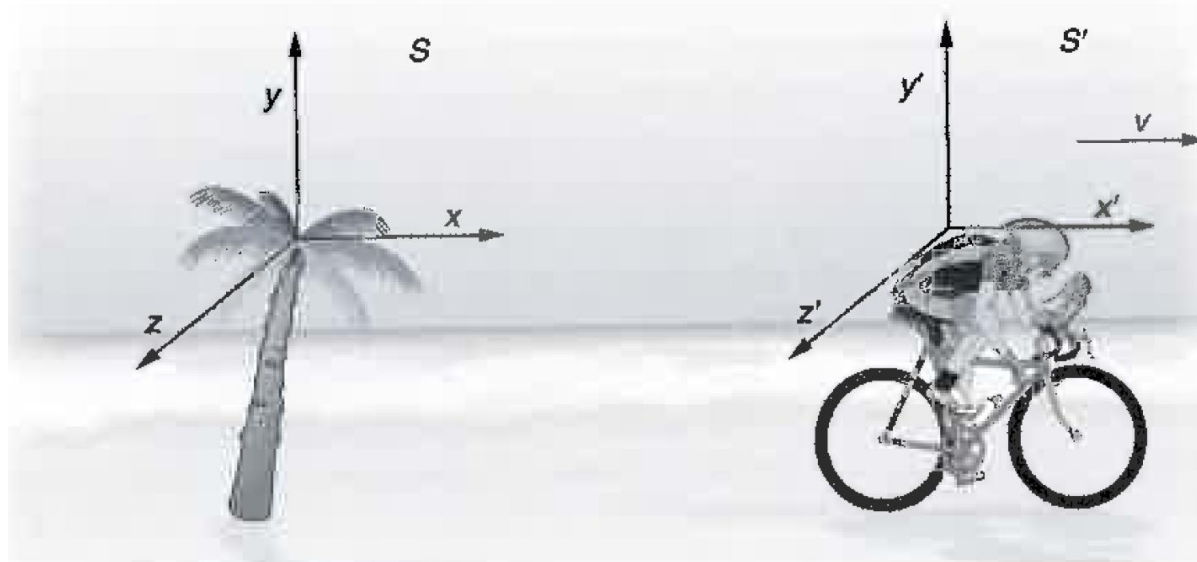# Chapter 1

## Relativity 1

# Classical Relativity – inertial vs noninertial reference frames



Inertial Reference Frames

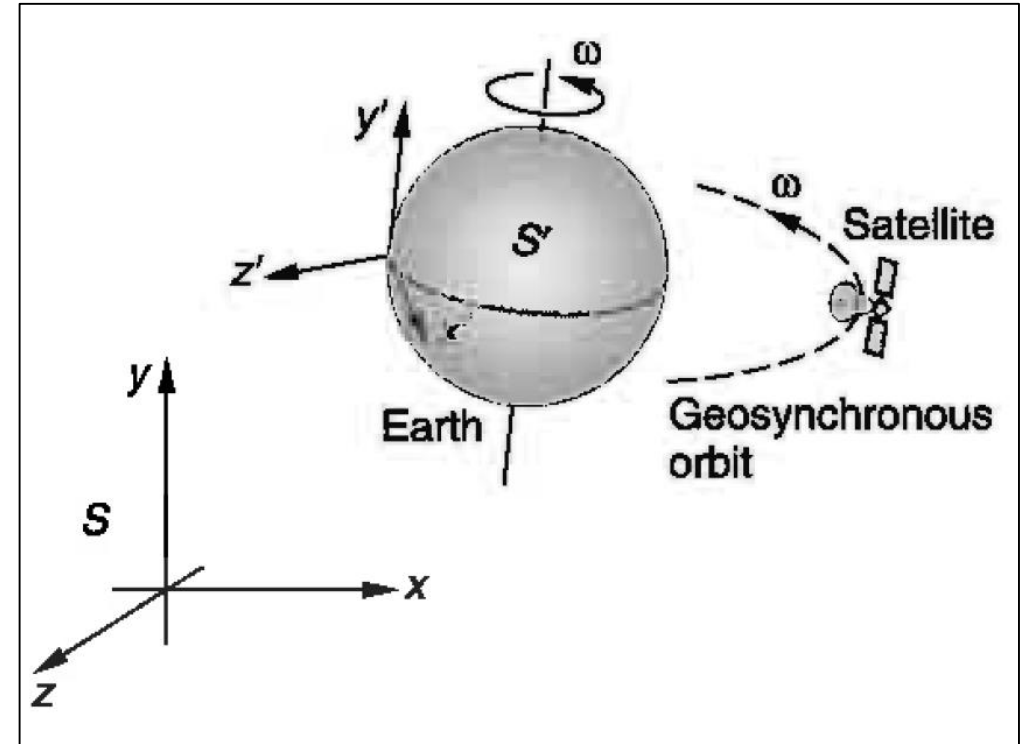Galilean transformation:   $x' = x - vt; y' = y; z' = z; t' = t$

$$u'_x = u_x - v; u'_y = u_y; u'_z = u_z$$

Any reference frame that moves at constant velocity with respect to an inertial frame is also an inertial frame. Newton's laws of mechanics are invariant in all reference systems connected by Galilean transformation.

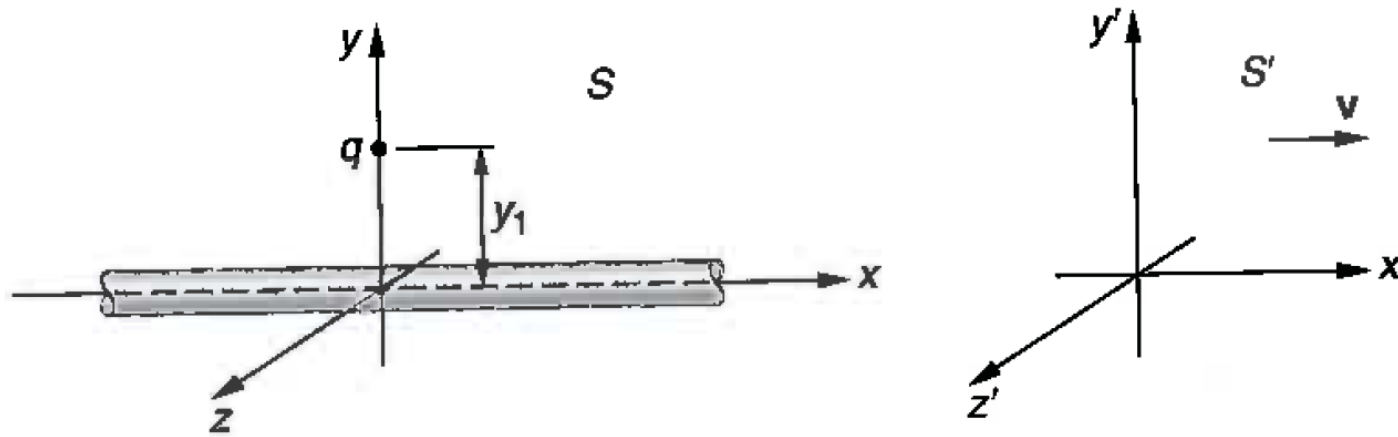# Classical Relativity – inertial vs noninertial reference frames
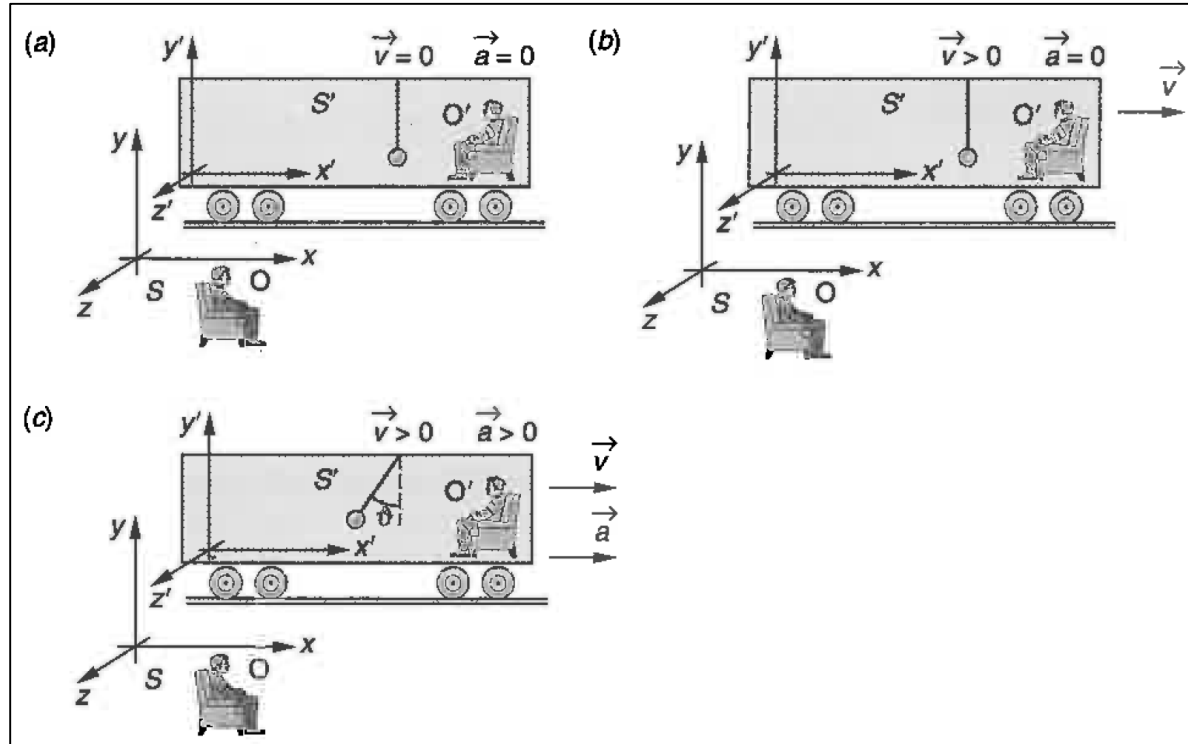


$$F' \neq F$$



$$F' \neq F$$

# Maxwell Eqs.



$$F = qE = \frac{q2k\lambda}{y_1}$$

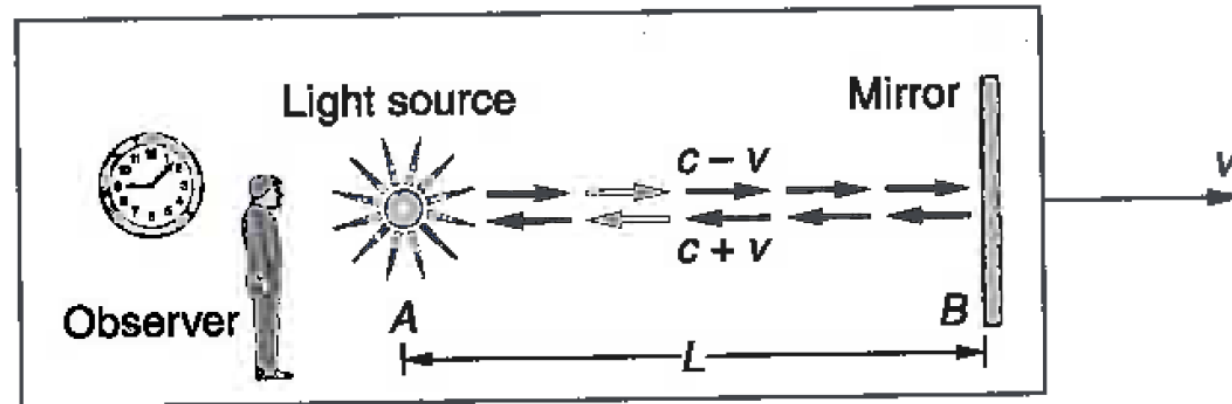$$F' = qE + q\vec{v} \times \vec{B} = \frac{q2k\lambda}{y_1} - \frac{\mu_0 \lambda v^2 q}{2\pi y_1}$$

