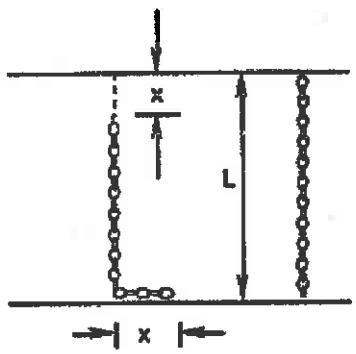
A

#4

Mechanics 1

A metallic chain of length L and mass M is vertically hung above a surface with one end just in contact with it. The chain is then released to fall freely. If x is the distance covered by the end of the chain, how much force (exerted by the bottom surface) will the chain experience at any instance during the process?



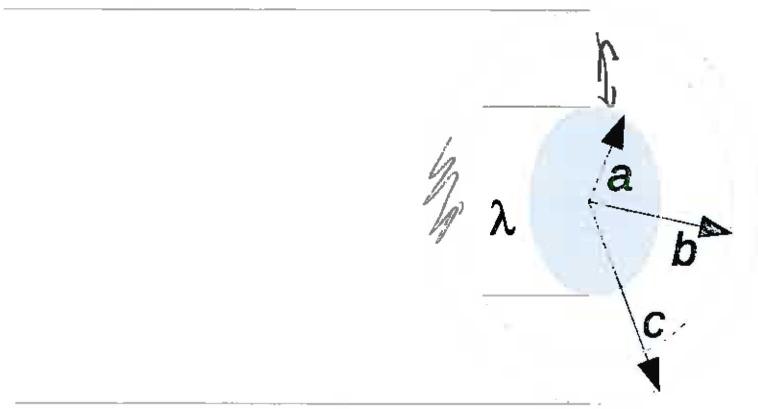
Quantum Mechanics Problem

Provide the wavefunction, ψ_0, for the lowest energy state of the hydrogen atom. Assume the electron mass is m_e, the electron charge is $-e$, the proton mass is infinite, and the normalized Planck constant is $\hbar$. Also, provide the energy of this lowest state, E_0, as well as to give its numerical value (in eV). Make sure to define any parameters used in addition to the ones given.

#4

Electricity and Magnetism

A long coaxial cable consists of an inner cylindrical conductor with radius a and an outer conducting coaxial cylinder with inner radius b and outer radius c. The outer cylinder is mounted on insulating supports and has no net charge. The inner cylinder has a positive charge per unit length λ.



1. Graph the charge per unit length as a function of the radial distance r from the axis of the cable for $0 < r < 2c$.

2. Calculate and graph the electric field for $0 < r < 2c$.

#4

Statistical Mechanics

For a simple model of a solid containing N atoms per mole:

a. How many harmonic oscillators are there per mole?

b. How much energy is there per oscillator? Explain.

c. How much E per mole (using the gas constant R)?

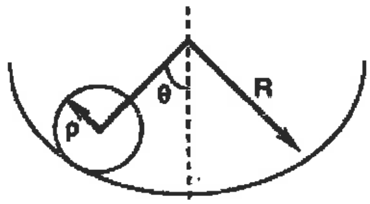
d. What is the molar heat capacity (using R? using k?)

B

#4

Mechanics 2

A sphere of radius r and mass m is constrained to roll without slipping on the lower half of the inner surface of a hollow, stationary cylinder of inside radius R. Find the Lagrangian function!



(B) # 4

Quantum Mechanics Problem.

For a particle of mass, m, with an energy level of E, in an infinite 1-D domain, $-\infty < x < \infty$, in a potential $V(x)$, with two sides (where $V = 0,$ for $x < 0;$ and $V = V_0,$ for $x > 0$),

determine the general wavefunction solution, $\psi(x)$, for $E > V_0$;

specifically where $\psi = \psi_1(x),$ for $x < 0;$ and $\psi = \psi_2(x),$ for $x > 0$. Make sure to define any parameters, such as the wavenumber, k, in terms of given parameters (such as the normalized Planck constant, $\hbar$) that you use in the analysis (where you do not need to connect the solution at $x = 0$).

