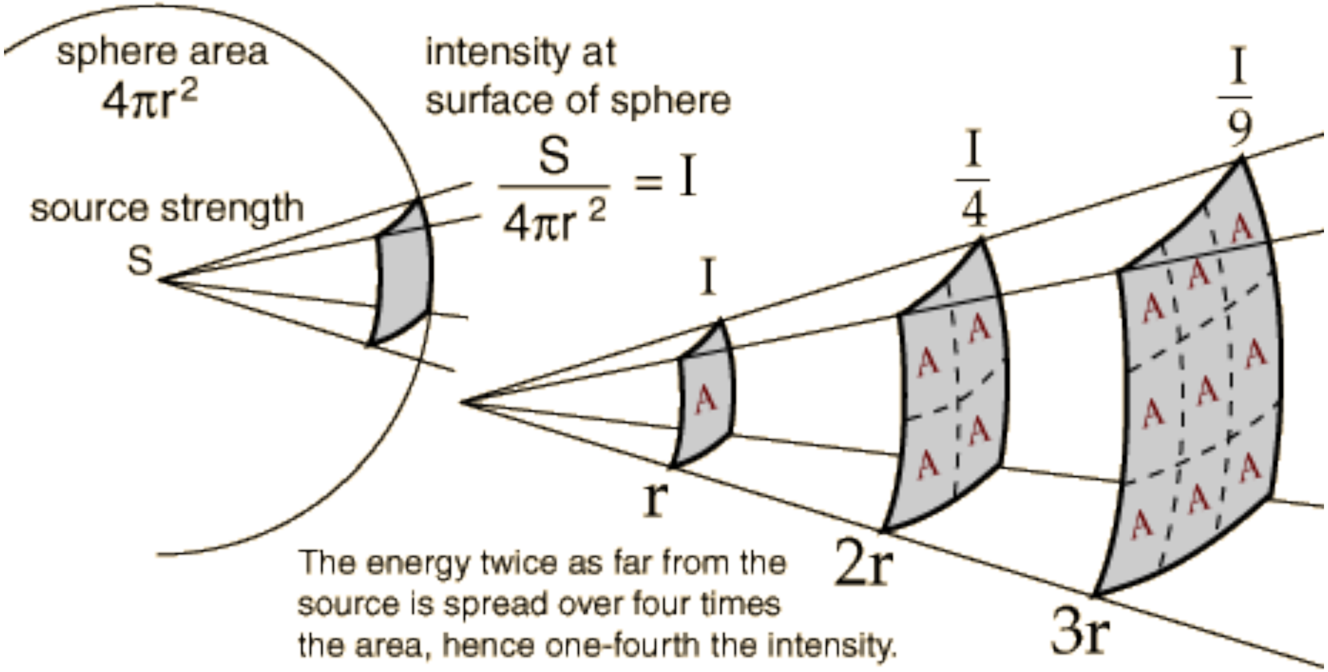
- Nature of Color
  - Sun’s Light is White (by definition)
  - Newton Discovered the Nature of Color
    - Prism shows that light is composed of different colors
    - Prisms separate light by bending (refracting) light according to its wavelength (color)
    - Newton showed that colors can be recombined to produce white light
    - Colors also result from preferential reflection of white light



Behavior of Light: Inverse Square Law

- Inverse Square Law
  - The light from an isotropically radiating source uniformly illuminates an imaginary sphere that encircles it.
  - The energy/unit area (I) must fall as $1/r^2$ since the total energy falling on the sphere = I x Area:

    $\mathbf{S = 4\pi r^2 I}$



The energy twice as far from the source is spread over four times the area, hence one-fourth the intensity.

Behavior of Light: Doppler Effect

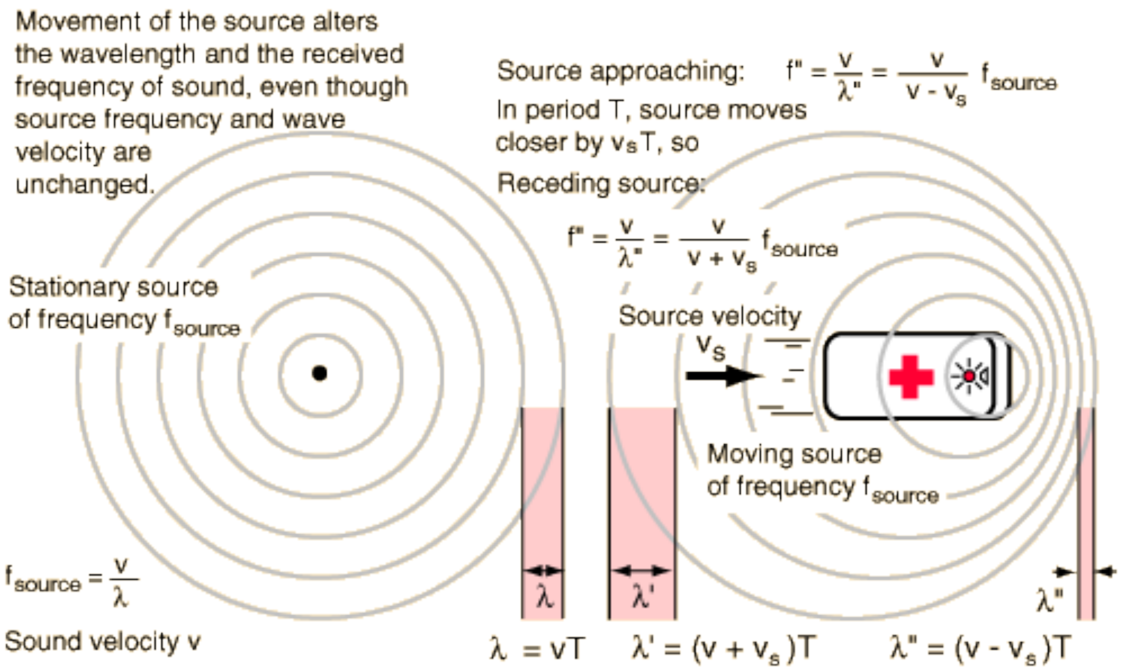
- **Doppler Effect**
  - Sources of sound undergo change in pitch if they are moving (see figure)
    - Pitch is high if approaching
    - Pitch is low if receeding.
  - For light:

$\Delta\lambda/\lambda_0 = v/c$ (classical Doppler shift, approximate)

$\lambda = \lambda_0\ (1 +- v/c)$ where λ_0 is the rest wavelength

$\lambda = \lambda_0\ (1 + z)$ where z is the redshift

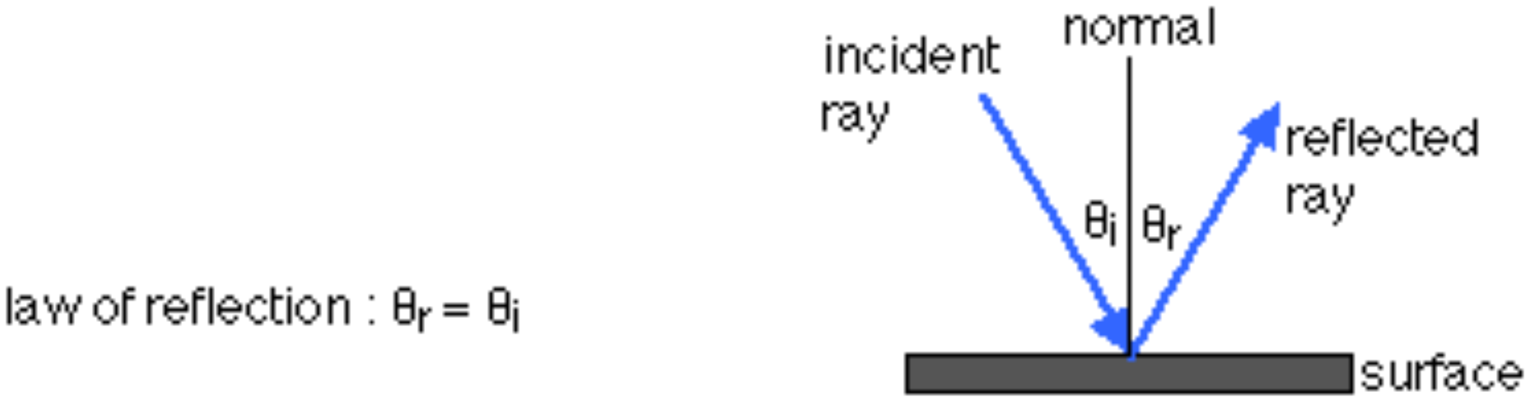
$\lambda = \lambda_0/\mathrm{sqrt}(1 +- v^2/c^2)$ (relativistic Doppler shift, exact)



Behavior of Light: Reflection of Light

- **Behavior of light can be described as "rays" characterizing the direction of motion**
- **Reflection of Light**
  - **Upon reflection of a light ray from a surface, the angle of incidence (θ_i) equals the angle of reflection (θ_r):**