Inertial Reference Frames, but... $\quad F' \neq F$

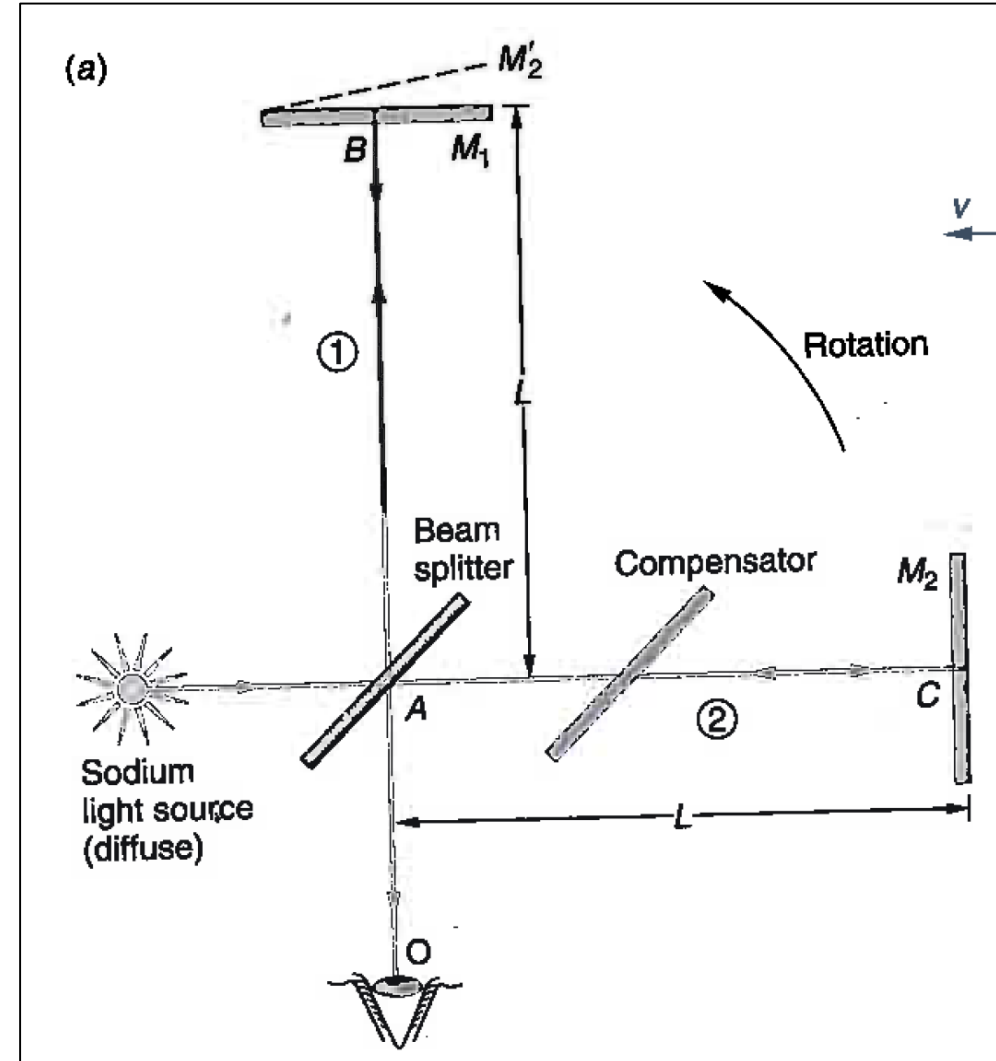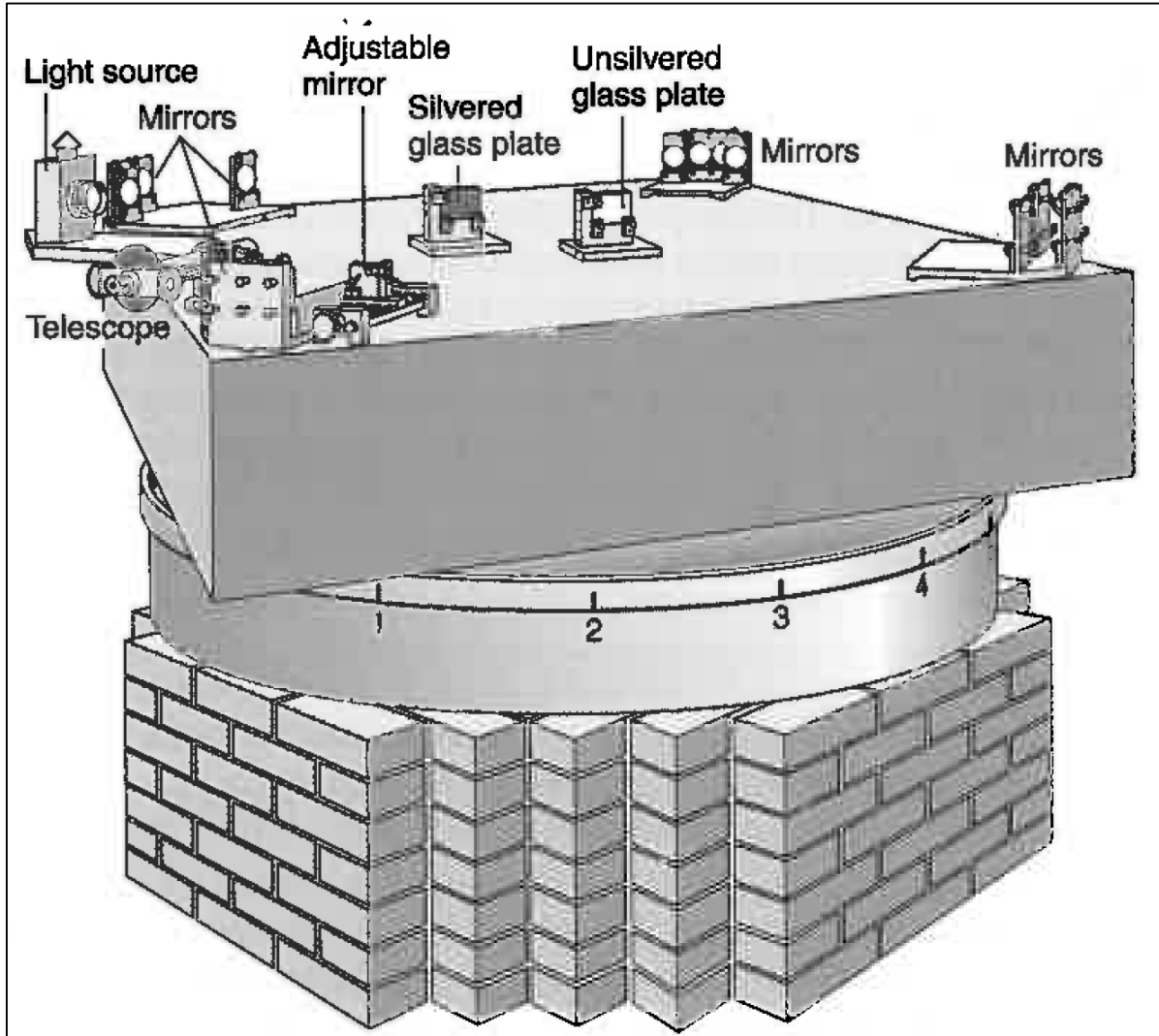➔Maxwell Eqs. Do not follow Galilean transformation

# E&M wave (light) and Maxwell Eqs.

Light is E&M wave, well understood by Maxwell Eqs., has a velocity of light: $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 m/s$
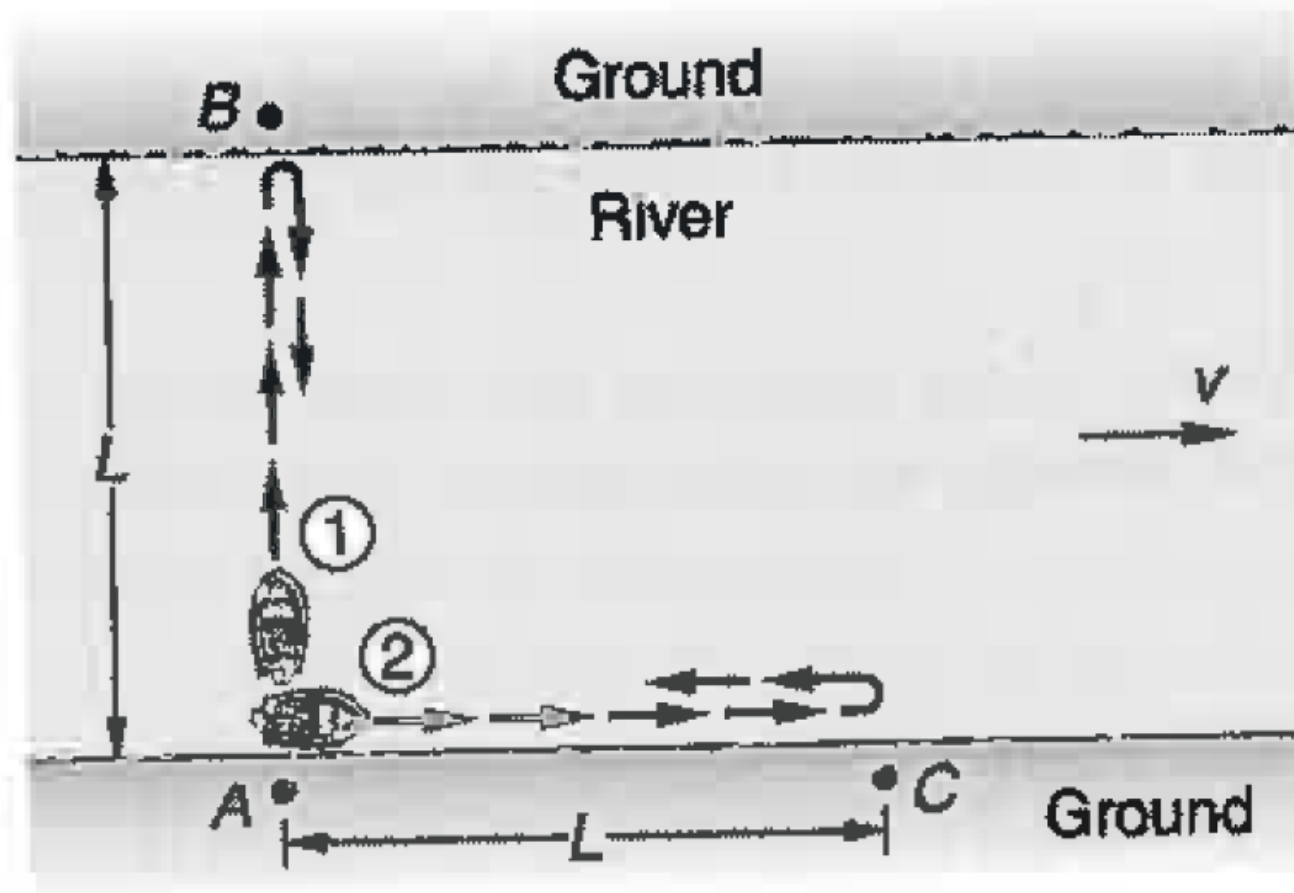
In 19$^{th}$ century, all waves are understood as needing media to propagate: ether (media of E&M wave)

