1) Same by symmetry, which you learned in Kinematics (Ch 2).

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2) Lose energy to friction

$$K_0 = K_f + W_{friction}$$

$$|V_0| > |V_f| \qquad V_0 \text{ is bigger}$$

---

8) $$\vec{F} = -\left(\frac{\partial U}{\partial x}\right)\hat{i} - \left(\frac{\partial U}{\partial y}\right)\hat{j} - \left(\frac{\partial U}{\partial z}\right)\hat{k}$$

$$= -(6x + 2y + 0)\hat{i} - (0 + 2x + 8yz)\hat{j} - (0 + 0 + 4y^2)\hat{k}$$

$$\boxed{\vec{F} = (-6x - 2y)\hat{i} + (-2x - 8yz)\hat{j} + (-4y^2)\hat{k}}$$

---

9) $$U = -\int F dx$$

$$= +(+k)\int x^{-3} dx$$

$$= k\left(-\frac{x^{-2}}{2}\right) + C$$

$$U = \frac{-k}{2x^2} + C$$

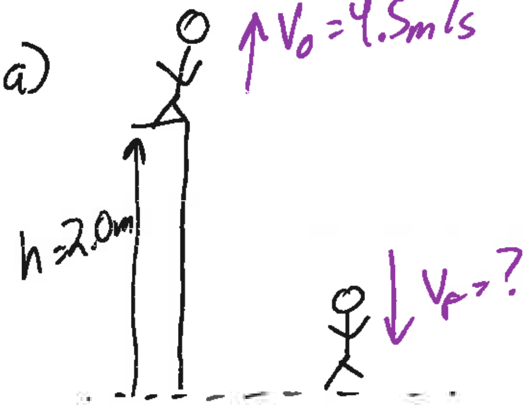
$$0 = \frac{-k}{2 \cdot 2^2} + C$$

$$C = +k/8$$

$$\boxed{U = \frac{-k}{2x^2} + \frac{k}{8}}$$

16) This is a case where before & after pictures are useful, to help ID forms of energy.

a)



Initial	Final
U_{g0}	
K_0	K_f

$$U_{g0} + K_0 = K_f$$

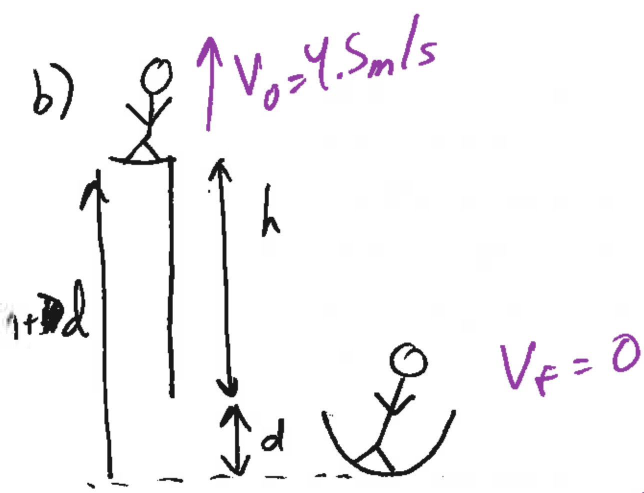
$$\cancel{m}gh + \tfrac{1}{2}\cancel{m}v_0^2 = \tfrac{1}{2}\cancel{m}v_f^2$$

$$V_f = \sqrt{2gh + v_0^2}$$

$$= 7.7104$$

$$= \boxed{7.7\,m/s}$$

b)



Initial	Final
U_{g0}	U_{sf}
K	

$$U_{g0} + K = U_{sf}$$

$$mg(h+d) + \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kd^2$$

$$\tfrac{1}{2}kd^2 - mgd - \tfrac{1}{2}mv_0^2 - mgh = 0$$

$$0.5 \cdot 5.8E4 \cdot d^2 - 72 \cdot 9.8 \cdot d - (.5 \cdot 72 \cdot 4.5^2 + 72 \cdot 9.8 \cdot 2.0) = 0$$

quadratic, two roots

$$\boxed{d = 0.284\,m} \quad -0.260\,m$$

I'll want the positive root for d based on how I defined it.

c) I'd use $v_f^2 = v_0^2 + 2a\Delta x$

d) Without the "$-mgd$" term, you get

$$d = \pm 0.272\,m$$

which was off by ~4%.

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The loop-de-loop is my favorite 1210 problem! :)

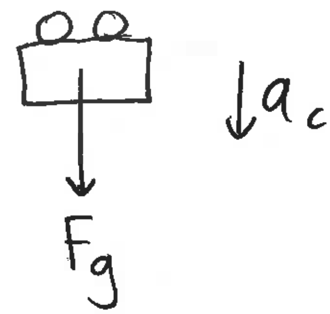
To "barely make it around", you will need normal force at the top = 0.

Approach: ① FBD & $a_c = v^2/r$ to find v at the top

② Energy to relate v to h.

① At the top, to barely make it, $F_N = 0$

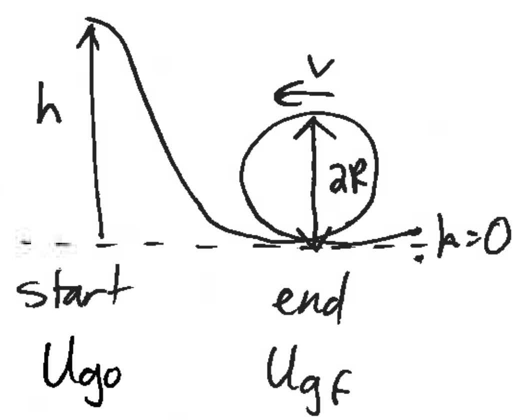
FBD to the right.



$$\Sigma F = m a_c$$

$$mg = F_g = \frac{mv^2}{R} \rightarrow v^2 = gR \rightarrow v = \sqrt{gR}$$

② $E_0 = E_f$



$$U_{g0} = U_{gf} + K$$

$$\cancel{m}gh = \cancel{m}g(2R) + \frac{1}{2}\cancel{m}v^2$$

$$gh = 2gR + \frac{1}{2}(gR)$$

$$h = 2R + \frac{1}{2}R$$

$$\boxed{h = \frac{5}{2}R}$$

sanity check: Units ✓

$h > 2R$ ✓