$\theta_i = \theta_r$ **the law of reflection**

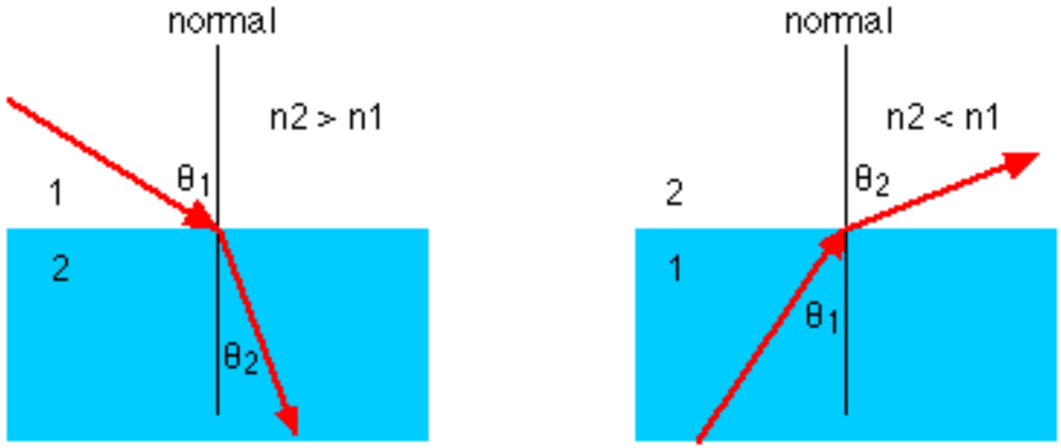


Behavior of Light: Refraction of Light

- **Refraction of Light**
  - The speed of light in a transparent medium decreases relative to its speed in a vacuum and characterized by the index of refraction:

    $n = c/v$ where v is speed in the medium and c that in the vacuum
  - Light can bend or refract as it traverses between two media with different n
  - The angle of the refraction (θ_2) is related to the angle of incidence (θ_1) and the index of refraction:

    $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ Snell’s Law



Snell's law : $n_1 \sin\theta_1 = n_2 \sin\theta_2$ or, equivalently, $\sin\theta_1 / \sin\theta_2 = v_1 / v_2$

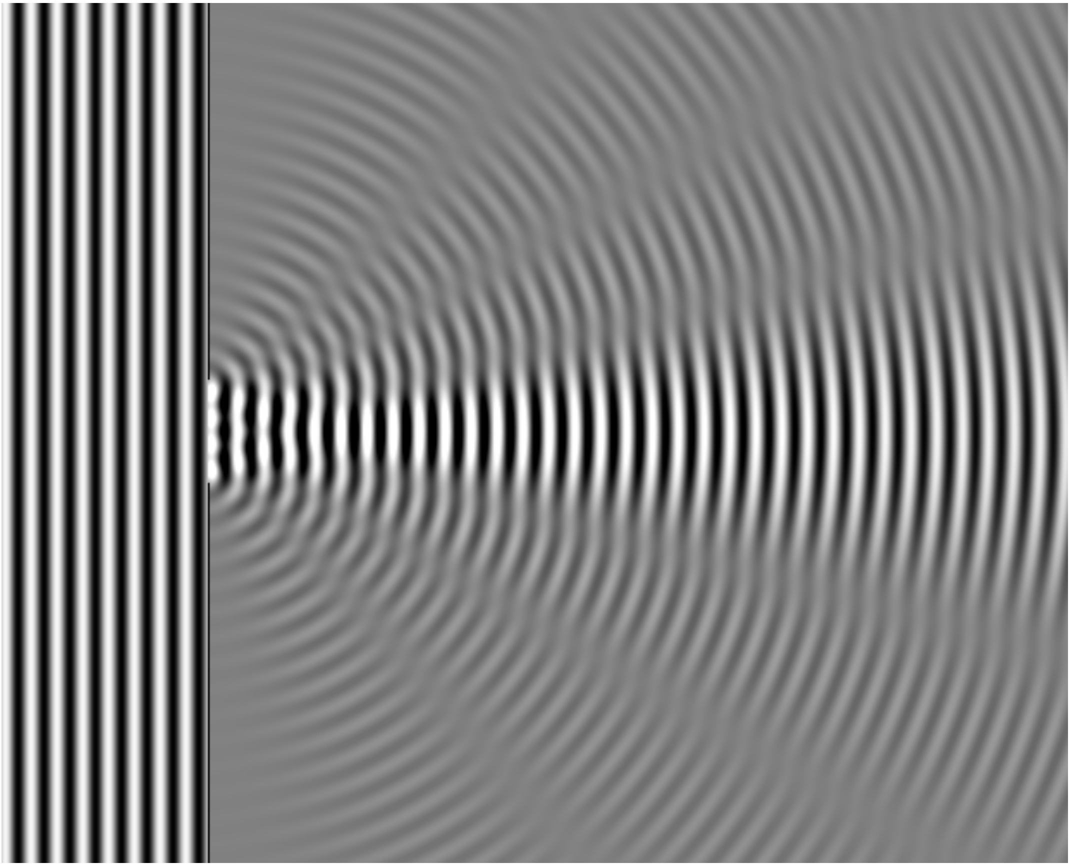
Behavior of Light: Diffraction of Light

- Diffraction of Light
  - When light encounters a sharp boundary or opening it interacts with itself as if the boundary or opening acts as a series of sources (wave-like behavior)

Square apertures (openings) produce square patterns

Circular apertures produce circular (ring-like) patterns

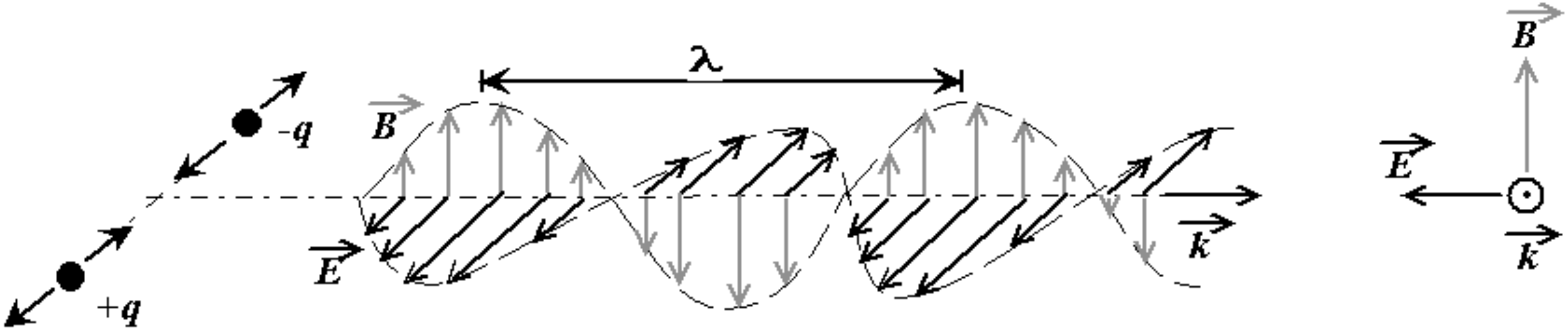
The latter is particularly relevant to telescope optics (more later).



Electromagnetic Radiation - III

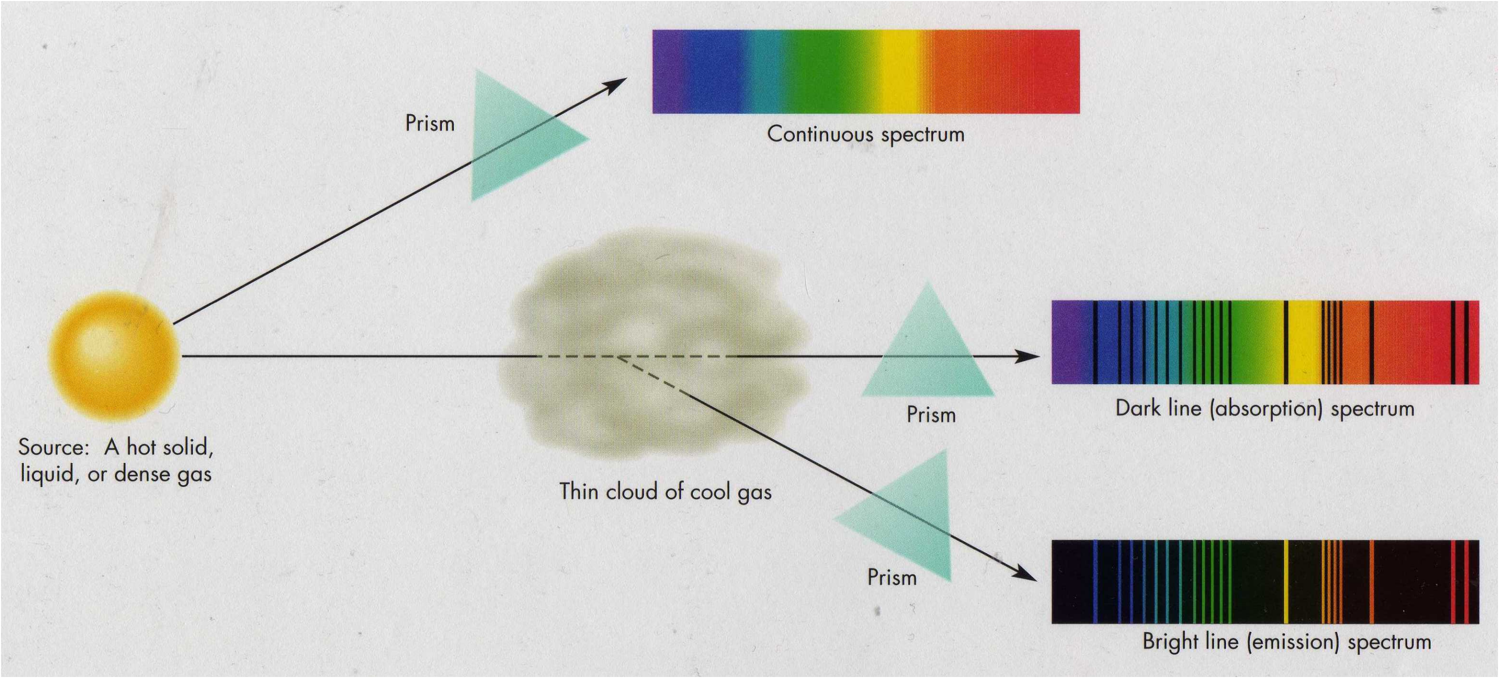
- Wave Nature of Light
  - Doppler shift, refraction, and diffraction provides evidence for the wave nature of light.
  - Light can be thought of as a propagating disturbance in the electromagnetic field of space-time.
  - The wavelength and frequency are related:

$\lambda\nu = c$ where the speed of light c = 299,792 km/s (constant in vacuum)



Kirchoff' s Laws

- Interaction of Light & Matter (Kirchoff' s Laws):
  - A hot solid, liquid, or dense gas will emit a continuous (Blackbody) spectrum
  - A hot gas produces an emission spectrum according to the elements present.
  - A continuum source shining through a cooler gas will absorb light at specific wavelengths according to the elements present to produce an absorption spectrum.