# Michelson-Morley Experiment

# How to understand the experiment

# Michelson-Morley Experiment - Meaning

- If ether exist, the relative velocity of the earth and the ether has a upper limit of 5 km/s (Michelson-Morley in 1887); or 1.5 km/s (Georg in 1930); and 15 m/s (recently)

- The speed of light (E&M wave) is the same in all inertial reference system.

- This implies, there must be some relativity principle that apply to E&M as well as to mechanics. This principle should be converged back to Galilean transformation in certain conditions.
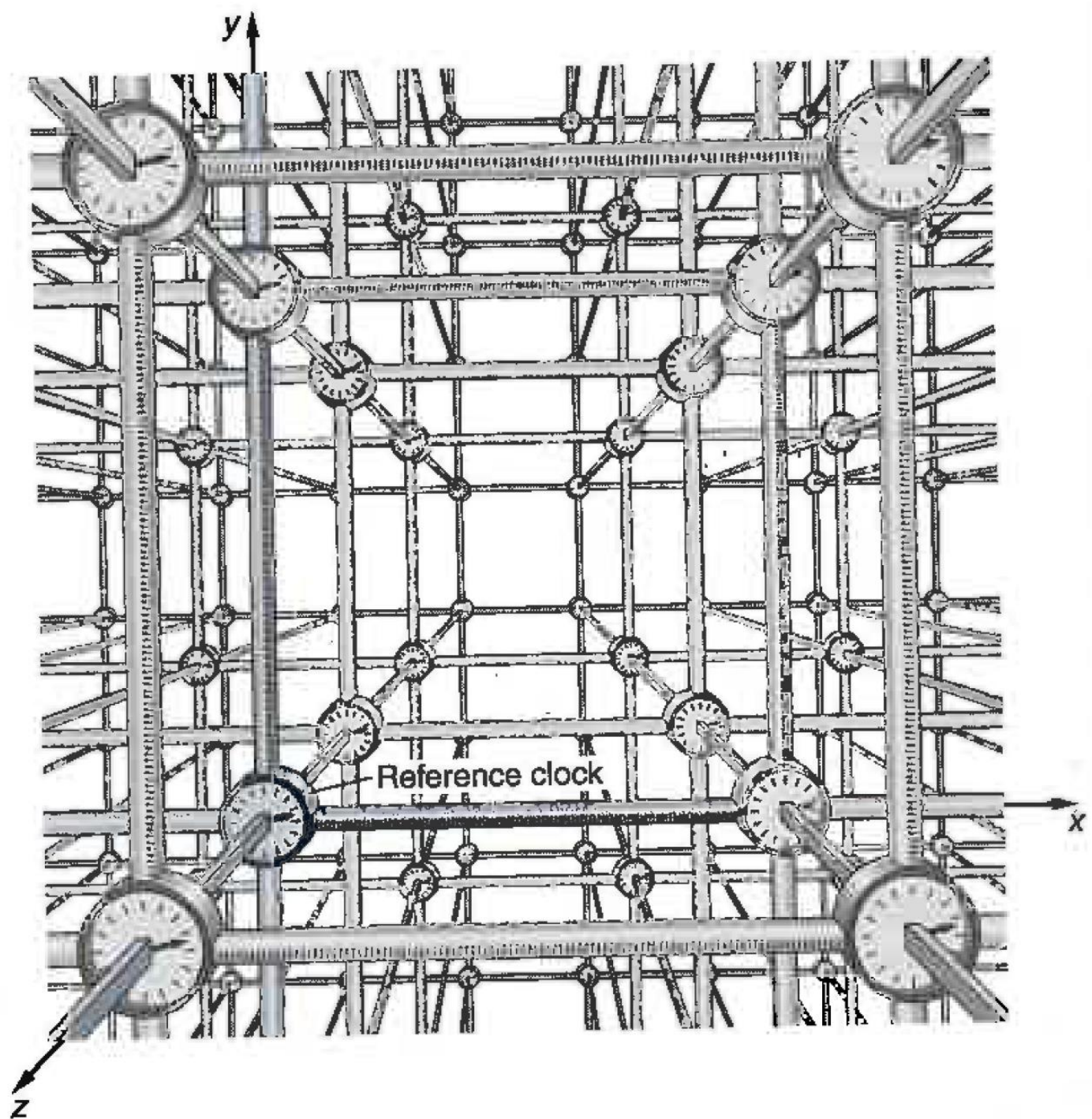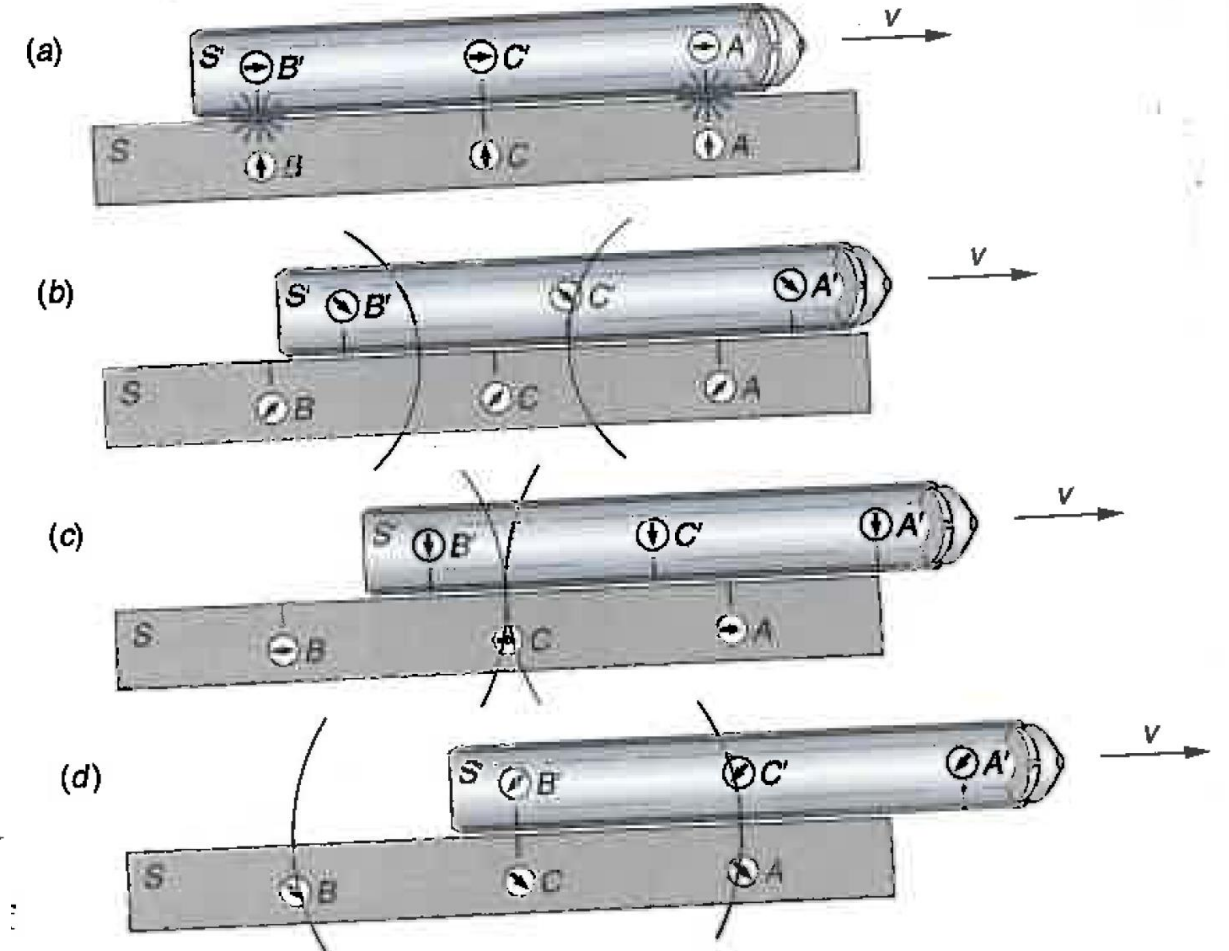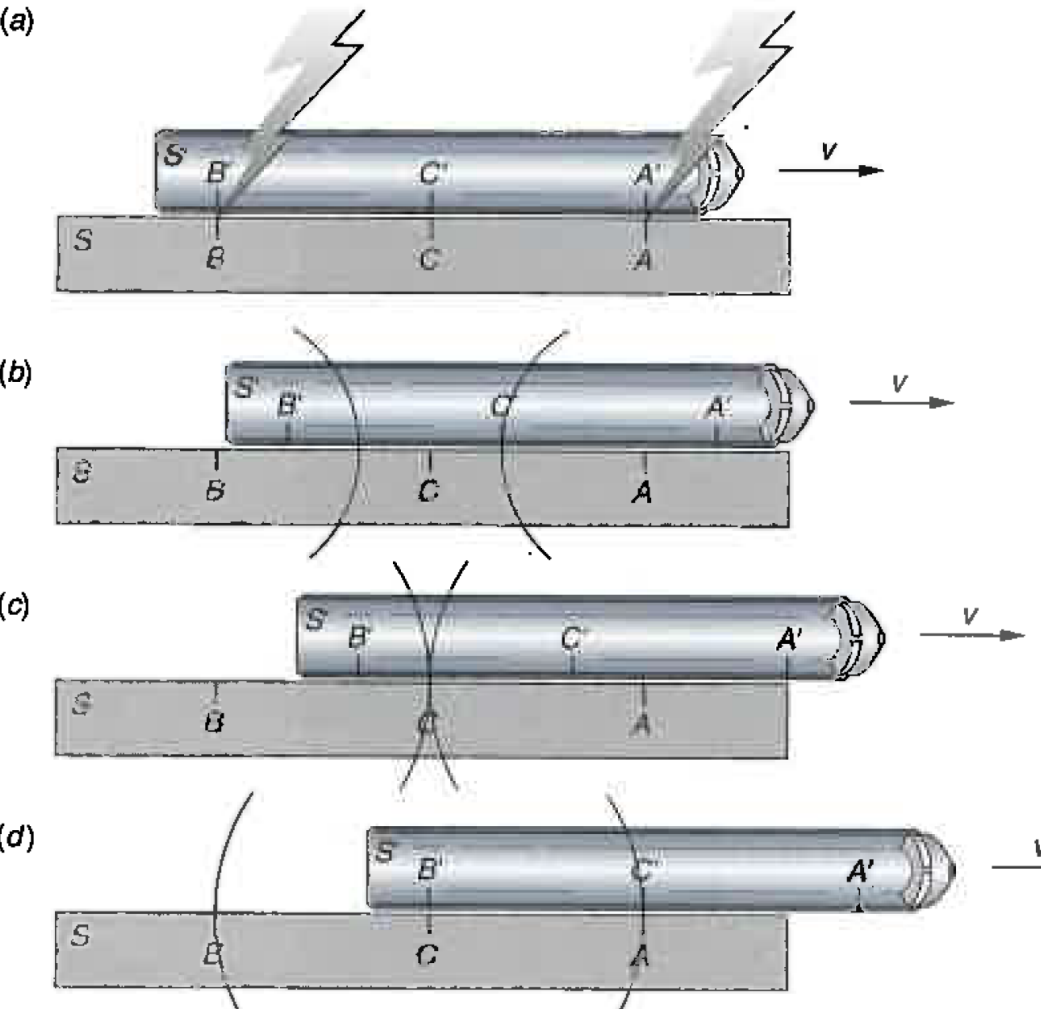
# Einstein's Postulates

