1 Intro: Simple Wollaston Methods

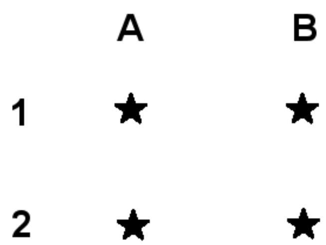
Consider a star intrinsically polarized where

$$q = \frac{Q}{I} = \frac{I_0 - I_{90}}{I_0 + I_{90}} = \frac{1-\beta}{1+\beta} \text{ so } I_{90} = \beta I_0 \text{ and } \beta = \frac{1-q}{1+q}$$

2 Photometric

2.1 Method 1

Now, introduce a throughput coefficient to account for the different response between the two sides of the Wollaston. Let the left side be A and the right side be B and let $B = \alpha A$ when $q = 0$. Now let SET 1 be HWP $= 0°$ and SET 2 be HWP $= 45°$, swapping I_0 and I_{90}. We are assuming α is not dependent on the HWP position!



Define the measured q_m as:

$$q_m = \frac{(A_1 - B_1) - (A_2 - B_2)}{A_1 + B_1 + A_2 + B_2} = \frac{1 - \alpha\beta - \beta + \alpha}{1 + \alpha\beta + \beta + \alpha}$$

Substituting in for β in terms of q and some algebra we have:

$$q_m = \frac{2q + 2\alpha q}{2 + 2\alpha} = q$$

Thus the measured q_m is the same value as the intrinsic q, independent of α.

2.2 Method 2

One might be tempted to form a q for each SET independently then take half the difference, but this does not work as well. For example:

For SET 1 we have:

$$q_1 = \frac{A_1 - B_1}{A_1 + B_1} = \frac{I_0 - \alpha I_{90}}{I_0 + \alpha I_{90}} = \frac{1 - \alpha\beta}{1 + \alpha\beta}$$

and for SET 2 we have:

$$q_2 = \frac{A_2 - B_2}{A_2 + B_2} = \frac{I_{90} - \alpha I_0}{I_{90} + \alpha I_0} = \frac{\beta - \alpha}{\beta + \alpha}$$

Now define the measured q_m as half the difference between these computed 'q's:

$$q_m = \frac{1}{2}(q_1 - q_2) = \frac{1}{2}\left[\left(\frac{1 - \alpha\beta}{1 + \alpha\beta}\right) - \left(\frac{\beta - \alpha}{\beta + \alpha}\right)\right] = \frac{\alpha(1 - \beta^2)}{\alpha(1 + \beta^2) + \beta(1 + \alpha^2)}$$

After substituting in for β in terms of q and some algebra:

$$q_m = \frac{4q}{(1 + \alpha)^2 - q^2 (1 - \alpha)^2}$$

Solving for q we have:

$$q = \frac{4 \pm \sqrt{16 + 4q_m^2 (1 + \alpha)^2}}{2q_m}$$

If $q_m << 1$ then:

$$q \approx \frac{1}{4} q_m (1 + \alpha)^2$$

If $\alpha \approx 1$ as well, then:

$$q \approx \alpha q_m$$

Thus, the measured value for the fractional polarization is not the intrinsic value and is not independent of α!

3 Non-Photometric

In the preceding, we assumed that SET 1 and SET 2 had the same response conditions. That is, if $q = 0$, then $A_1 = A_2$ and $B_1 = B_2$. What if the response changed between the two positions of the HWP due to, say, clouds? Here we introduce yet another factor, γ, to represent the change in throughput between SET 1 and SET 2. The definition of q_m from Section 2.1 is now:

$$q_m = \frac{(1 - \alpha\beta) - \gamma(\beta - \alpha)}{(1 + \alpha\beta) + \gamma(\beta + \alpha)}$$

Substituting for β in terms of q and solving for q we have the following big mess:

$$q = \frac{1 - \alpha - \gamma + \alpha\gamma - q_m(1 + \alpha + \gamma + \alpha\gamma)}{q_m(1 - \alpha - \gamma + \alpha\gamma) - (1 + \alpha + \gamma + \alpha\gamma)} = \frac{C_1 - q_m C_2}{q_m C_1 - C_2}$$

where

$$C_1 = 1 - \alpha - \gamma + \alpha\gamma$$
$$C_2 = 1 + \alpha + \gamma + \alpha\gamma$$

Note that if either $\alpha = 1$ or $\gamma = 1$, then $C_1 = 0$ and $q = q_m$, as expected. However, if this is not the case and $\beta = 1$, i.e. no intrinsic polarization, then q_m is:

$$q_m = \frac{(1 - \alpha)(1 - \gamma)}{(1 + \alpha)(1 + \gamma)} \neq q = 0$$

which is VERY BAD if α **and** γ are significantly different from 1. So, lets reconsider Method 2 from Section 2.2 with our factor of γ now included. We have

$$q_m = \frac{1}{2}(q_1 - q_2) = \frac{1}{2}\left[\left(\frac{1 - \alpha\beta}{1 + \alpha\beta}\right) - \left(\frac{\gamma\beta - \gamma\alpha}{\gamma\beta + \gamma\alpha}\right)\right]$$

The factor of γ of course cancels and we are left with the same value for q_m as before using Method 2. If the weather sucks and you can roughly estimate α, you are better off with Method 2! In fact, simply using $\alpha = B_1/A_1$ has errors only in second order in q. You could also iterate on α by removing your first calculation of β (from q) from your initial estimate of α. This would proceed as follows:

Start with α_1

$$\alpha_1 = \frac{B_1}{A_1} = \alpha\beta$$

Compute q_1 and get β_1 from this value. Now set a new value for α_2

$$\alpha_2 = \frac{B_1}{\beta_1 A_1}$$

Using the value for α from just this one iteration will result in errors that now appear only in third order in q. Note this only works for weakly polarized, high S/N sources. However, you could use such a source to get a handle on α in the first place. Of course, you can also use observations of unpolarized standards plus some sort of flat fielding to get a value for α as well. Whatever the case, Method 2 will be your best bet if the sky transmission is iffy.