Orange You Glad . . . ?
## A Lesson with Sixth, Seventh, and Eighth Graders

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In this lesson, students investigate and construct the formula for the surface area of a sphere using an orange. Using such a model to physically discover how many square units it takes to cover the surface of a sphere helps students to understand the formula and, if needed, to re-create it. Orange You Glad . . . ? *is excerpted from* Sizing Up Measurement: Activities for Grades 6–8 Classrooms, *by Ann Lawrence and Charlie Hennessy (Math Solutions Publications, 2007).*

Instructions

1. Bring to class one orange for each small group of students, plus a few extras. Before class, cut all but the extra oranges in half, keeping the two halves of each orange together. Alternately, help students cut the oranges once you have modeled the activity and distributed the materials and recording sheets. (See last page for recording sheet.)

2. Ask students to offer definitions for the terms *surface area, radius,* and *great circle of a sphere.* Verify correct answers. Then have the students relate their definitions to an orange.

3. Explain that each small group of students will use an orange to investigate and make a conjecture about the formula for finding the surface area of a sphere.

4. Demonstrate the main steps of the investigation:

   a. Using a whole, uncut orange, make the orange model a sphere: squeeze and roll the fruit into a shape as close to a sphere as possible.

   b. Cut the orange into two hemispheres to reveal a great circle of the orange. Ask a volunteer to point out to the class the great circle.

   c. Draw a great circle of the orange on a paper towel by tracing around the circular face of either hemisphere.

orange half

great circle
(traced from
cross-section
of orange)
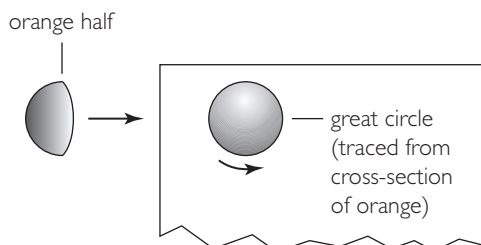
Ask students to individually estimate how many great circles can be covered with the pieces of your orange peel. Ask for volunteers to share their thinking. Tear off pieces of the orange peel, each about 3/4 to 1 square inch in size. Place the pieces, one at a time, in the great circle you drew, pointing out that you are being careful not to overlap pieces or leave gaps. Fill at least three-fourths of your circle. Emphasize that each circle must be covered entirely with no gaps or overlaps. Ask each student to reconsider his or her estimate silently.

5. Provide students with the recording sheets, which include instructions for the activity and questions that they should answer in writing. Emphasize that each student should be prepared to explain the group's findings and thinking in a whole-class discussion. Tell them to leave the display of their findings out until the class discussion has been completed.

6. Allow small groups of students time to set up their work space with the appropriate materials. Have students wash their hands if you intend to let them eat the oranges at the end of the investigation. Distribute the cut oranges or help groups prepare their uncut oranges.

7. Each group performs the investigation, including writing responses to the questions.

8. Have each group summarize its findings and conclusions on an overhead transparency.

9. When everyone is ready, have groups share their findings and conclusions. Discuss any differences among the groups and help students to reconcile those differences.

10. Consider allowing students to eat their oranges before cleaning up their work space. They always want to eat some or all of their oranges!

Notes to the Teacher

Research shows that students commonly have difficulty remembering formulas unless they discover those formulas themselves. This activity offers students the opportunity to find the formula for the surface area of a sphere through a guided investigation.

When you use this activity with your students for the first time, you may find the following comments useful.

- Student definitions for surface area should include the following:

    ❖ Surface area is the number of square units needed to cover the entire outside of a solid figure.

    ❖ The correct unit of measure for surface area is square units.

    ❖ Several partial surface areas can be added to find the total surface area for a

solid. (If necessary, use a rectangular solid as an example to illustrate this point: surface area = $2ba + 2bh + 2ah$, or the total surface area is the sum of the surface areas of all the faces.)

If these ideas are not volunteered, ask questions to solicit them.

- Help students realize that the surface area of the orange equals the surface area of the peel. If they do not discover this on their own, ask questions to elicit this understanding.

- Help students realize that the radius of a great circle of a sphere is the same as the radius of the sphere itself. With an orange, the radius of a great circle is also the radius of the orange. If they do not discover this on their own, ask questions to elicit this understanding.

- To reveal a great circle of an orange, students need to cut the orange exactly in half. To help them do this, suggest that they cut through the "equator" of the orange to create two hemispheres. Students can use a piece of string to find the largest measure around the orange to increase their accuracy.

- If students cut their orange into two significantly unequal pieces, they can trim the larger piece to approximate half the original orange, but they still need to use all the pieces of the peel to cover the circles they draw.

- If a group comes up with the wrong formula, such as surface area = $3\pi r^2$, help them find their error. Most often students leave gaps between pieces of peel or overlap them. Waiting until after the class discussion to clean the work space will allow them to see and correct their mistake.

- A quick way to determine whether students understand how to use the formula they have derived is to have each group find the surface area of its original orange. Ask the group to use a ruler to determine the radius of the great circles they have drawn and then substitute it in their formula to find the numerical value for the surface area of their orange.

Orange You Glad . . . ?

Group Members: _____

_____

_____

Part 1

Follow the steps below to make a conjecture about how to find the surface area of a sphere.

1. Squeeze and mold your orange to make it model a sphere as closely as possible.

2. Cut your orange in half to expose a great circle of the orange.

3. Trace a great circle of your orange on a paper towel.

4. Estimate how many of these great circles you think you can cover with pieces of your orange's peel.

Record: _____

5. Trace a few more great circles than you think you will need on your paper towels.

6. Tear off pieces of the orange peel, each about $\frac{3}{4}$ to 1 square inch.

7. Use the pieces of peel to cover as many circles as possible. Each circle must be covered entirely without any gaps or overlaps.

Part 2

Use your results to answer the following questions in complete sentences. Work together to develop your answers. Then create a transparency summarizing your findings and conclusions. Each of you needs to be prepared to explain your group's thinking to the whole class.

1. Write a mathematical expression that tells the amount of area you covered for each circle. What does r in your expression stand for?

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2. How many circles did you cover in all, using the entire peel of the orange? Based on your results, write an equation that tells the surface area of your orange.

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_____

3. Do you think your equation (formula) will work to find the surface area of any sphere? Explain your reasoning.

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