- Postulate 1: The laws of physics are the same in all inertial reference frames.

- Postulate 2: The speed of light in a vacuum is equal to the value c, independent of the motion of the source.

- Fact: Speed of light (no media) is independent of the inertial reference frames.

# Events and Observers

- Events: Physical Event is something that happens.

- Observers: Someone or something that see/detect the events in a certain inertial reference frame.

- Information needs time to propagate. What is "simultaneity"?
  - *The spatially separated events simultaneous in one reference frame are not, in general, simultaneous in another inertial frame moving relative to the first.*
  - *Clocks synchronized in one reference frame are not, in general, synchronized in another inertial frame moving relative to the first.*

Reference clock

# Thought Experiments

# Lorentz transformation

Galilean transformation:

$$x' = x - vt; y' = y; z' = z; t' = t$$
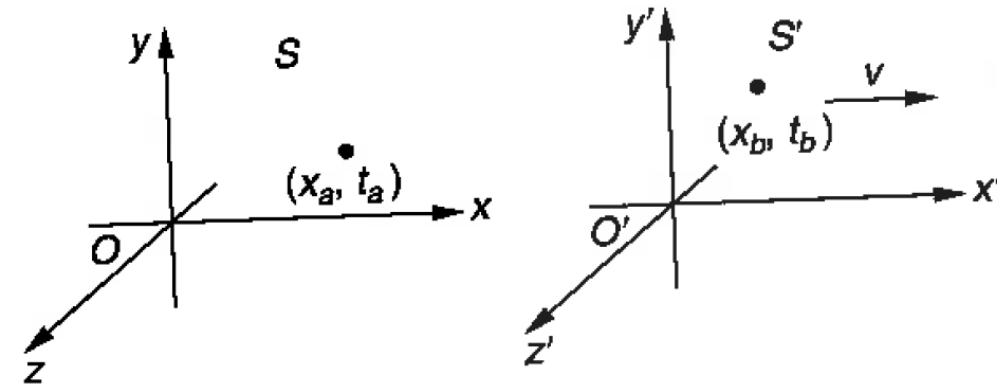
$$x = x' + vt; y = y'; z = z'; t = t'$$

Lorentz transformation: should satisfy Einstein's postulates and converge back to Galilean when v is small.

$$x' = \gamma(x - vt)$$

$$\gamma \to 1 \ when \ v/c \to 0$$

$$x = \gamma(x' + vt)$$

$$t' = \gamma \left[ t + \frac{(1 - \gamma^2)}{\gamma^2} \frac{x}{v} \right]$$

# Obtaining $\gamma$

A flash of light in S at t = 0 at origin while defining the same point in S' as the origin in S' at t' = 0.

$$x^2 + y^2 + z^2 = c^2 t^2$$

$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}$$

# Lorentz transformation

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left[t - \frac{v}{c^2}x\right]$$

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left[t' + \frac{v}{c^2}x'\right]$$

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}$$

# Example

- The arrivals of two cosmic ray $\mu$ mesons (muons) are recorded by detectors in the laboratory, one at time $t_a$ at location $x_a$ and the second at time $t_b$ at location $x_b$ in the laboratory reference frame, S in Fig. 1-17. What is the time interval between those two events in system S', which moves relative to S at speed v?
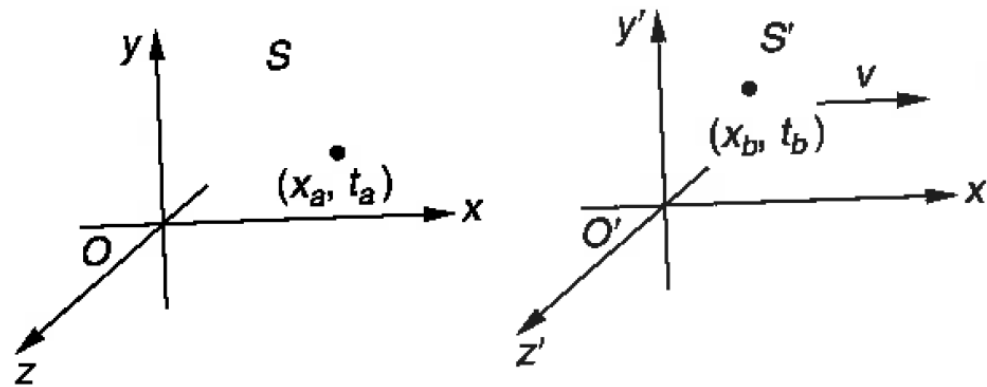