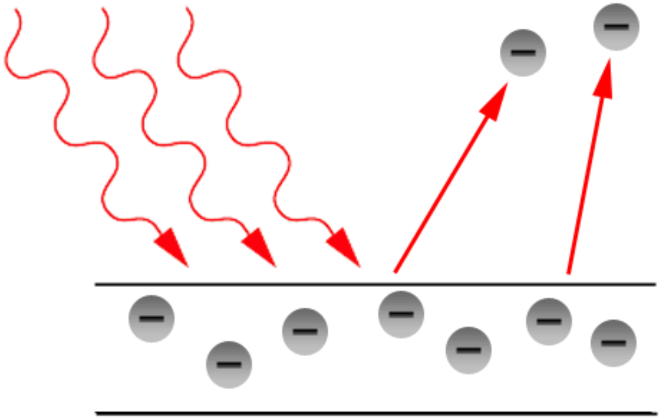


Quantum Nature of Light

- Light as a Particle (Photon)
  - **Light illuminating some substances can eject electrons (photoelectric effect)**
    - **When light intensity is reduced, single electrons can be ejected.**
    - **Light must be composed of packets of energy (photons)**
    - **The energy of each quanta (photon) of light is proportional to its frequency (ν) and is given by:**

$E = h\nu$ where h is Plank's constant (6.626×10^{-34} J sec)

Light has the properties of both waves and particles. This is central to quantum mechanics.



Atomic Structure & Spectra

- Rutherford had shown that the electrons were found in a "cloud" outside a small, dense nucleus.
- Bohr Atom
  - One of the big mysteries of 20-th century physics was how gasses produced emission lines.

  Bohr derived the energy spectrum of Hydrogen by assuming that the angular momentum of the electron was quantitized:

  $mvr = n(h/2\pi)$ where h is Plank's constant and n = 1, 2, 3, 4 …

  Setting the centripital force = Coulomb force:

  $mv^2/r = k(Ze)e/r^2$ where $k = 1/4\pi\varepsilon_0$

  Combining we get:

  $r = n^2(h^2/4\pi^2 me^2 kZ)$ the radius of the allowed orbits

  From the total energy (K.E. + P.E.) we get (see text):

  $E(n) = -(2\pi^2 me^4 k^2 Z^2)/n^2 h^2$

  Grouping the constants together:

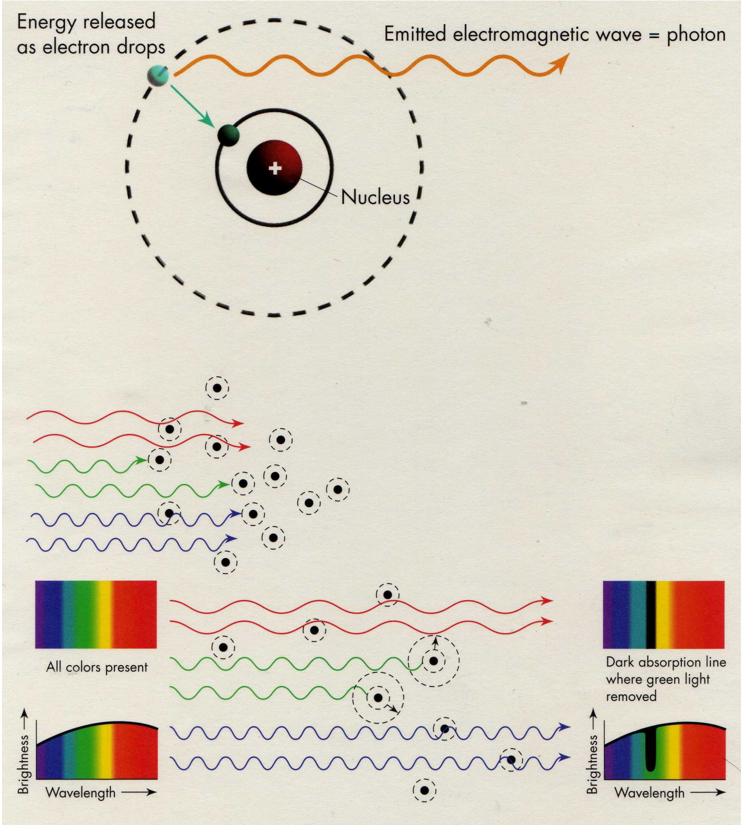
  $E(n) = R' Z^2(1/n^2)$ where $R' = 2.18 \times 10^{-18}$ J

  But since energy is emitted or absorbed when n changes we only care about energy differences and when expressed in terms of wavelength ($E = hc/\lambda$):

  $1/\lambda = RZ^2(1/n^2 - 1/m^2)$ where $m > n$ and $R = 10.96776\ \mu m^{-1}$

Atomic Structure & Spectra

- **Atomic Spectra**
  - Spectrum of Hydrogen
    - Sets of series (n) with names like Lyman (n=1), Balmer (n=2), etc. representing transitions into and out of a specific level. As given by changes in m.
  - Empirically described by Rydberg but now explained theoretically by Bohr.
  - Spectral lines are discrete and correspond to energy emitted or absorbed when electrons “jump” between allowed orbits within an atom.



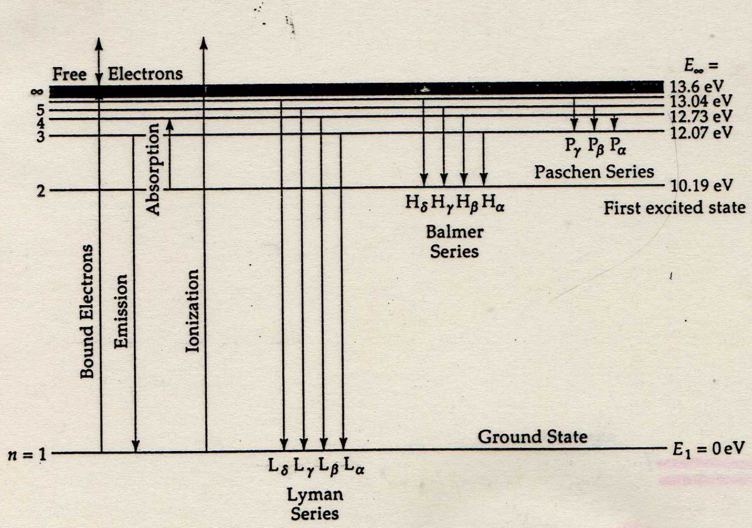
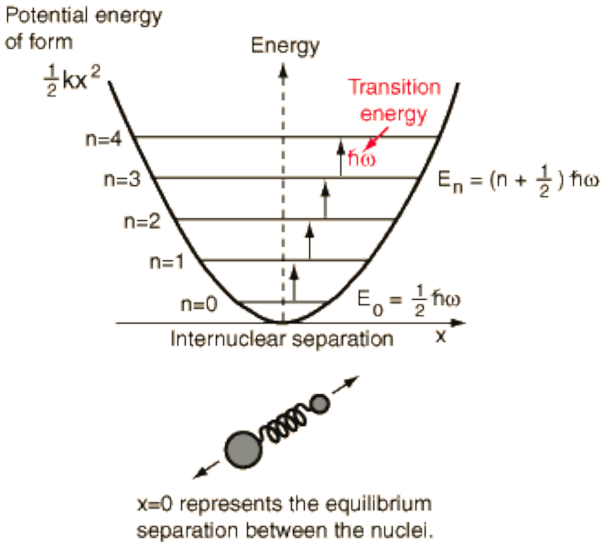


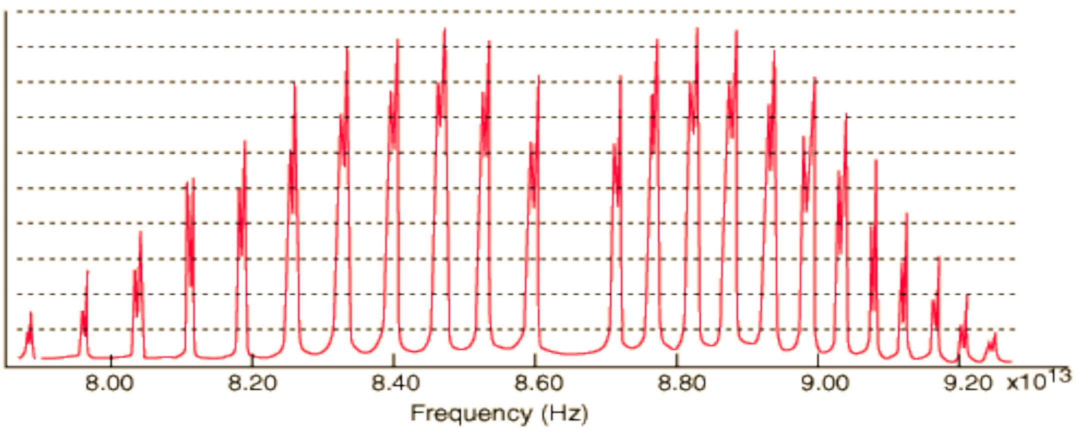
Figure 8–9 Hydrogen energy-level diagram. Transitions for the first three hydrogen series are shown in emission. On the left are shown the general types of transitions for absorption, emission, and ionization. The levels here are drawn to scale.

