

$$\vec{\tau} = \vec{r} \times \vec{F} \rightarrow \tau = r_{\perp} F = r F_{\perp}$$

CCW +

CW -

$$\tau_1 = 5\text{ m} \cdot 8\text{ N} = 40\text{ N m} \text{ CW -}$$

$$\tau_2 = 2\text{ m} \cdot \underbrace{(12\text{ N} \cos 60^\circ)}_{F_{\perp}} = 12\text{ N m} \text{ CCW +}$$

$$\tau = -40\text{ N m} + 12\text{ N m} = \boxed{-28\text{ N m}}$$

↑
CW

↓ Forces thru the axis
don't count!

$$\text{B) } \tau_1 = 0\text{ m} \cdot 8\text{ N} = 0$$

$$\tau_2 = 3\text{ m} \cdot (12\text{ N} \cos 60^\circ) = \boxed{-18\text{ N m}}$$

↑
CW

17) Energy to find V_f . Biggest V_f hits first

$$E_0 = E_f$$

$$U_g = K_{\text{Trans}} + K_{\text{rot}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(\#mr^2)\left(\frac{v}{r}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\#mv^2$$

$$2gh = v^2 + \#v^2$$

$$\frac{2gh}{1+\#} = v^2$$

Biggest v for smallest $\#$ (in $I = \#mr^2$)

	Object shape	I	$\#$
small v	Hoop	mr^2	1
	tennis ball	$\frac{2}{3}mr^2$	$\frac{2}{3} = 0.\bar{6}$
	D-cell	$\frac{1}{2}mr^2$	$\frac{1}{2} = 0.5$
	marble	$\frac{2}{5}mr^2$	$\frac{2}{5} = 0.4$
Big v	Box	0	0

empty can somewhere be between hoop & D-cell

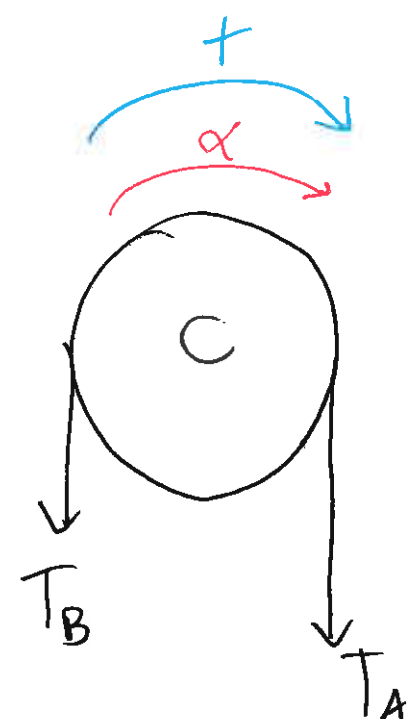
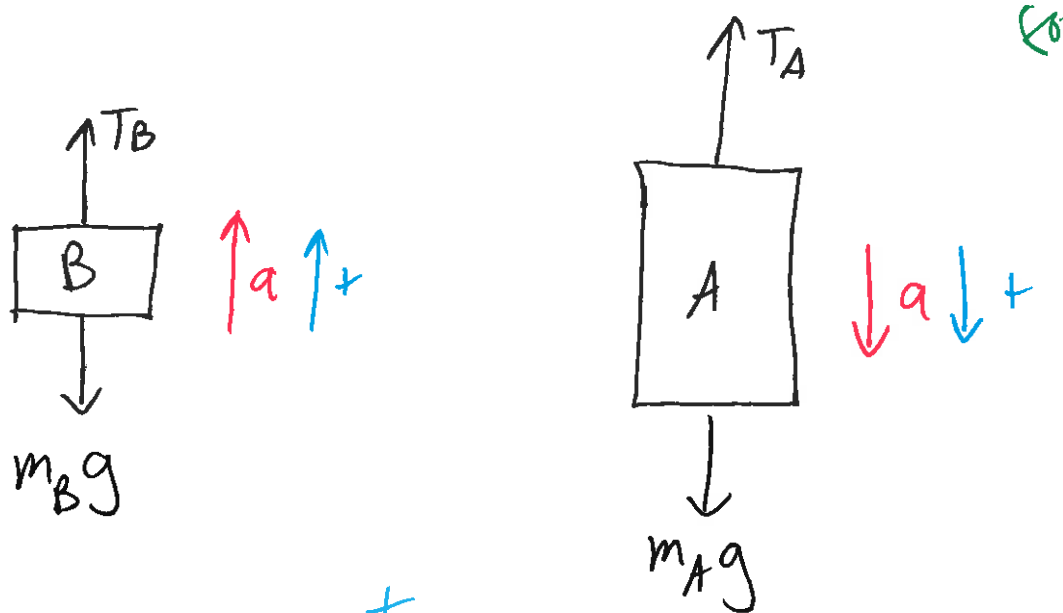
67 | Approach:

- ① FBD for each object
- ② $\Sigma F = ma$ for A & B
- ③ $\Sigma \tau = I\alpha$ (rotational equivalent) for C
- ④ relate a & α
- ⑤ look for 3 eqⁿs & 3 unknowns

Note: This one is a great review! Do it again before the final for practice. ;)

Note: These steps are the same as for any force problem from Ch 4-5.