Fig. 1-17

# Relativistic velocity transformations

$$u'_x = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}$$

$$u_x = \frac{u'_x + v}{1 + \frac{vu'_x}{c^2}}$$

$$u'_y = \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}$$

$$u_y = \frac{u'_y}{\gamma\left(1 + \frac{vu'_x}{c^2}\right)}$$

$$u'_z = \frac{u_z}{\gamma\left(1 - \frac{vu_x}{c^2}\right)}$$

$$u_z = \frac{u'_z}{\gamma\left(1 + \frac{vu'_x}{c^2}\right)}$$

# Example

- Suppose that two cosmic-ray protons approach Earth from opposite directions as shown in Fig. 1-18a. The speeds relative to Earth are measured to be $v_1 = 0.6c$ and $v_2 = -0.8c$. What is Earth's velocity relative to each proton, and what is the velocity of each proton relative to the other?
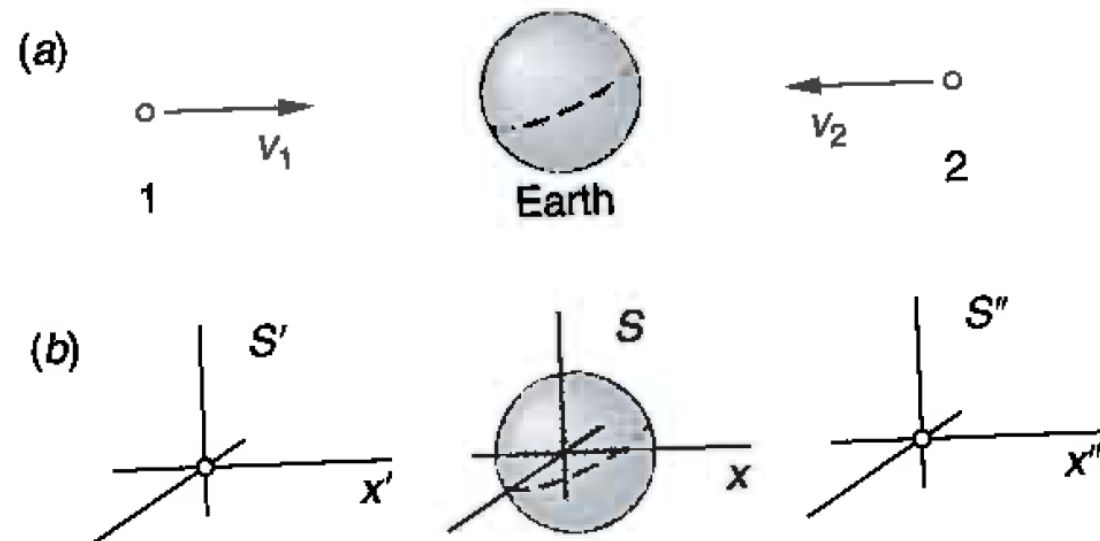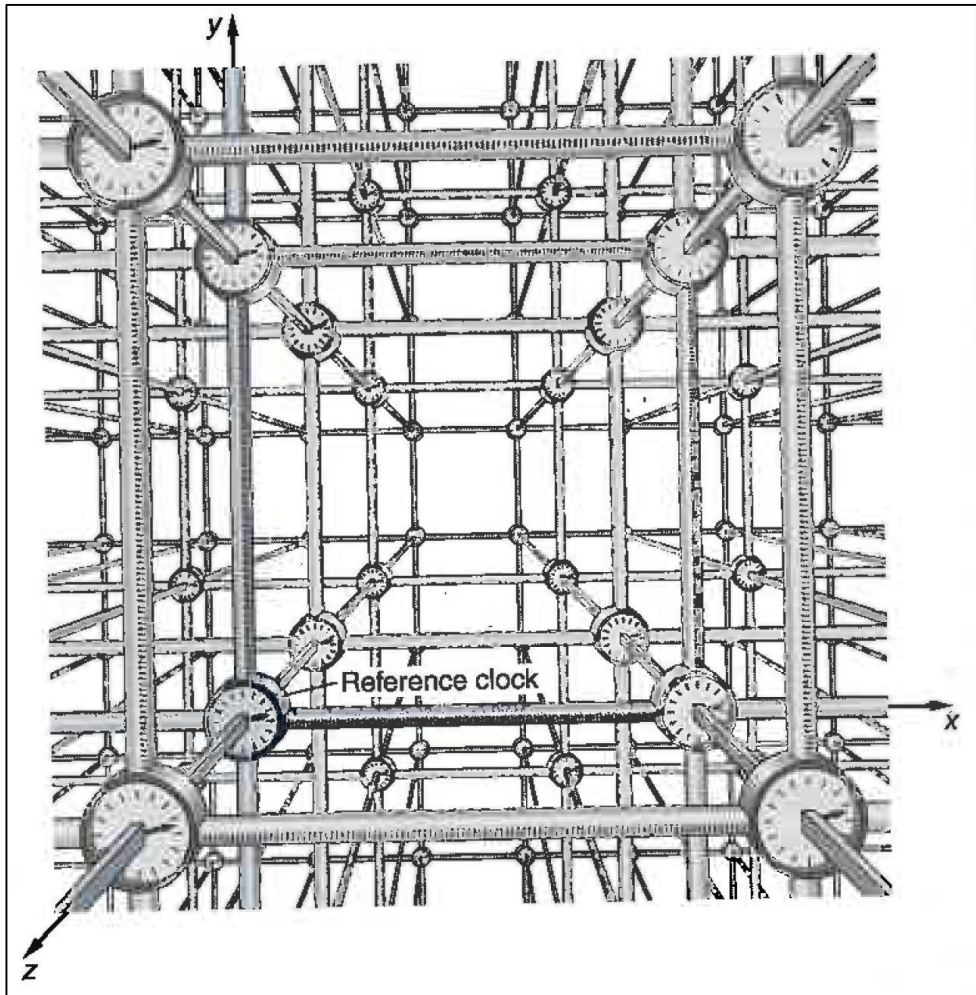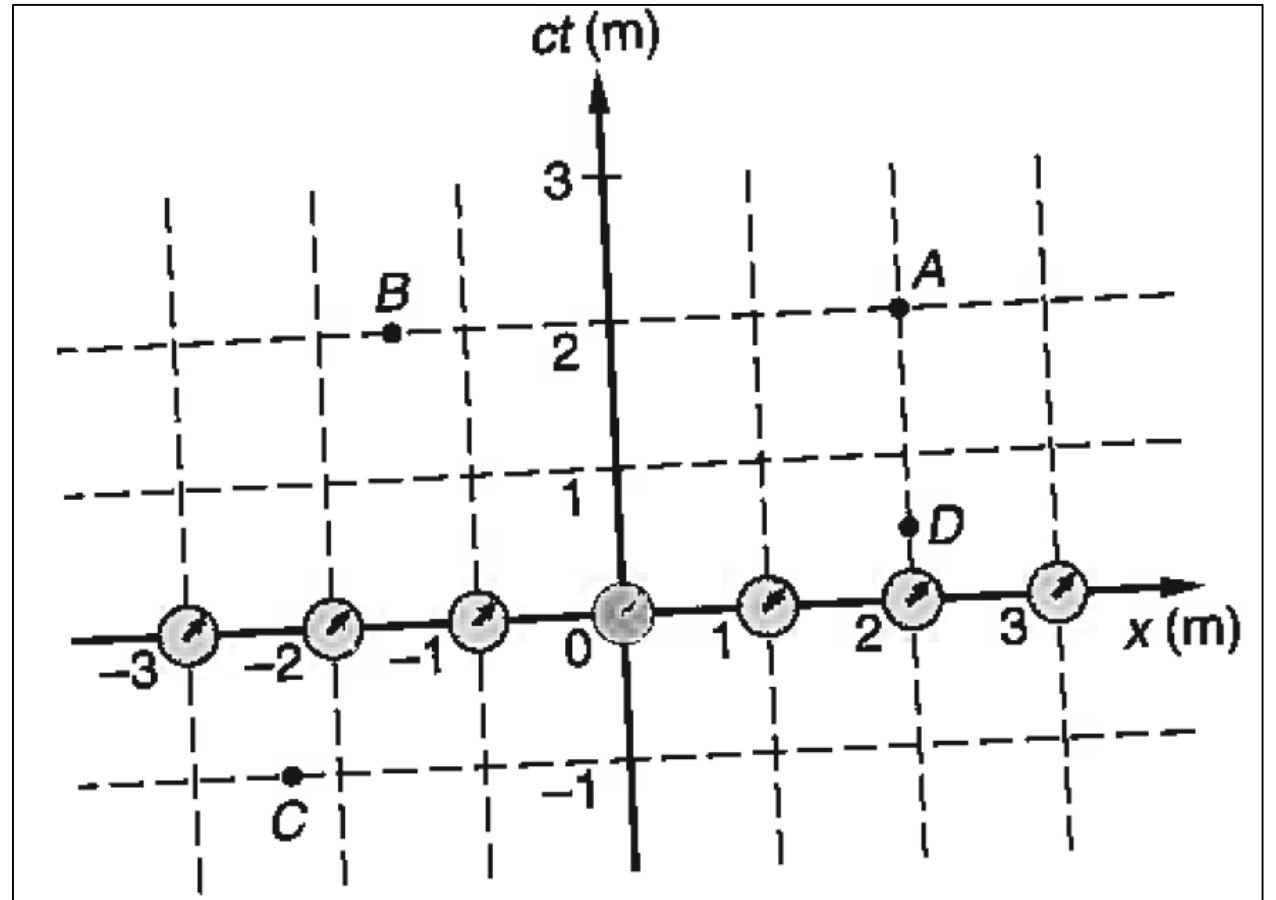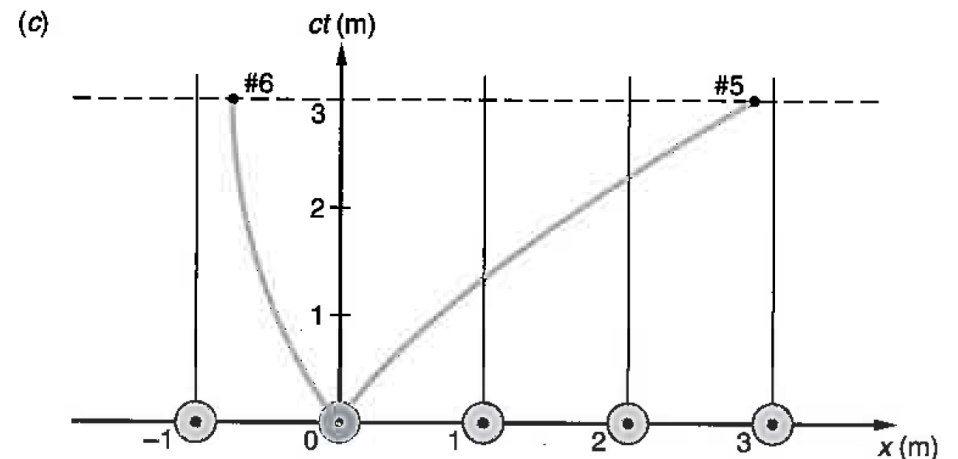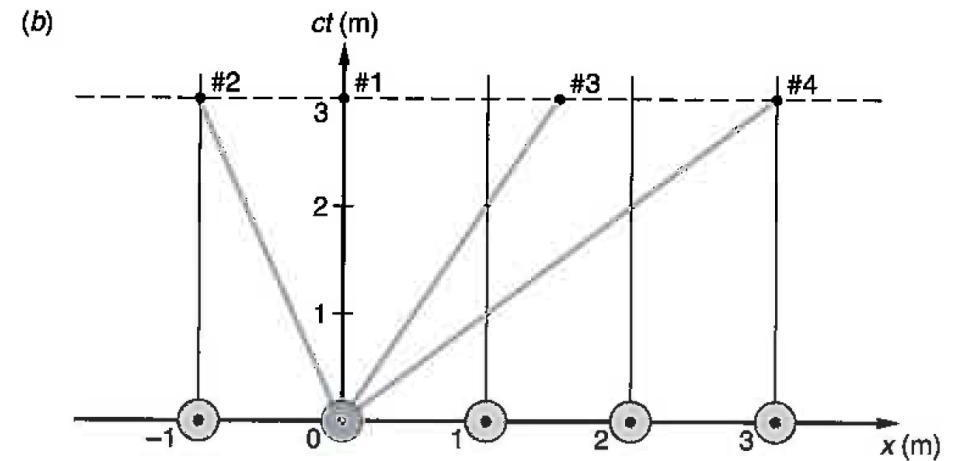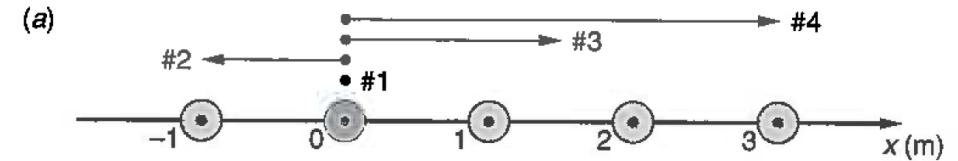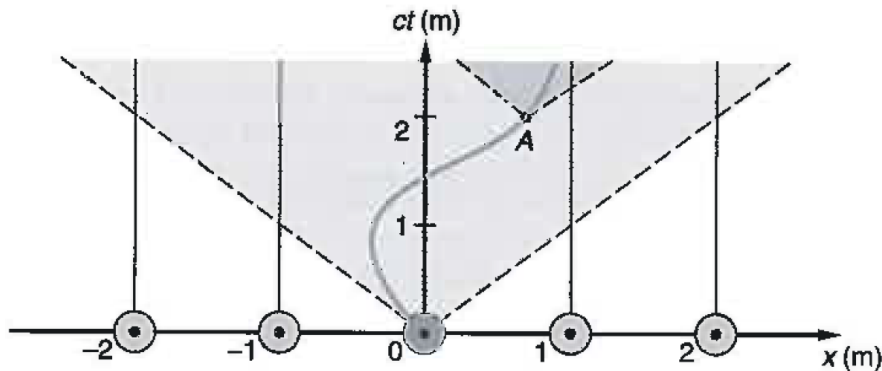


Fig. 1-18

# Spacetime Diagrams



3D in space



1D in space

# Worldlines in Spacetime Diagrams



- Find the speed u of particle 3 in the figure to the right.

- The speed of light is the limit of the moving speed of particle. In spacetime diagram, the dashed line (speed of light) limit the particles' trajectory in spacetime diagram at any point.