Atomic Structure & Spectra

- **Molecular Spectra**
  - **Molecules have additional degrees of freedom**
    - **Rotational and vibrational states**
    - **Both effect the energy of the molecule and are associated with emission/absorption of radiation since the electric potential the electrons see changes as atom rotates and vibrates.**
    - Quantum physics is complicated but progress is made via high-performance computing. Lots of empirical lab data is available.



Infrared spectrum of HCl



Atomic Structure & Spectra

- **Blackbody Spectra**
  - **Plank derived the spectrum of a Blackbody by assuming that the “oscillators” are distributed according to a Boltzman distribution and that their energy is quantitized. The result is:**

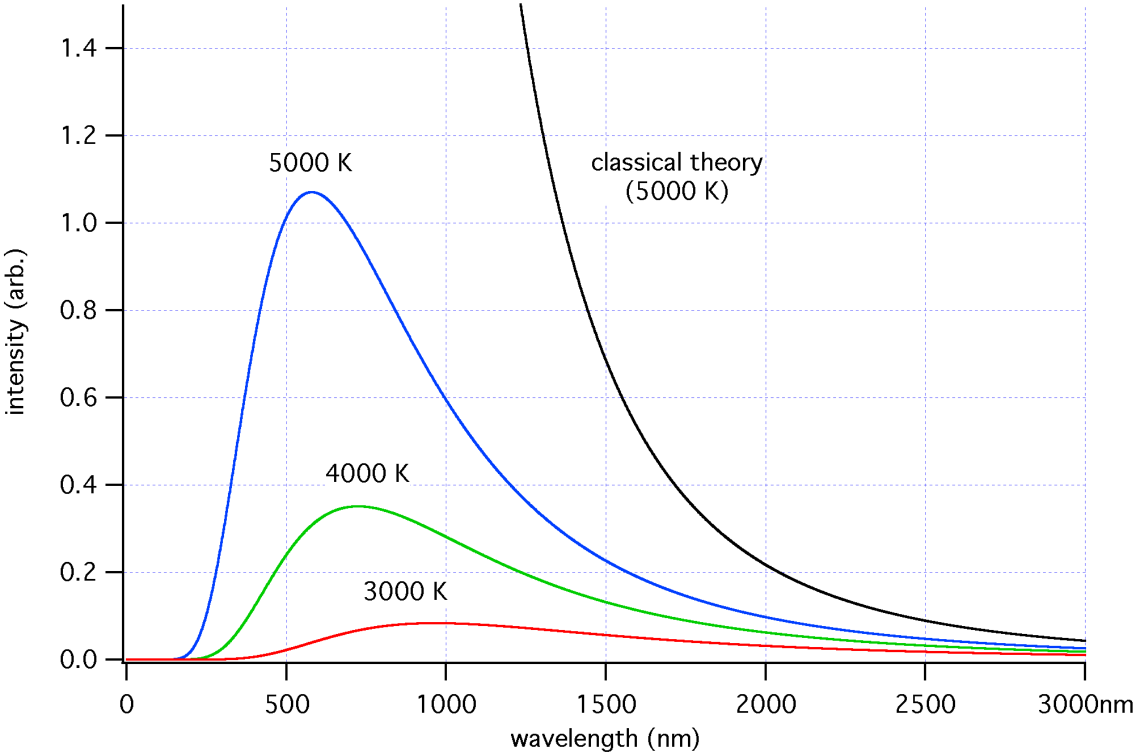
$$E(\lambda,T) = 2hc^2/\lambda^5\ (1/e^{hc/\lambda kT} - 1)$$

Differentiate to find maximum and get Wein’s law:

$$\lambda_{max} = 2898\ \mu m/^{o}K$$

Integrate and we have Stefan-Boltzman law:

$$I(T) = \sigma T^4$$



Radiative Transfer - I

- Intensity vs. Flux
  - Intensity: the energy emitted by a region/surface per unit area. Per unit time, per unit frequency (or per unit wavelength), per unit solid angle (fraction of the sky).
  - Solid angle is the 3-d equivalent to the arc length:
  - $\Delta\Omega = \Delta area/r^2$ where is the fraction of the sky x 4π subtended by an area Δarea at a distance r from the origin.
  - Note that the entire sky subtends 4π steradians and an angle of 1 radian corresponds to a solid angle of 1 steradian.
  - Consider now a particular object which emits some energy spectrum $I(\lambda)$. If we integrate over the entire surface and over the spectrum (λ, or ν), we can compute the luminosity. In the case of a spherical object with uniform $I(\lambda)$:

$$L = (4\pi)4\pi r^2 \ \text{Integral}\ [I(\lambda)]\ d\lambda$$

  - We can now define the flux (F) as the energy per unit time and per unit wavelength which passes through a unit area. If an objects emits uniformly in all directions then:

$$F = L/4\pi r^2 \qquad \text{(the inverse square law)}$$

  - Absorption of Light
- Transfer Equation
- Simple Solutions & the Source Function
- Spectral Line Formation

Radiative Transfer - II

Consider a source emitting a beam with spectrum $I(\lambda)$, which encounters an absorbing medium. In this simple case:

$dI/I = - \chi\, dl$ where χ is the extinction coefficient and describes the absorbing properties density, etc. and dl represents the path length through the medium. Integrating we have:

$I = I_0\, e^{-\tau}$ that is the intensity declines exponentially.

The more general form of this differential equation allows for emission within the medium, not just absorption. This is called the transfer equation:

$dI_\lambda/dl = \eta_\lambda - \chi_\lambda I_\lambda$ where is called the emisssion coefficient and represents the energy emitted by the gas per unit volume, per unit steradian, per unit wavelength, per second. It is customary to consider an observer looking into the source. Thus we introduce a minus sign since now $l = - l$. In addition, we note that any emitting region is seen in projection ($\mu = \cos\theta$) from an arbitrary location. So if we divide through by χ_λ we have:

$\mu dI_\lambda/dl = I_\lambda - S_\lambda$ where S_λ is the ratio of emission to absorption for the gas and is known as the source function.

- See the text for some simple solutions.

Boltzman & Saha Equation

- Thermal Equilibrium
  - Recall that the Boltzman distribution describes the number of atoms or molecules as a function of energy.
    - On average:

      $<mv^2/2> = 3kT/2$
  - Collisions between these can change their *internal* energy
    - Electrons can be excited into upper levels
    - Electrons can be collisionally de-excited as well.
- Boltzman Equation
  - Describes the number of atoms with electrons in a given state according to the energy difference between the excited levels:

    $$N_B/N_A = (g_B/g_A)e^{-(E_A - E_B)/kT}$$

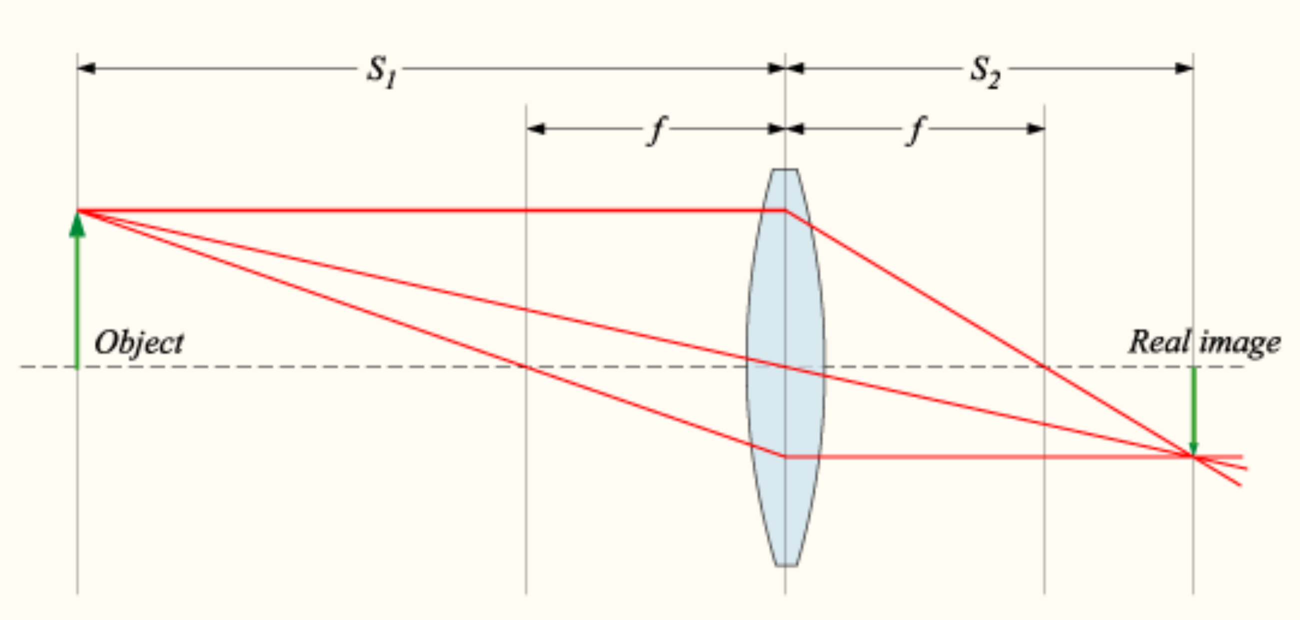
    where the g's are the statistical weights and contain the multiplicity of the levels (statistical weights) and the transition probabllities (quantum mechanics).
- Saha Equation
  - Saha showed that a similar relation exists for atoms with differing ionization states:

$$N_+/N_0 = [(2\pi mkT/h^2)^{3/2} /N_e]B_+/B^0\ e^{-\chi//kT}$$

Where N_e is the number density of electrons. So in the textbook A = $[(2\pi mkT/h^2)^{3/2}$

