# Solution I

1) When comparing two satellites of the same object, the ratio form of Kepler's third law (K3, periods) is good.

$$\left(\frac{P_1}{P_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3 \quad \text{or} \quad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

In this form, you can use any units for $P_1$ & $P_2$ (period or time of one orbit) as long as both are the same, and any units for $a_1$ & $a_2$ (semi-major axis, orbital radius, or avg. distance).

$$P_1 = \frac{27.32 \text{ days}}{} \left| \frac{24 \text{ hrs}}{1 \text{ day}} \right| \frac{60 \text{ min}}{1 \text{ hr}} = 3.93408 \times 10^4 \text{ min}$$

$$P_2 = 92.69 \text{ min}$$

$$a_1 = D_{EM} = 3.844 \times 10^8 \text{ m}$$

$$a_2 = ?$$

Note: $h = a_2 - R_E$

$$\left(\frac{P_1}{P_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3 \rightarrow a_2 = a_1 \left(\frac{P_2}{P_1}\right)^{2/3} = 3.844E8 \left(\frac{92.69}{3.93408E4}\right)^{2/3}$$

$$a_2 = 6.8062E6 \rightarrow h = 6.8062E6 - 6.371E6 = \boxed{4.35E5 \text{ m}}$$

Solution II

Look at ISS's orbital velocity

$$V_{orbit} = \sqrt{\frac{GM}{r}}$$

$$\frac{2\pi r}{t} = \sqrt{\frac{GM_E}{r}}$$

$$\frac{4\pi^2 r^2}{t^2} = \frac{GM_E}{r}$$

$$r^3 = \frac{GM_E}{4\pi^2} t^2$$

$G = 6.67E{-11}$

$M_E = 5.972E24 \text{ kg}$

$t = 92.69 \text{ min} \cdot \frac{60 s}{1 \text{ min}} = 5{,}561.4 \text{ s}$

$$r = \left(\frac{GM_E}{4\pi^2} t^2\right)^{1/3}$$

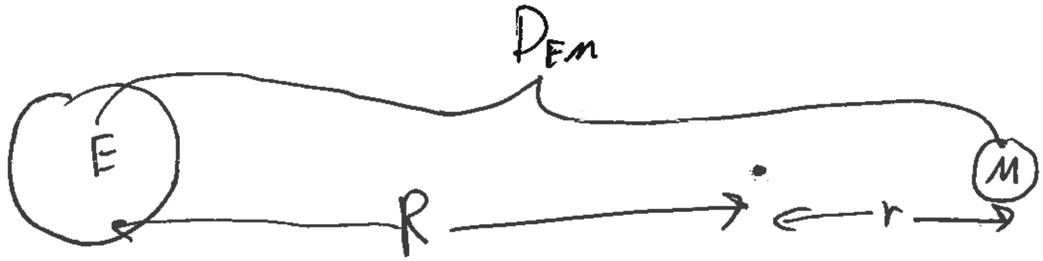
$$r = 6.78 \times 10^6 \text{ m}$$

$$h = r - R_E = (6.78 - 6.371) \times 10^6 \text{ m}$$

$$= \boxed{4.09 \times 10^5 \text{ m}}$$

difference of <10% from other answer is reasonable, probably from rounding.

2)




I'm going to guess in this picture that the point is closer to the Moon. Earth to the point is R, Moon to the point is r ($= D_{EM} - R$). At this point, the Gravities Cancel out, so:

$$F_{gE} = F_{gM}$$

$$\frac{\cancel{G} M_E \cancel{m}}{R^2} = \frac{\cancel{G} M_m \cancel{m}}{r^2}$$

$$\frac{M_E}{R^2} = \frac{M_m}{(D_{EM} - R)^2}$$

you can now do algebra to find a quadratic equation, or use solver or Wolfram Alpha.

$$\frac{M_E}{R^2} - \frac{M_m}{(D_{EM} - R)^2} = 0$$

$M_E = 5.972\text{E}24\text{ kg}$ $\quad M_m = 7.348\text{E}22\text{ kg}$

$D_{EM} = 3.844\text{E}8\text{ m}$

solver... $\boxed{R = 3.460 \times 10^8\text{ m}}$

Sanity check: Is it closer to E or M?

Or if you want an algebraic solution (and it seems like the NSpire calucculator solver struggles with this),...

$$\frac{M_E}{R^2} = \frac{M_M}{(D_{EM}-R)^2}$$

$$\frac{(D_{EM}-R)^2}{R^2} = \frac{M_M}{M_E}$$

$$\left(\frac{D_{EM}-R}{R}\right)^2 = \frac{M_M}{M_E}$$

$$\left(\frac{D_{EM}}{R} - \frac{R}{R}\right)^2 = \frac{M_M}{M_E}$$

$$\left(\frac{D_{EM}}{R} - 1\right)^2 = \frac{M_M}{M_E}$$

$$\frac{D_{EM}}{R} - 1 = \sqrt{\frac{M_M}{M_E}}$$

$$\frac{D_{EM}}{R} = \sqrt{\frac{M_M}{M_E}} + 1$$

$$\frac{D_{EM}}{R} = \frac{\sqrt{M_M}}{\sqrt{M_E}} + \frac{\sqrt{M_E}}{\sqrt{M_E}}$$

$$\frac{D_{EM}}{R} = \frac{\sqrt{M_M} + \sqrt{M_E}}{\sqrt{M_E}}$$

$$\frac{R}{D_{EM}} = \frac{\sqrt{M_E}}{\sqrt{M_M} + \sqrt{M_E}}$$

$$R = D_{EM}\left(\frac{\sqrt{M_E}}{\sqrt{M_M} + \sqrt{M_E}}\right)$$

3) a) $V_{esc} = \sqrt{\dfrac{2GM}{r}}$

$G = 6.67E{-11}$, $M_E = 5.972E24$, $R_E = 6.371E6\ m$

$\boxed{V_{esc} = 1.118E4\ m/s} = \sim 11\ km/s$

b) $V_{esc} = \sqrt{\dfrac{2GM}{r}}$

$M_S = 1.989E30\ kg$, $R = D_{ES} = 1.496E11\ m$

$\boxed{V_{esc} = 4.211 \times 10^4\ m/s}$

Or could be asking how much speed added to Earth's orbital speed

$V_{esc} = V + V_{orbit}$

$V = V_{esc} - V_{orbit}$

$= \sqrt{\dfrac{2GM}{r}} - \sqrt{\dfrac{GM}{r}}$

$= \sqrt{\dfrac{GM}{r}} = \boxed{2.978 \times 10^4 m/s}$