# Two inertial reference systems in spacetime diagram

$$x' = \gamma(x - vt)$$

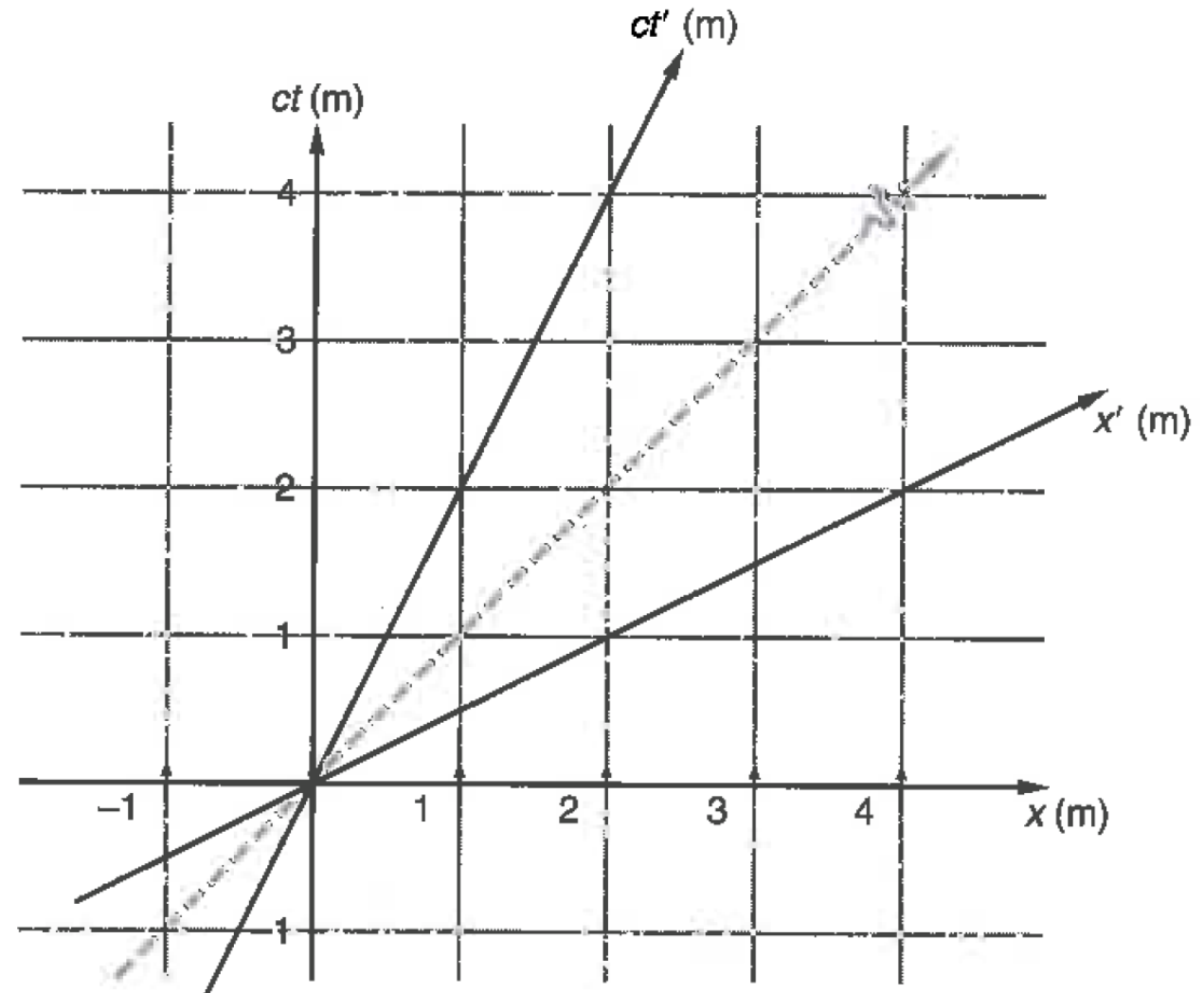$$t' = \gamma\left[t - \frac{v}{c^2}x\right]$$

x′ axis: t′ = 0

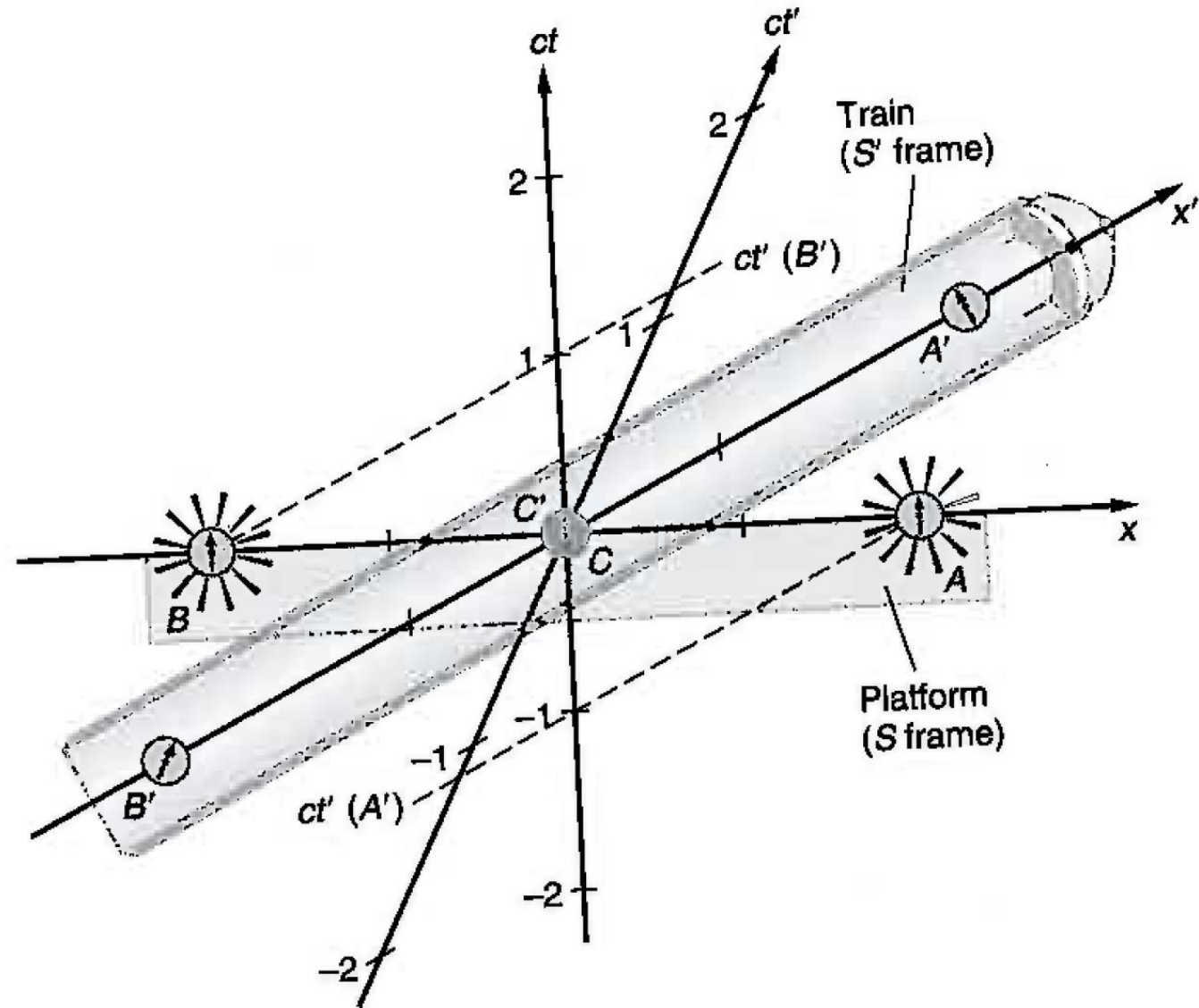$$t' = \gamma\left[t - \frac{v}{c^2}x\right] = 0$$

$$ct = \beta x$$

t′ axis: x′ = 0

$$x' = \gamma(x - vt) = 0$$

$$ct = \frac{1}{\beta}x$$
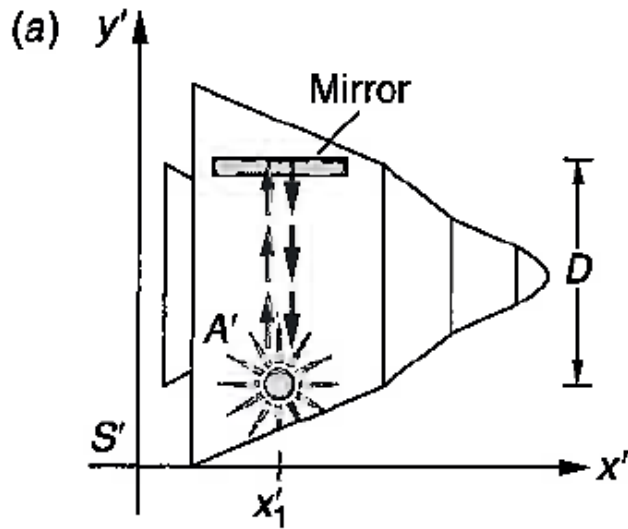
# Spacetime Diagram for Train

If v = 0.5 c, the two flashes of light are simultaneous in S. What is the time interval in S'?

# Time Dilation

**(a)**



**(b)**



**(c)**
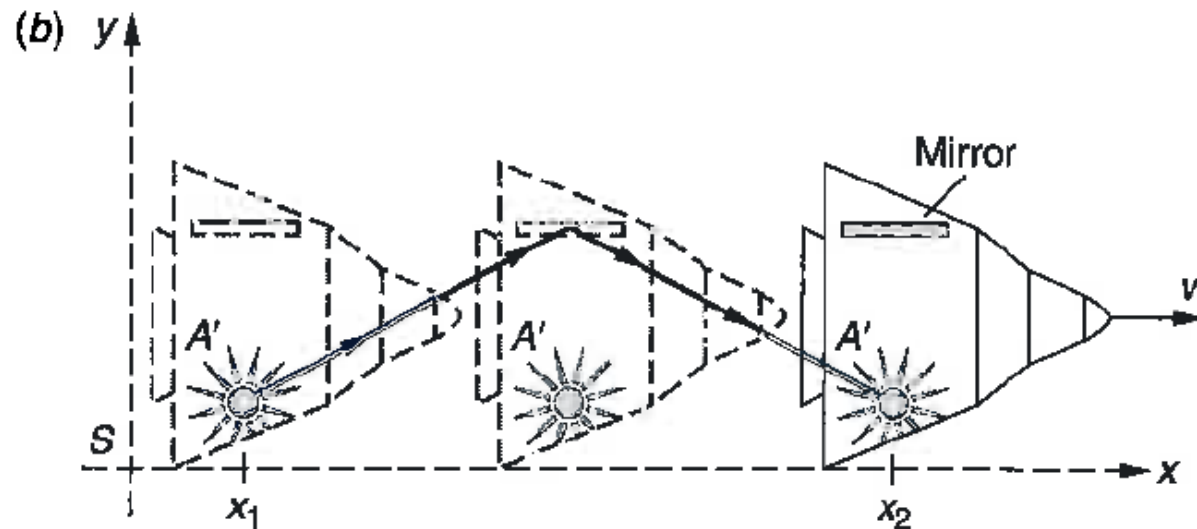


$$\Delta t' = \frac{2D}{c}$$

$$\Delta t = \frac{2\sqrt{D^2 + \left(v\frac{\Delta t}{2}\right)^2}}{c}$$

$$\Delta t = \frac{2D}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t = \gamma\Delta t' = \gamma\tau$$

$\tau$ is "proper time", shortest measurable time in all frames, which could be done when the measurements of the two events are at the same location in this frame.