Simple Optics – positive (convex) lenses

Simple Optical Components Behave According to the Laws of Reflection and Refraction:



Positively curved (convex) lenses refract light to form an image by bending the light to form an image. The focal length (f) is the image distance (s2) for an object located at infinity. For any other locations the thin lens equation applies:

$$1/s_1 + 1/s_2 = 1/f$$

Simple Optics – negative (concave) lenses

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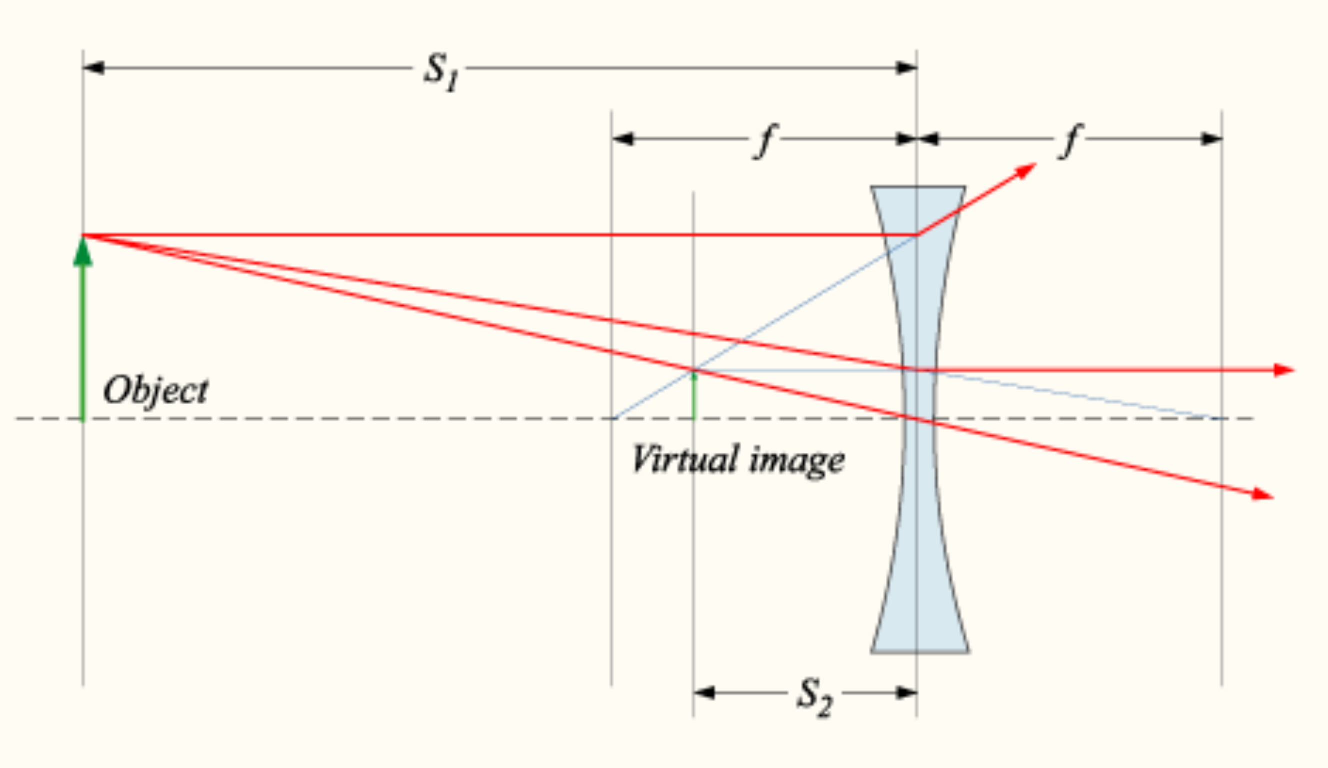


Negatively curved (concave) lenses refract light to form a virtual image. The thin lens equation still applies but f is negative:

$$1/s_1 + 1/s_2 = 1/f$$

Chapter 6: Astronomical Detection of Light

- Telescope as a Camera
  - Telescope objective (lens or concave mirror)
    - Images like a camera (thin lens equation with object at ~infinity)
    - We don't (in general) know object's distance so we speak of its angular extent
    - Size of the image depends on the focal length of objective

      $s = f\theta$ where θ is angular extent in radians, f is the focal length (mm) and s is object's linear size within the image (eg., mm). The plate scale (radians/mm) is just 1/f and multiplying by 206265 gives scale in arcsec/mm.
    - Larger objective (D) collects more light: $\sim D^2$
    - Maximum resolution is limited by diffraction ($\theta_{min} = 1.22\ \lambda/D$)

      or $\theta_{min} = 206265\ \lambda/D$
  - Detectors
    - Response to light depends on:
      - collecting area of the objective (bigger telescope collects more light) $\sim D^2$
      - the source brightness (brighter objects emit more light),
      - Detector quantum efficiency, and
      - the exposure time (longer detector is exposed, the more light it collects)

    - Detector size limits size of image
      - Since image scale is arcsec/mm the field of view FOV = scale x detector size
      - Pixel size determines image sampling

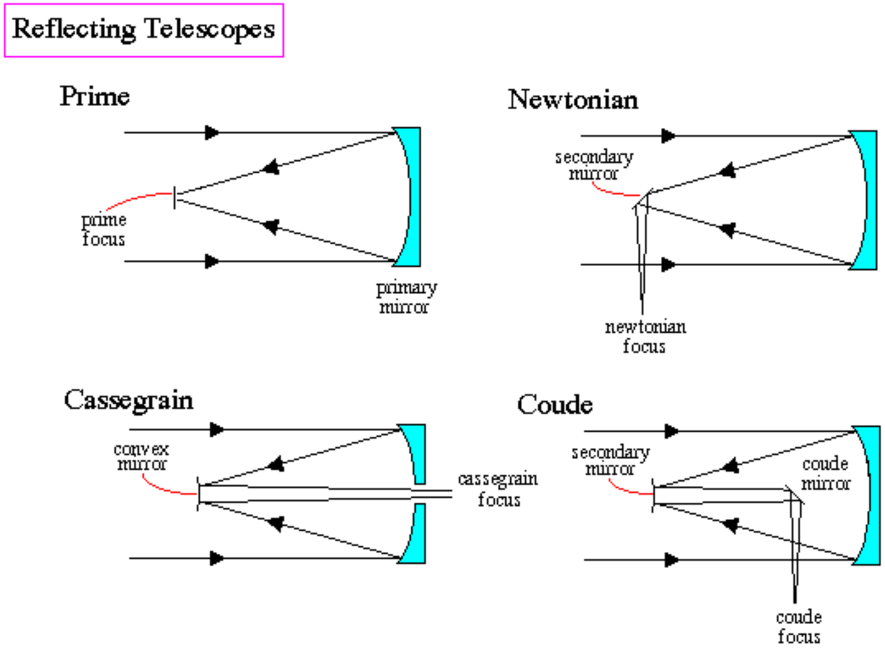
Astronomical Detection of Light - II

- **Refracting Telescopes**

  The refracting telescope forms an image using a lens. Inexpensive telescopes but research refractors are of historical interest only. Requirement of edge support limits lenses to about 1-meter. Still used within instruments but chromatic aberration (dependence of focal length on wavelength) requires multiple lenses for use over broad wavelengths.

