PHYS 1220, Engineering Physics, Chapter 31 – Electromagnetic Wave

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Goal of this chapter is to learn the nature of Electromagnetic wave and the relations to the Maxwell's equations.

- From Ampere's Law and Faraday's law in free space:

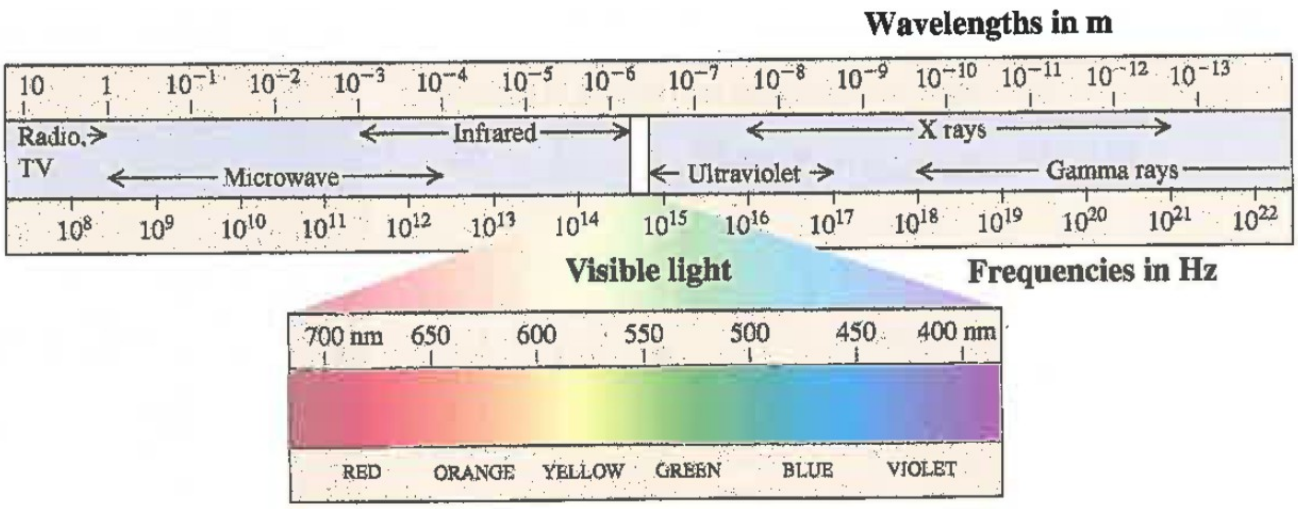
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere's Law)

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

- Electric and magnetic fields are mutually induced. An electric field wave will induce a magnetic wave and vice versa. This is the physics behind the E&M wave.

- The electromagnetic wave (E&M wave) is a wave that is composed of electric and magnetic waves (simultaneously).

- E&M wave has other names (in the order of wave length from short to long):
  - **Gamma ray**
  - **X-ray**
  - **Ultraviolet**
  - **(visible) light** (when the wavelength is in the range of ~400 nm to ~700 nm).
  - **Infrared**
  - **Microwave**
  - **Radio wave**



- Let's look at the Maxwell's equations in free space (no charge and no current):