① FBDs



There's also gravity down and a rope up, but don't affect torque.
From this picture, T_A can't be equal to T_B , or else the pulley wouldn't speed up.

$$\underline{A}: \Sigma F = m a$$

$$\textcircled{1} +m_A g - T_A = m_A a$$

$$\underline{B}: \Sigma F = m a$$

$$\textcircled{2} +T_B - m_B g = m_B a$$

$$\underline{C}: \Sigma \tau = I \alpha$$

$$R T_A - R T_B = I_c \alpha$$

$$\textcircled{3} R T_A - R T_B = I_c a / R$$

These steps are similar to Disc 4, 5.15, except $T_A \neq T_B$

R is \perp to each T , so $\sin \theta = \sin 90^\circ = 1$

$$\textcircled{4} \alpha = a / r$$

Three equations with 3 unknowns! $\ddot{\smile}$
(T_A, T_B, a)

$$\textcircled{1} m_A g - T_A = m_A a \rightarrow T_A = m_A g - m_A a$$

$$\textcircled{2} T_B - m_B g = m_B a \rightarrow T_B = m_B a + m_B g$$

$$\textcircled{3} R T_A - R T_B = I_c \frac{a}{R} \Rightarrow T_A - T_B = \frac{I_c a}{R^2}$$

$$(m_A g - m_A a) - (m_B a + m_B g) = \frac{I_c a}{R^2}$$

get a's all on same side.

$$m_A g - m_A a - m_B a - m_B g = \frac{I_c}{R^2} a$$

$$m_A g - m_B g = \frac{I_c}{R^2} a + m_A a + m_B a$$

$$(m_A - m_B) g = \left(\frac{I_c}{R^2} + m_A + m_B \right) a$$

$$\frac{(m_A - m_B) g}{\frac{I_c}{R^2} + m_A + m_B} = a$$

$$a = \left(\frac{m_A - m_B}{m_A + m_B + \frac{I_c}{R^2}} \right) g$$

if $I_c = \frac{1}{2} m_c R^2$,
then
↓

$$a = \left(\frac{m_A - m_B}{m_A + m_B + \frac{1}{2} m_c} \right) g$$

Part
B
Solved!

Part A) plug in

$$m_A = 4 \text{ kg}$$

$$m_B = 2 \text{ kg}$$

$$I_c = 0.300 \text{ kg m}^2$$

$$R = 0.120 \text{ m}$$

$$g = 9.80 \text{ m/s}^2$$

$$a = 0.730 \text{ m/s}^2$$

$$\alpha = \frac{a}{R} = 6.08 \text{ rad/s}^2 = \alpha$$

From ①

$$T_A = m_A g - m_A a = m_A (g - a) = 36.3 \text{ N} = T_A$$

From ②

$$T_B = m_B (a + g) = 21.1 \text{ N} = T_B$$

$$\text{Part D) } I_{\text{tot}} = I_A + I_B + I_c$$

$$= m_A R^2 + m_B R^2 + I_c$$

$$= 0.3864 = 0.386 \text{ kg m}^2$$

Part E) This is Kinematics, like Ch 2.

want V_f from a , t :

$$V_f = V_0 + at$$

$$= 0 + 0.730 \cdot 0.5 = \boxed{0.365 \text{ m/s}}$$

$$\text{Part F) } \omega = \frac{V}{R} = 3.0416 = \boxed{3.04 \text{ rad/s}}$$

$$\begin{aligned} \text{Part G) } K_{\text{tot}} &= K_A + K_B + K_C \\ &= \frac{1}{2} m_A V^2 + \frac{1}{2} m_B V^2 + \frac{1}{2} I \omega^2 \\ &= 1.78744 = \boxed{1.79 \text{ J}} \end{aligned}$$

$$\text{Part H) } L_{\text{tot}} = L_A + L_B + L_C$$

Note: I wrote one of the relevant formulae wrong!

$$\vec{L} = I \vec{\omega} \text{ is correct}$$

~~$$\vec{L} = m \vec{v}$$~~

Sanity check:

$$\begin{array}{ccc} \vec{p} = m \vec{v} & & \\ \downarrow & \downarrow & \downarrow \\ \vec{L} & I & \vec{\omega} \end{array}$$

for linear momentum

$$L_{\text{tot}} = Rm_A v + Rm_B v + I_C \omega$$

$$= Rv(m_A + m_B) + I_C \omega$$

$$= 1.17530$$

$$= \boxed{1.18 \text{ kg}\cdot\text{m}^2/\text{s}}$$

all perpendicular
so cross-
product's
 $\sin \theta = 1$

Units: $R m v$
 $\downarrow \quad \downarrow \quad \downarrow$
 $m \quad \text{kg} \quad \text{m/s} \rightarrow \text{kg}\cdot\text{m}^2/\text{s}$