- **Reflecting Telescopes**

  All modern research telescopes use a mirror to collect light. Several different types (see



Optical Aberrations and Image Quality

- **Spherical Aberration**
  - **Spherical surfaces are easy to make but don’t produce the best images**
  - **Why concave telescope mirrors are usually parabolic.**
- **Coma**
  - **Off-axis images are “fuzzy” since outer rays don’t strike lens at same angle.**
- **Astigmatism**
  - **Off-axis images images “violate symmetry” of lens so rays in different planes don’t focus in the same plane.**
  - **Curvature of Field**
  - **Focal plane should lie on a spherical surface (radius = lens focal length) so a flat focal plane results in a distortion of the image (magnification is a function of position)**
- **Chromatic Aberration**
  - **Index of refraction of glasses depends on the wavelength**
  - **Focal length of a single lens depends on wavelength**
  - **Two element achromatic lens used in refractors corrects to some extent.**

Physics of Optical Detection

- **Photon Nature of Light**
  - **Light waves are finite in extent (photons)**
  - **Energy of photon: Eγ = hν**

    (where h = 6.626 x 10^{-34} J seconds and ν = photon frequency)
  - **Detection of photons is discrete (you can't detect half a photon)**
  - **Signal flux (watts/m^2 /μ) from a source of photons (f_ϖ) is "grainy" so follows Poisson's counting statistics.**
- **Signal to Noise of Detection (text: 6.5)**
  - **Photon noise from source itself**
  - **Detector read-out noise**
  - **Sky background noise**
  - **Resulting Signal-to-Noise Ratio (CCD equation)**
  - **To reach a given S/N ratio you solve for the number of photon you need to detect (μ_st) given the various noise sources and then the exposure time (t). If the exposure time is impractically large you need a bigger telescope!**

Binomial theorm can be used to derive Poisson distribution:

$P(x,\mu)=\frac{\mu}{x!}e^{-\mu}$ (probability of detecting x photons in a given time interval if μ is the average measured over large amount of time (see text for proof).

The width, or spread of the distribution measures how x varies with multiple sets of observations, i.e., the deviation (σ) from μ. Specifically, 66% of the time x will be within $\mu \pm \sigma$ with:

$$\sigma = \sqrt{\mu}$$

So a photon source will have an uncertainty (photon noise) of $\pm \mu_s^{1/2}$

There is often a sky background signal that needs to be subtracted but there is still a photon noise associated with the sky flux:

$$\sigma_b = \sqrt{\mu_b}$$

The detector usually has an electronic read noise (σ_r in electrons). The noise deviates either positive or negative from μ so the total is:

$$\sigma_T^2 = \sigma_s^2 + \sigma_b^2 + \sigma_r^2$$

The CCD equation gives the signal-to-noise ratio of a detection:

$$\left(\frac{S}{N}\right)_s = \frac{\mu_s t}{\left(\sigma_s^2 + \sigma_b^2 + \sigma_r^2\right)}$$

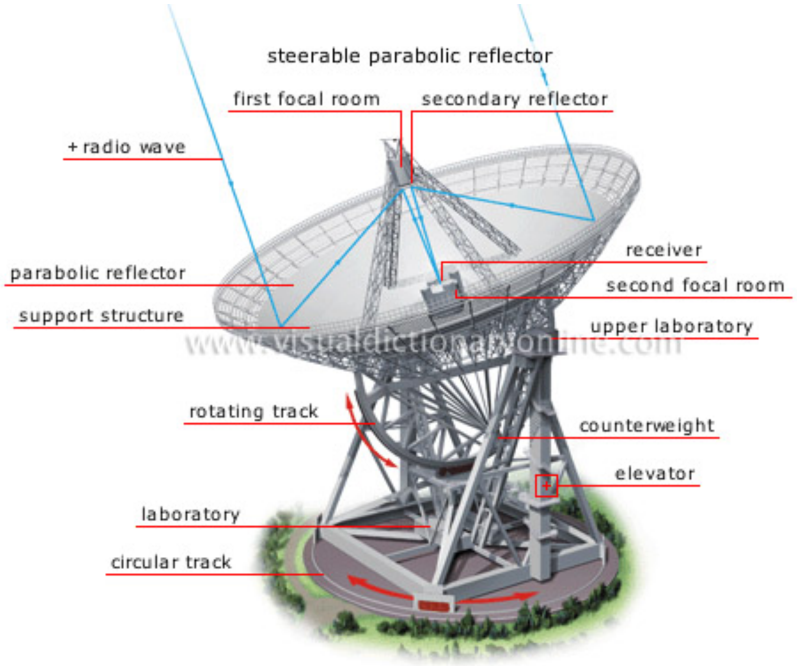
where t is the exposure time. The source signal (μ_s) depends on its intrinsic flux (f_λ), the telescope aperture (D), and detector efficiency(ε_λ). Specifically:

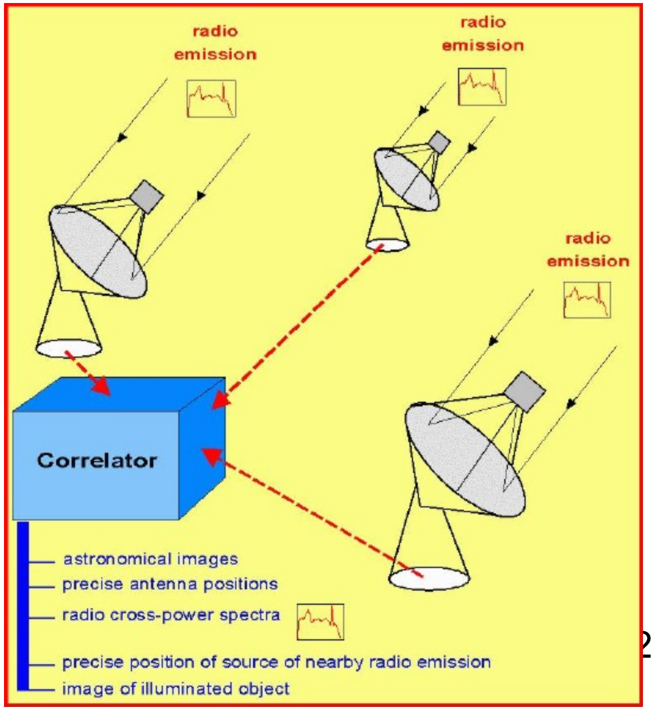
$$\mu_s = 4\pi D^2 f_\lambda \varepsilon_\lambda$$

Astronomy at Other Wavelengths

- **Ground-based Radio Astronomy**
  - Radio atm. window allows ground-based radio astronomy
    - Radio technology is well-developed
    - Diffraction limit of largest radio telescopes is huge
      - Consider a 300-meter dish at $\lambda = 10$ cm

        $\theta_{min} = \lambda/D = 0{,}1/300 = 3.33 \times 10^{-4}$ radian = 1 arcmin
    - Radio Interferometry (aperture synthesis)
      - Use signal delay between multiple telescopes to simulate a bigger aperture



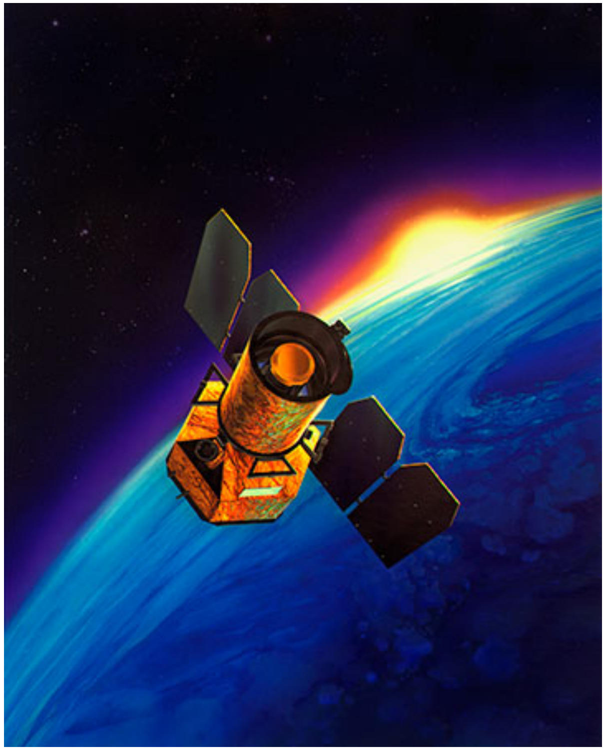


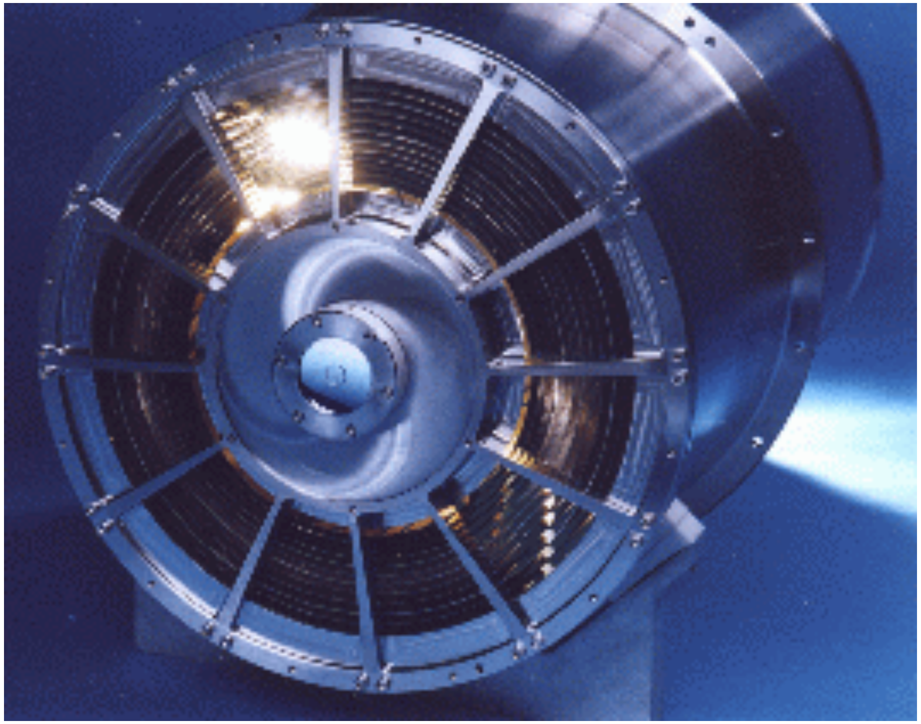
Ground-based and Space-based Infrared Astronomy