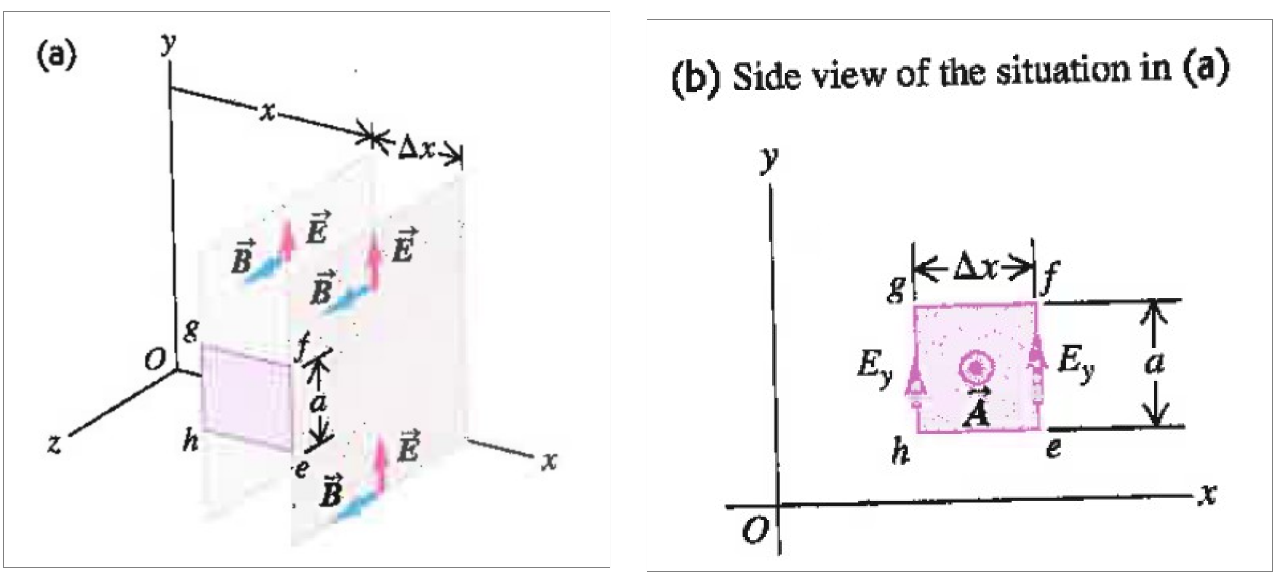
$\oint \vec{E}\cdot d\vec{A}=0$ (Gauss's Law for $\vec{E}$)

$\oint \vec{B}\cdot d\vec{A}=0$ (Gauss's Law for $\vec{B}$)

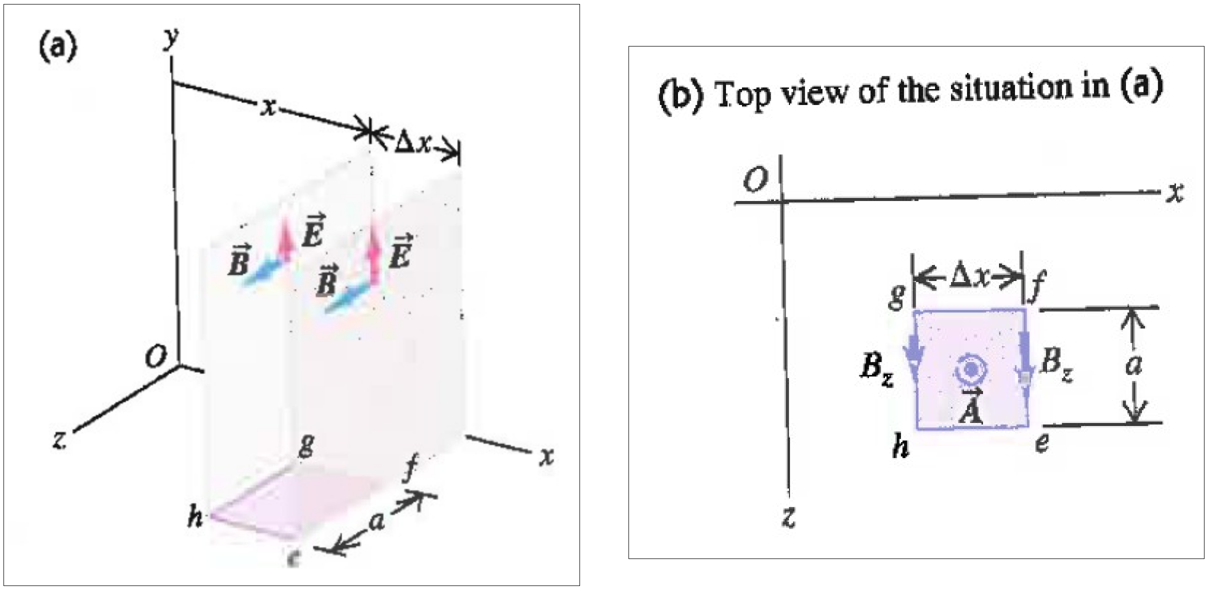
$\oint \vec{B}\cdot d\vec{l}=\mu_0\epsilon_0\frac{d\Phi_E}{dt}$ (Ampere's Law)