B

(#4)

E&M

The electric field of a plane electromagnetic wave travelling along the z-axis is

$$E = \left(E_{0x}\hat{x} + E_{0y}\hat{y}\right)\, \sin(\omega t - kz + \phi).$$

Find the magnetic field B.

#4

Statistical Mechanics

A system consists of N weakly interacting particles, each of which can be in one of two states with energies E1 and E2, with E1 < E2.

a. Calculate the mean energy as a function of T.

b. Calculate the heat capacity at constant volume as a function of T.

(1)

Graduate Exams Part I - A

Saturday, September 8, 2012

Mechanics

A solid sphere of mass m hangs by a rope. A lighter solid sphere of mass $m/4$ hangs from it, connected by a massless spring of spring constant k. The rope suspending the top sphere is cut, allowing the two spheres to drop in free-fall.

a. Immediately after the rope is cut, what is the acceleration of the top sphere? $5/4\, g$

b. Immediately after the rope is cut, what is the acceleration of the bottom sphere? 0

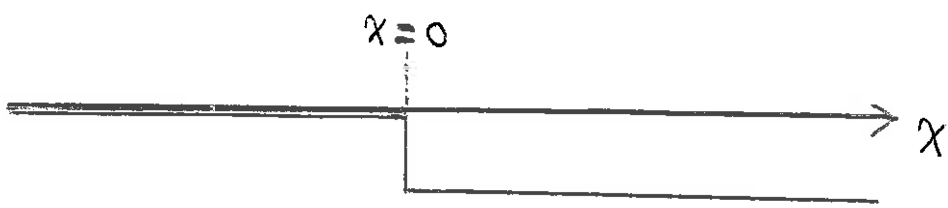
c. Immediately after the rope is cut, what is the acceleration of the center of mass of the system of the two spheres? g

Quantum Mechanics

Calculate the transmission probability for scattering through a downward step ($V_0 > 0$)

$V = 0$ for $x < 0$

$V = -V_0$ for $x > 0$



Thermodynamics

As a physics grad student left the building late at night, she heard a loud "thud" behind her. She turned around to see that a 20-kg bag of gravel had fallen to the concrete from the top of the building, 40 m above the ground.

$\Delta U = 8000\,J$

a. What was the internal energy change ΔU of the bag of gravel as it landed on the ground? 0

b. What was the internal energy change ΔU of the concrete on which it landed? Ignore any work done by air resistance. 0

c. What was the entropy change ΔS of the bag of gravel as it landed on the ground? Assume that the specific heat capacity of the gravel is 0.75 kJ/(kg·K), and that the ambient temperature (and the initial temperature of the gravel) was 280 K. $\sim 32\, J/K$

$28.5\, J/K$

Over time, the gravel cooled back to ambient temperature by transferring heat to the surroundings.

d. Assuming that the surroundings are infinite, what was the entropy change of the surroundings as it absorbed this heat from the gravel? 32 J/K

e. What was the entropy change of the gravel as it transferred heat to the surroundings? -32 J/K

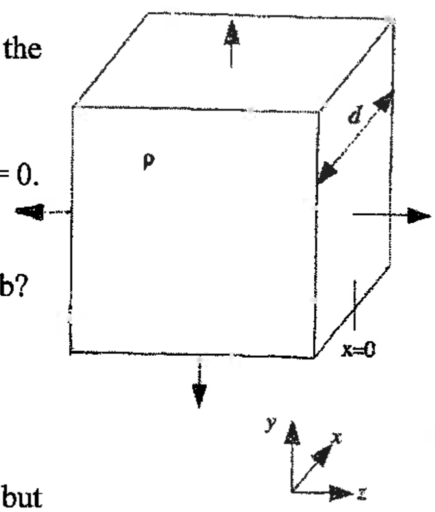
f. What was the total entropy change of the universe as the gravel transferred heat to the surroundings? 0

g. What was the total entropy change of the universe from before the bag fell to after it came to thermal equilibrium with the surroundings? 32 J/K