- Infrared Windows (between the water absorption bands) allows ground-based infrared astronomy
  - Lower extinction in the infrared
    - Star forming regions
    - Center of the Galaxy
  - Cool stars and dust
  - Redshift of distant objects in the expanding universe
    - Visible light redshifted into infrared
- Longer Wavelength Infrared must be Observed from Space

Space-based Ultraviolet and X-ray Astronomy

- Earth’s atmosphere absorbs ultraviolet and x-ray photons
  - Ultraviolet telescopes use conventional technology (e.g., GALAX)
    - Hottest stars emit in UV
    - Accreting gas within interacting binary stars
    - Quasars and other active galactic nuclei
    - X-ray telescopes require grazing incidence reflecting optics
  - X-ray telescopes require grazing incidence reflecting optics to focus light (grazing light doesn’t penetrate the mirror)



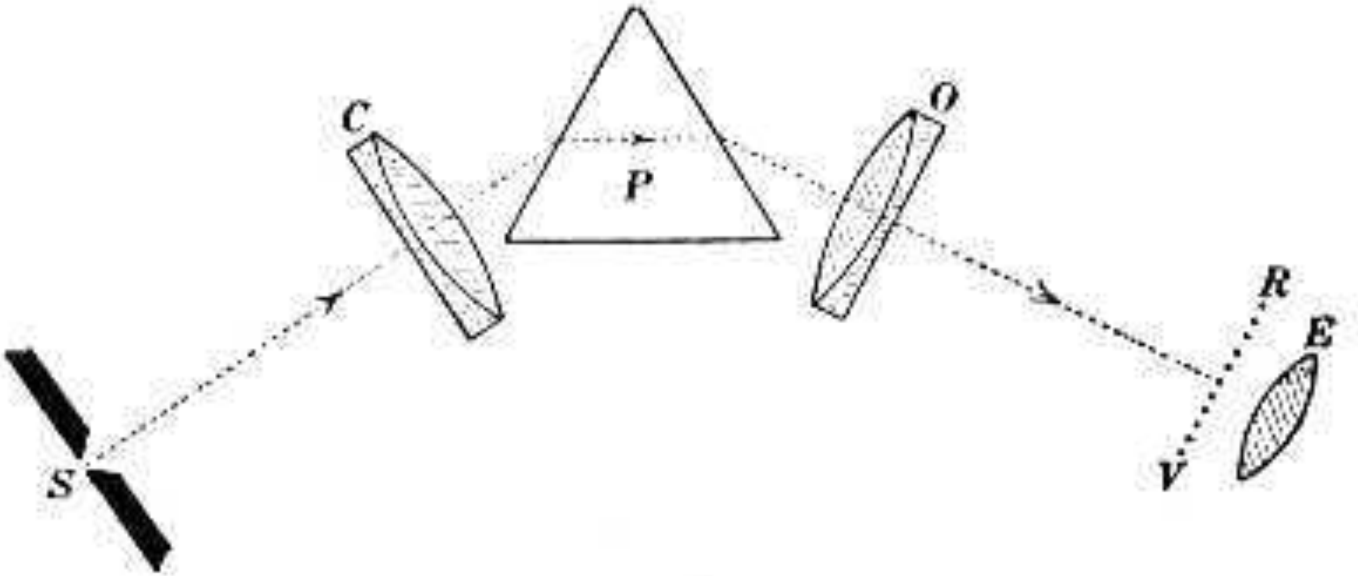


Pan-Starrs Camera

Largest mosaic under development is the 1.4 G pixel Pan-Starrs Camera

Spectroscopy (conceptual)

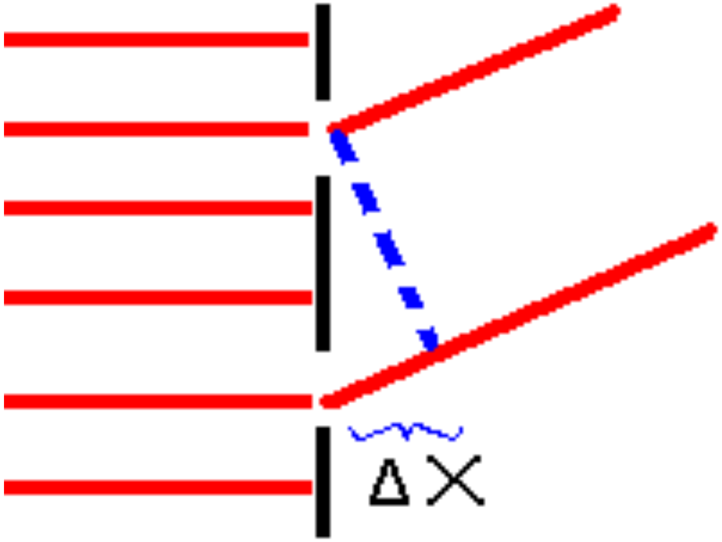
- Prism Spectrograph
  - Dispersion of light by a prism can be used to make a low resolution spectrometer
  - Slit isolates region of telescope's image
  - First lens makes light parallel (collimated)
  - Prism disperses light by color (refraction changes angle according to wavelength)
  - Second lens images slit onto a focal plane but at different positions according to wavelength (a spectrum)

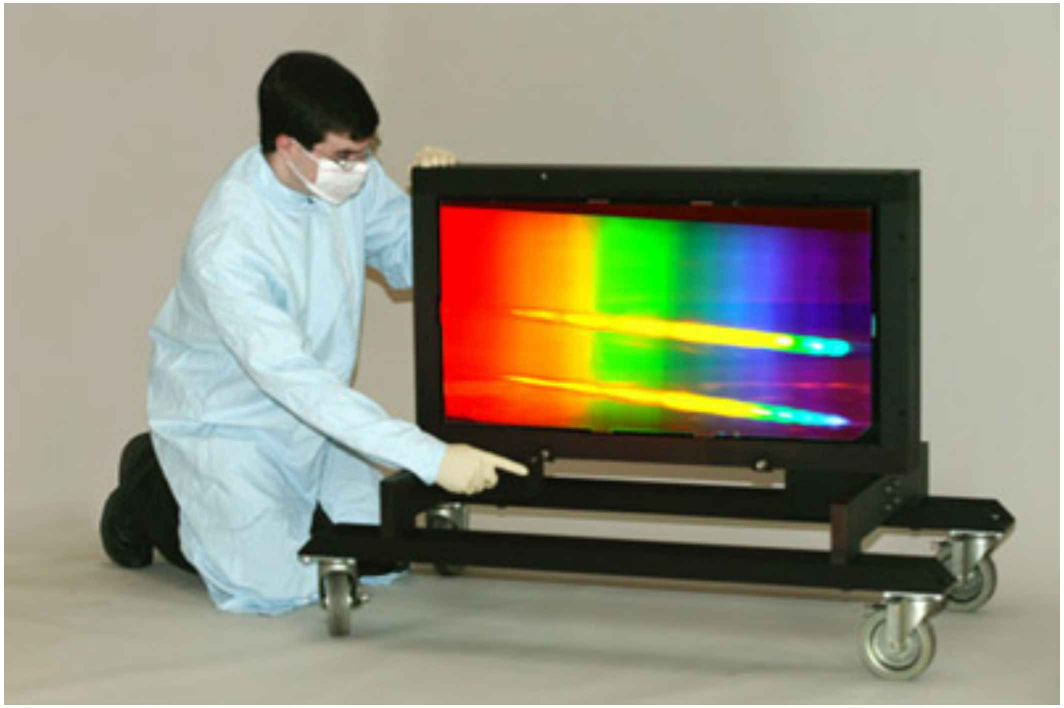


Diffraction Grating

- Parallel groves act like multiple slits
  - Reflected light interferes constructively when path difference is an integer number of wavelengths.
  - Parallel light incident on surface reflects and interferes with itself
    - Angle of reflected light depends systematically with wavelength