$\oint \vec{E}\cdot d\vec{l}=-\frac{d\Phi_B}{dt}$ (Faraday's Law)

- Start with Faraday's law, it could reach: $\frac{\partial E_y(x,t)}{\partial x}=\frac{-\partial B_z(x,t)}{\partial t}$



- Start with Ampere's law, it could reach: $\frac{-\partial B_z(x,t)}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y(x,t)}{\partial t}$



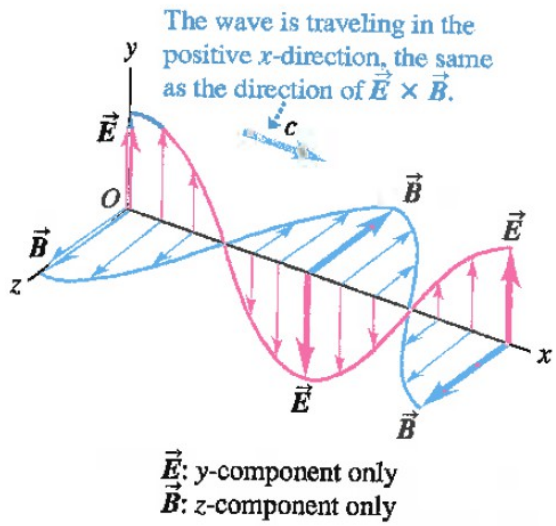
- Thus: $\frac{\partial^2 E_y(x,t)}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$
- Note: in general, wave function is: $\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2}$
- So: $v = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ (speed of light in vacuum)
- and: $v = \frac{1}{\sqrt{\epsilon \mu}}$ (speed of light in material)
- In Optics: $n = \frac{c}{v} = \frac{\sqrt{\epsilon \mu}}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{K K_m}$ (index of refraction equal to the ratio of speed of light in vacuum and in media) (K and K_m are relative permittivity and relative permeability of the material)

- Sinusoidal E&M Wave
  - $\vec{E}(x,t) = \hat{j} E_{max} \cos(kx - \omega t)$
  - $\vec{B}(x,t) = \hat{k} B_{max} \cos(kx - \omega t)$
  - from $\frac{\partial E_y(x,t)}{\partial x} = \frac{-\partial B_z(x,t)}{\partial t}$, we know that:

    $kE_{max} \sin(kx - \omega t) = \omega B_{max} \sin(kx - \omega t)$, hence: $E_{max} = c B_{max}$

- Notes for E&M wave:
  - The directions of electric field and magnetic field in an E&M wave are mutually perpendicular.

- The speed of the E&M wave is the speed of light: $c = 3\times 10^{8}\, m/s$
- The wave propagation direction is determined by $\vec{c} = \vec{E}\times\vec{B}$



- Energy propagation by E&M wave
  - energy density in fields (both electric and magnetic):
    $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 B^2 = \epsilon_0 E^2$
  - Poynting vector: $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ "how much energy is propagated at which direction by E&M wave per unit area per time (this is just like the definition of light intensity or light energy flux)"
  - Derivation: $|\vec{S}| = \frac{1}{A}\frac{dU}{dt} = \frac{1}{A}\frac{u\,dV}{dt} = \frac{1}{A}\frac{u\,A\,c\,dt}{dt} = uc = \epsilon_0 c\,E^2 = \epsilon_0 c^2 E\,B = \frac{1}{\mu_0} E\,B$

Math Preview for Chapter 17:

- Derivative

Questions to think about for Chapter 17:

- What is "heat"? What is "temperature"? What is the relationship between heat and temperature? How do we measure heat or temperature?