E&M

✓

A slab of uniform thickness d, from -d/2 to +d/2 along the x-axis, extends infinitely in the y and z directions, as shown. The slab has uniform volume charge density ρ. The electric field is zero in the middle of the slab, at x = 0.



a. Which of the following statement is true of the electric field at the surface of one side of the slab?
   1. The direction of E is constant, but its magnitude varies across the surface.
   2. Both the magnitude and direction of E are constant across the entire surface.
   3. The direction of E varies across the surface, but its magnitude is constant.
   4. Both the magnitude and direction of E vary across the surface.

b. The surface normal points in the +x direction on one side of the slab, and in the –x direction on the other. What is the angle θ that the field E makes with respect to the surface normal?

c. To calculate E on one of these surfaces, we will use Gauss' Law. Describe the location and shape of your choice for the Gaussian surface.

d. What is the total enclosed charge within your surface?

e. What is E at the surface?

f. Sketch the magnitude of the electric field as a function of x.

$$E = \begin{cases} \dfrac{\rho d}{2\varepsilon_0} & |x| > d/2 \\ \dfrac{\rho x}{\varepsilon_0} & |x| \le d/2 \end{cases}$$

Graduate Exams Part I - B

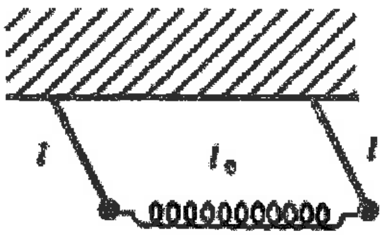
Saturday, September 8, 2012

Mechanics

✓ 7

The bobs of two simple pendula each have mass m and are attached to a string of length l as shown. If the two pendula are coupled by a massless spring of constant k, then find the higher frequency of oscillation of the system.

$\omega = \sqrt{\frac{k}{m}}$



$L = \frac{1}{2} m l^2 (\dot{\theta}_A^2 + \dot{\theta}_B^2) + mgl$

$- k l^2 + k l^2 \cos(\theta_1 + \theta_B)$

E&M

Consider an infinitely large x - y plane boundary between two uniform dielectric materials at $z = 0$, where the index of refraction is n_1 for $z < 0$ and it is n_2 for $z > 0$. Consider an incident plane polarized (in the x - z plane) electromagnetic wave, of frequency ω, which propagates from the domain $z < 0$ into the domain $z > 0$. If the incident wave vector (in the x - z plane) makes an angle of θ_1 with respect to the z axis, determine the angle, θ_2, that the wave vector makes with respect to the z axis in the $z > 0$ domain, expressed in terms of the given parameters. In addition, determine the precise description of the wave vector in the $z > 0$ domain, expressed in terms of the given parameters and the speed of light in vacuum, c. Finally, given the magnitude of the electric field, E_2 ,in the $z > 0$ domain, determine the magnetic field time, t , and space, x, y, z , dependent vector in the $z > 0$ domain.

$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2} \sin\theta_1\right)$

✗ **Quantum Mechanics**

There are two spin 1/2 particles in a 1-D box and their interaction Hamiltonian is

$\hat{H}_{int} = C_1\vec{S}_1 \bullet \vec{S}_2 + C_2(\hat{S}_{1z} + \hat{S}_{2z})$, where $\vec{S}_1, \hat{S}_{1z}$ and $\vec{S}_2, \hat{S}_{2z}$ are corresponding spin operators of the two particles.

a. If the two particles are distinguishable, what are the possible eigenvalues and corresponding spin eigenstates of the interaction Hamiltonian?

b. If these two particles are identical and we cannot distinguish them, and suppose the space wavefunction is

$$\varphi(x_1, x_2) = \frac{1}{\sqrt{2}}[Sin(\frac{\pi x_1}{a})Sin(\frac{3\pi x_2}{a}) - Sin(\frac{3\pi x_1}{a})Sin(\frac{\pi x_2}{a})],$$

what are the possible eigenvalues and corresponding eigenstates of the interaction Hamiltonian?