# Example

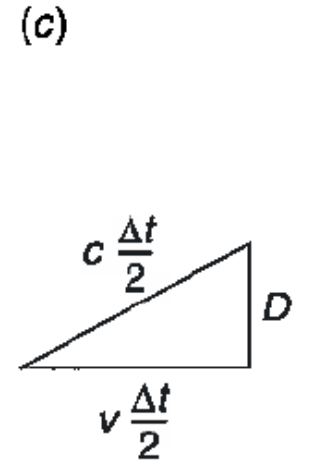- Elephants have a gestation peiod of 21 months. Suppose that a freshly impregnated elephant is placed on a spaceship and sent toward a distant space jungle at v = 0.75 c. If we monitor radio transmissions from the spaceship, how long after launch might we expect to hear the first squealing trumpet from the newborn calf?
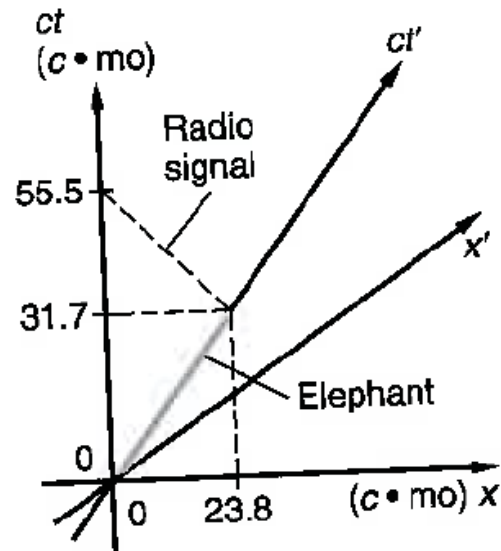
# Example

- Elephants have a gestation peiod of 21 months. Suppose that a freshly impregnated elephant is placed on a spaceship and sent toward a distant space jungle at v = 0.75 c. If we monitor radio transmissions from the spaceship, how long after launch might we expect to hear the first squealing trumpet from the newborn calf?

# Length Contraction

$\Delta x' = L_P = x'_2 - x'_1$ measured when $\Delta t' = 0$

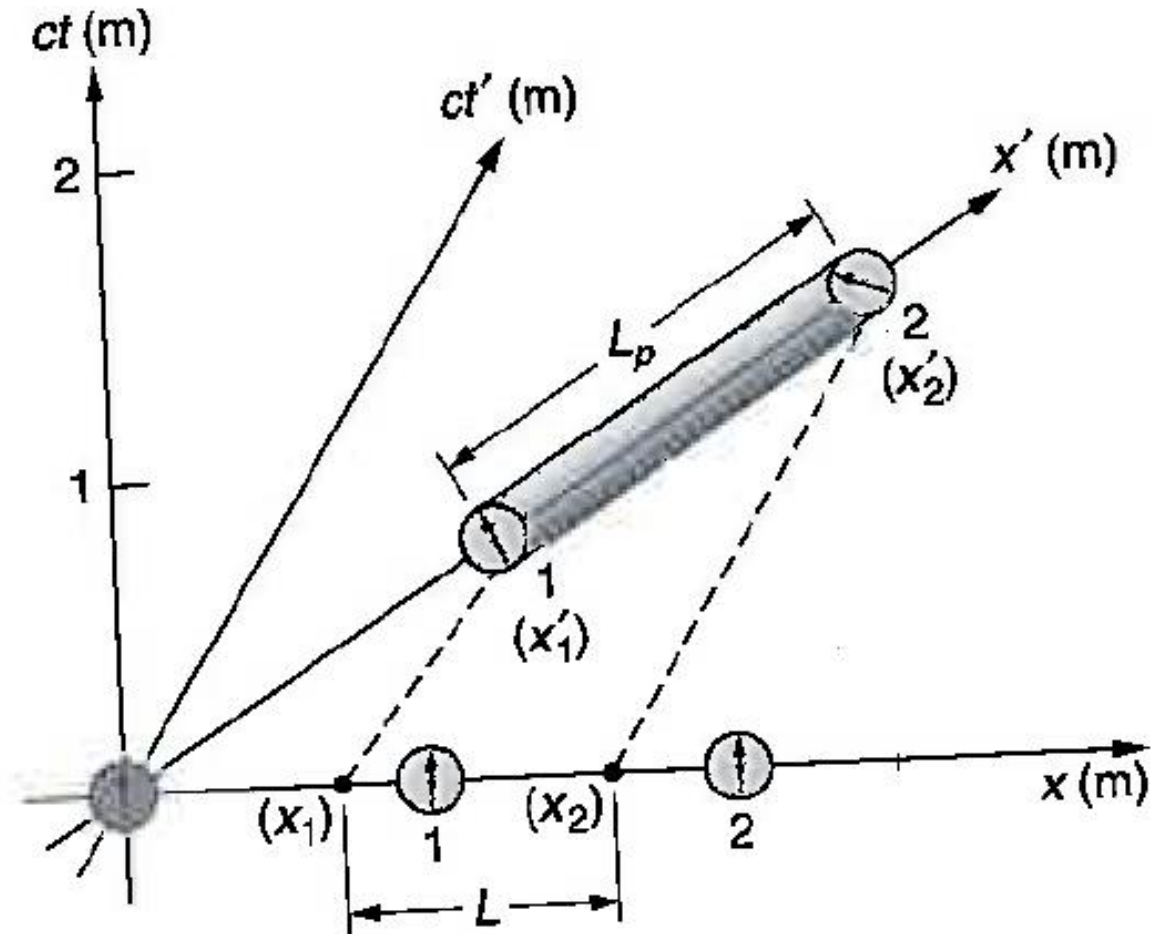$\Delta x = L = x_2 - x_1$ measured when $\Delta t = 0$

$x'_2 = \gamma(x_2 - vt_2)$

$x'_1 = \gamma(x_1 - vt_1)$

$\Delta x = \dfrac{1}{\gamma}\Delta x' \qquad\qquad L = \dfrac{1}{\gamma}L_P$

$L_P$ is "proper length", longest measurable length in all frames, which could be done when the measurements of the two ends are at rest in this frame.

# Example

- A stick that has a proper length of 1 m moves in a direction parallel to its length with speed v relative to you. The length of the stick as measured by you is 0.914 m. What is the speed v?

# Muon decay

$$N(t) = N_0 e^{-t/\tau}$$

Proper time:  $\tau = 2\mu s$                    $h = 600m$

$v = 0.998c$

Proper length (height):  $H = 9000m$

$\tau' = 30\mu s$

$N_0 = 10^8$

Non-relativistic                                        Relativistic

N=30.6                                                    N= 3.68$\times 10^7$



(a) Muon

9000 m

(b)