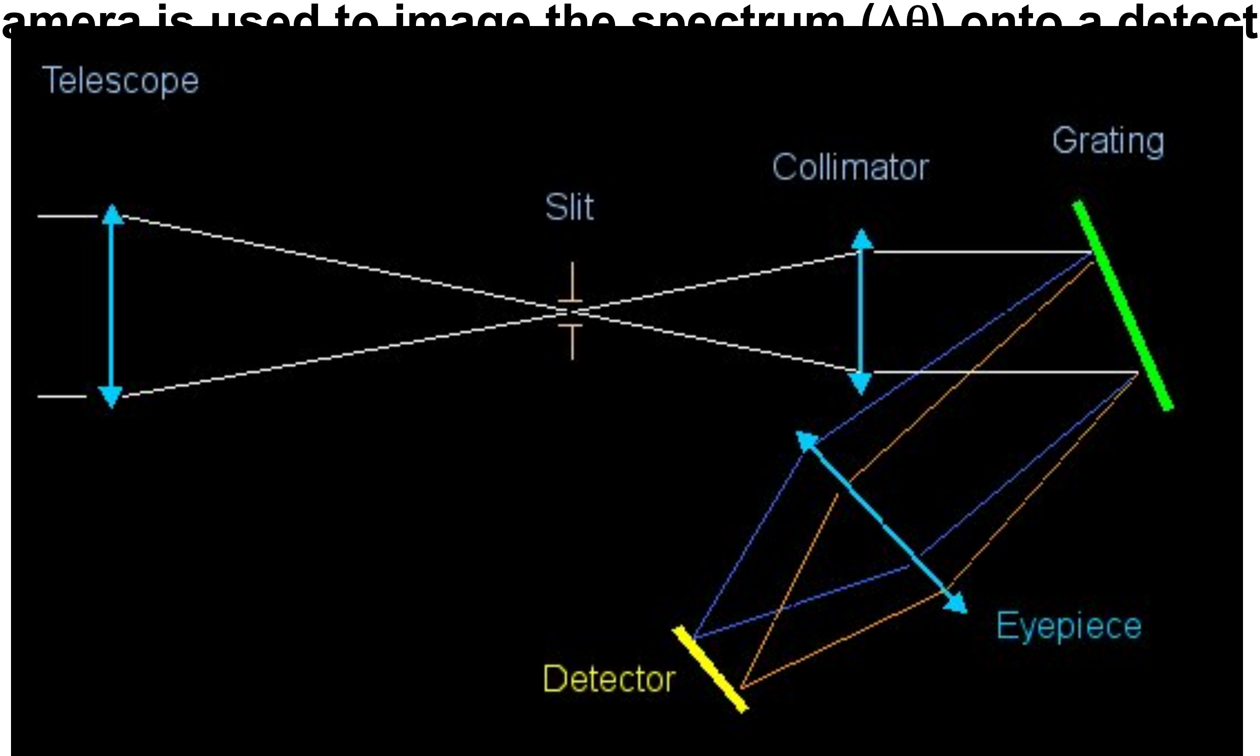
      $n\lambda = d \sin \theta$ where n is the order (1, 2, ..), λ is the wavelength, and θ is angle





Grating Spectrometer

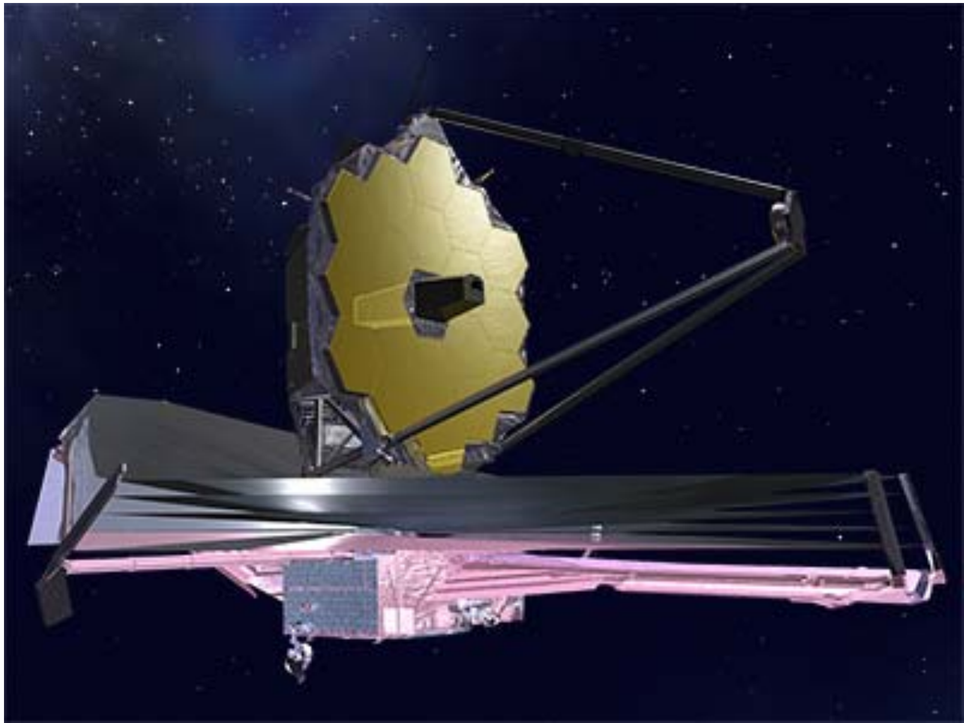
- **Grating spectrometers offer more versatility than prism spectrographs and are now standard. Note the grating (reflection or transmission) replaces prism.**
- **A sit is used to isolate a position in the telescope's image plane**
- **A collimating lens is used to form parallel (collimated) light (all light strikes grating at same angle)**
- **The collimated light reflects from the grating with an angle that is a function of wavelength (grating equation: mλ = dsinθ)**
- **A camera is used to image the spectrum ($\Delta\theta$) onto a detector (s = fθ)**



or camera lens

Next Generation Telescopes

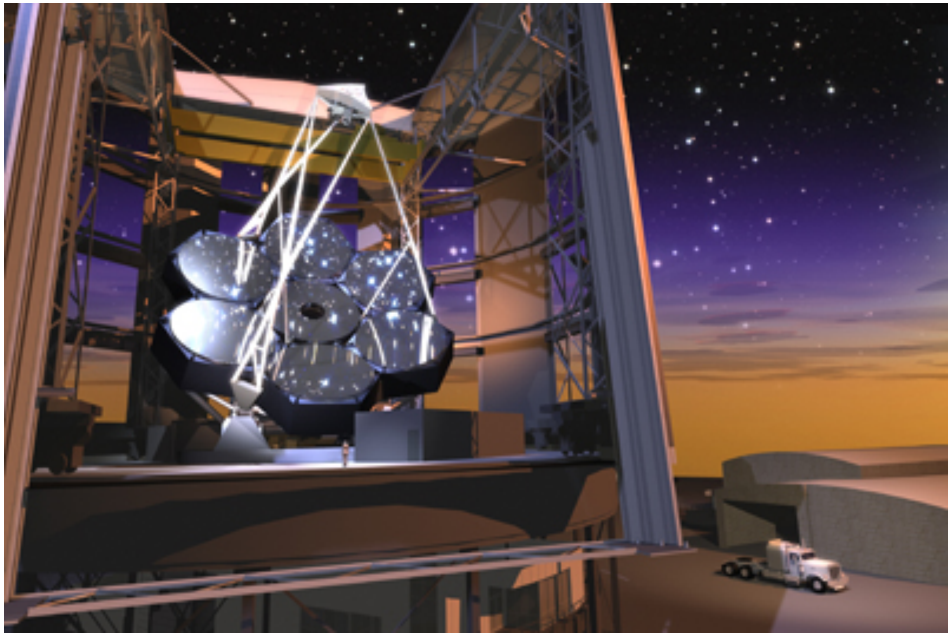
- Hubble Space Telescope
  - Though old, Hubble is the first large space-based telescope
- James Webb Space Telescope
  - The next generation of space-based telescopes (3 times Hubble)
- Next Generation Ground-based Telescopes
  - Ground-based telescopes will use adaptive optics to achieve diffraction limited images



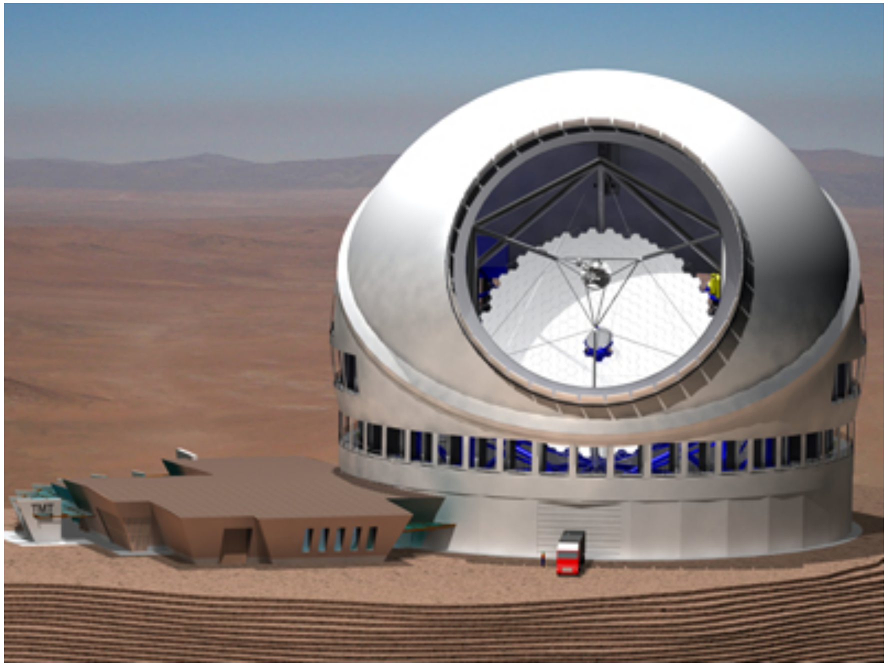
Next Generation Ground-based Telescopes

- Ground-based telescopes will use adaptive optics to achieve diffraction limited images
- Can be much larger than space-based telescopes
- Primary goal is too characterize:
  - extrasolar planets,
  - star formation within our galaxy, and
  - the evolution of the universe

Giant Magellan 24-meter Telescope



Thirty Meter Telescope



Chapter 5: Homework

Chapter 5: #1, 2, 3, 4, 5, 6, 7, 8

- Due Tuesday March 8