Muon

600 m
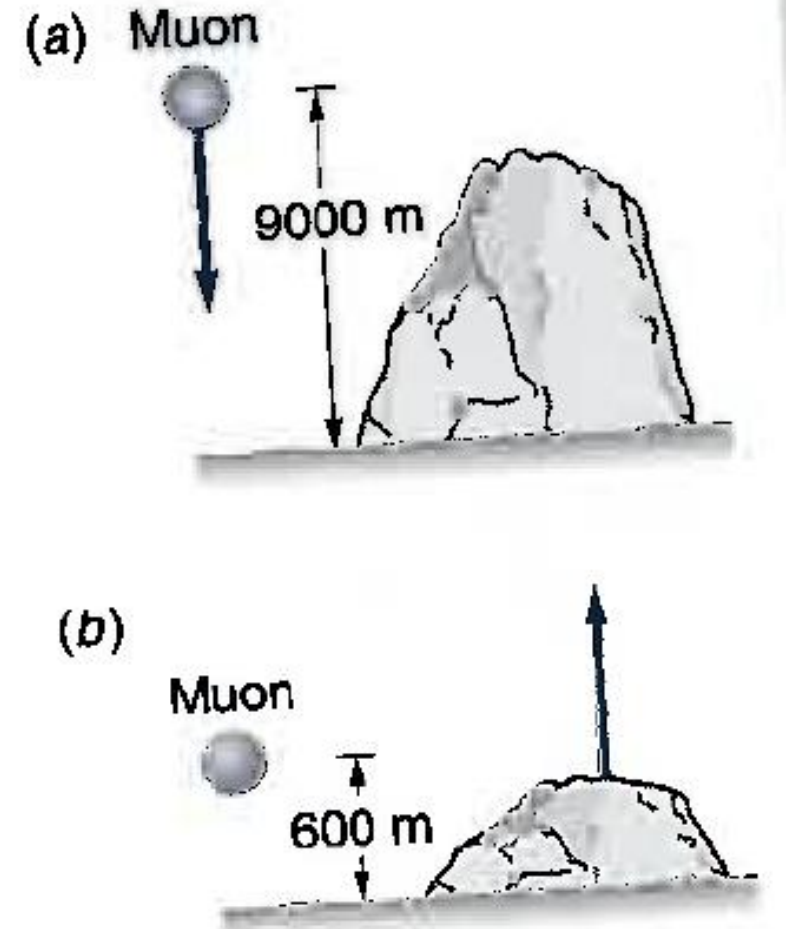
# Spacetime Interval

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x^2 + \Delta y^2 + \Delta z^2)$$
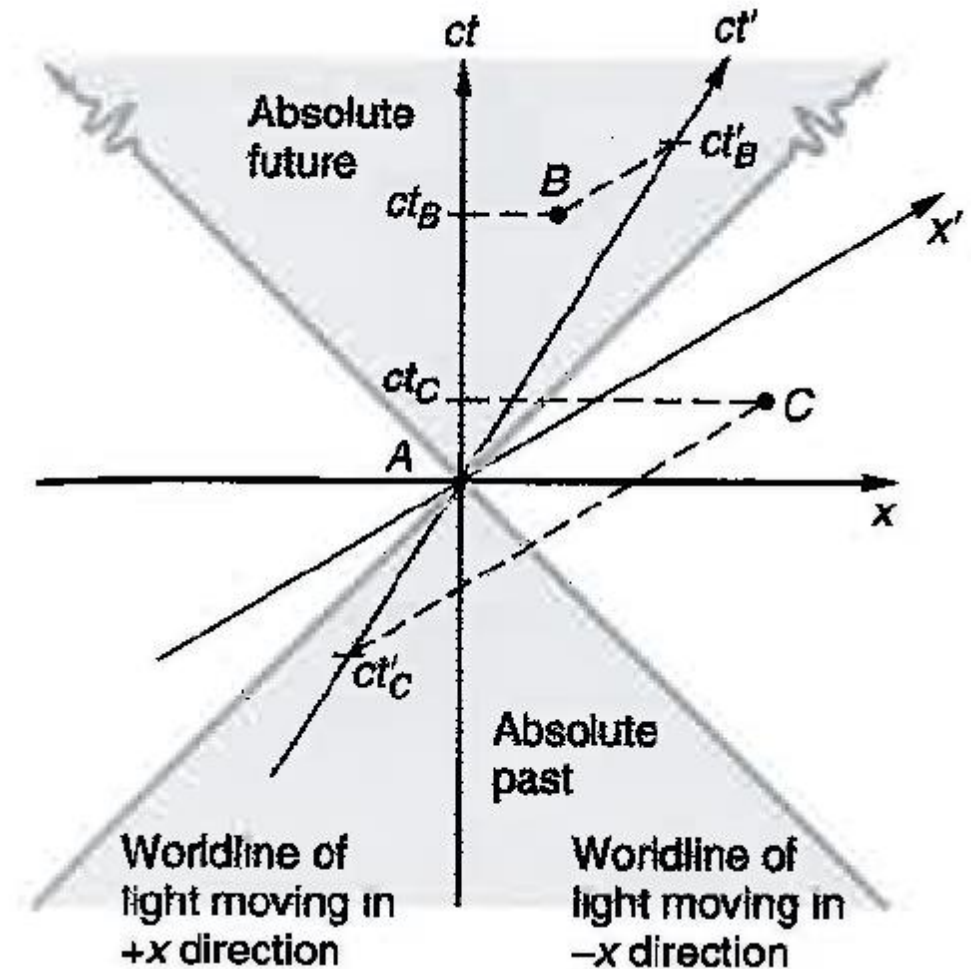
Timelike interval

$$(c\Delta t)^2 > (\Delta x^2 + \Delta y^2 + \Delta z^2)$$

Spacelike interval

$$(c\Delta t)^2 < (\Delta x^2 + \Delta y^2 + \Delta z^2)$$

Lightlike interval

$$(c\Delta t)^2 = (\Delta x^2 + \Delta y^2 + \Delta z^2)$$

# Doppler Effect

$$f = \sqrt{\frac{1+\beta}{1-\beta}} f_0 \qquad \text{approaching}$$

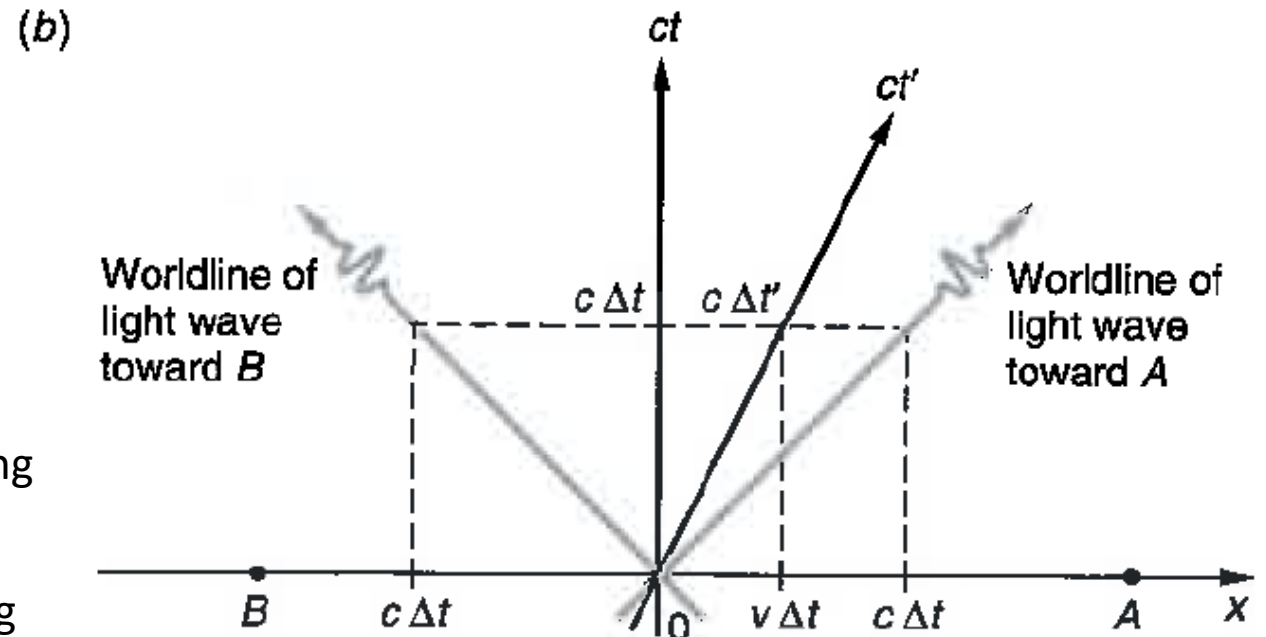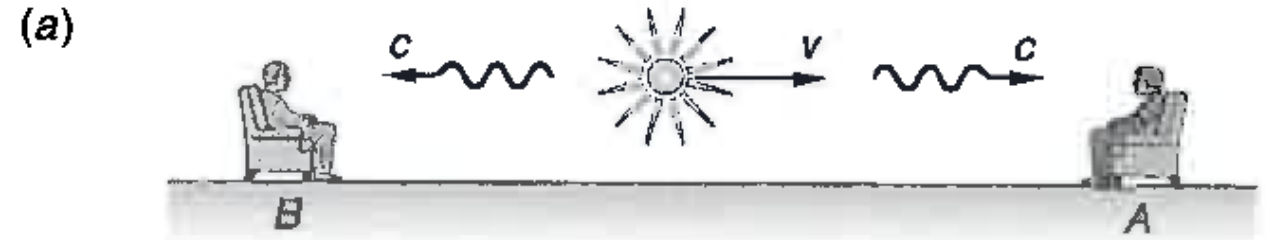$$f = \sqrt{\frac{1-\beta}{1+\beta}} f_0 \qquad \text{receding}$$

Useful approximations

$$f \approx (1+\beta)f_0 \qquad \Delta f = f_0 - f \approx -\beta f_0 \quad \text{approaching}$$

$$f \approx (1-\beta)f_0 \qquad \Delta f = f_0 - f \approx \beta f_0 \qquad \text{receding}$$

# Example

- The longest wavelength of light emitted by hydrogen in the Balmer series (see Chapter 4) has a wavelength of $\lambda_0 = 656 nm$. In light from a distant galaxy, this wavelength is measured as $\lambda = 1458 nm$. Find the speed at which the galaxy is receding from Earth, assuming the shift to be due to Doppler effect.
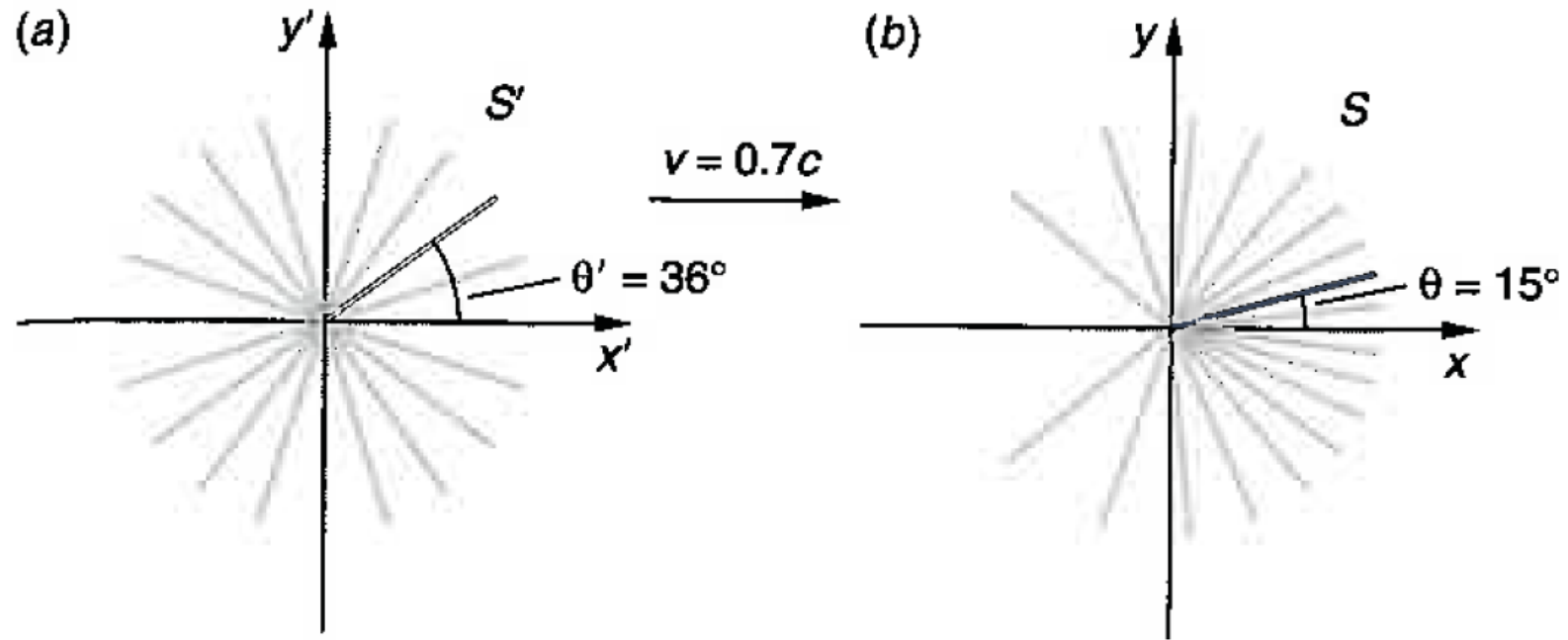
# Twin Paradox

- Twin: Homer and Ulysses. Ulysses traveled from Earth in a ship with 0.8 c away from Earth and then come back to Earth with -0.8 c. Homer see Ulysses 10 years after the launch.

# Pole and Barn Paradox

- Runner with 10 m pole in his hand, moving toward a 5 m barn with front door opened and rear door closed. Farmer (same frame with barn) sees the pole is entirely enclosed in the barn at certain moment.

# Headlight Effect



(a) S' frame: $y'$, $x'$ axes, angle $\theta' = 36°$

(b) S frame: $y$, $x$ axes, angle $\theta = 15°$, $v = 0.7c$

$$cos\theta = \frac{\Delta x}{\Delta(ct)}$$

$$cos\theta' = \frac{\Delta x'}{\Delta(ct')}$$

$$cos\theta = \frac{cos\theta' + \beta}{1 + \beta cos\theta'}$$