Thermodynamics

X

It is possible to make crystalline solids that are only one layer of atoms thick. Such "two-dimensional" crystals can be created by depositing atoms on a very flat surface.

a) What is the (molar) heat capacity near room temperature? Express your answer in terms of R.

$C = T\frac{dS}{dT} + nR$ or $\frac{C}{n} = \frac{TdS}{ndT} + R$

b) Explain how your answer would change if the crystal was cooled to very low temperatures (i.e., close to zero Kelvin).

$C = nR$

Qualifying exam, undergraduate material section

29 August 2009

1) Mechanics I

Consider a 2D (in $x-y$) projectile motion of a point mass, m, under the influence of gravitational acceleration, g, in the vertical direction, assuming no air resistance.

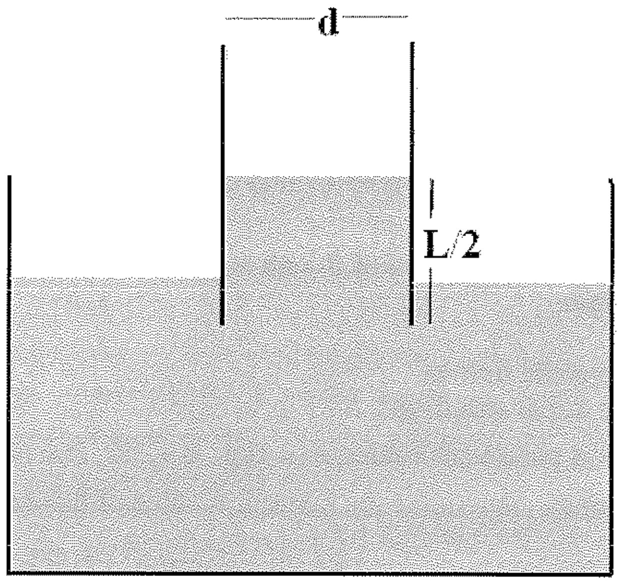
Calculate the minimum initial speed of the projectile, v_0, which is necessary for it to just clear a fence at a vertical height, $y=h$, a horizontal distance, D, from the origin, $x=0$.

Assume the projectile is fired at an angle of θ with respect to the horizontal, at an initial height y_0, above the origin.

a. Sketch the projectile motion.
b. Determine the initial speed, v_0.
c. Given numerical parameter values, $m = 1\text{kg}$, $\theta = 35°$, $D = 60\text{m}$, $h = 2\text{m}$, $y_0 = 0.7\text{m}$, $g = 9.8\text{m/s}^2$, determine the numerical value of the initial speed.

2) Electromagnetism I

A parallel plate capacitor with square plates of side L and plate separation d is charged to a potential V and disconnected from the battery. It is then vertically inserted into a large reservoir of dielectric liquid, with relative dielectric constant ε and density ρ, until the liquid fills half the space between the capacitor plates as shown in the figure below.



a) What is the capacitance of the system?
b) What is the electric field strength between the capacitor plates?
c) What is the distribution of charge density over the plates?
d) What is the difference in vertical height between the level of liquid within the capacitor plates and that in the external reservoir?

3) Quantum Mechanics I

One thousand free particles, $N_0 = 1000$, are contained in a one-dimensional box, between infinite walls at $x = 0$ and $x = a$, and initially, at $t = 0$, the wavefunction for each particle, of mass m, is $\Psi(x,0) = Ax(x - a)$.

a. Normalize the wavefunction, to determine A, and write down the normalized initial individual particle wavefunction, $\Psi(x,0)$.

b. Write down the initial individual spatial probability density, $P(x)$, and determine the number of particles, N, that are initially contained in the domain .

4) Thermodynamics I

If 2 [kg] of liquid water at 90[°C] is mixed adiabatically and at constant pressure with 3[kg] of liquid water at 10[°C], what is the total entropy change resulting from this process?

Hint: Take the heat capacity of water to be constant at $C_p = 4184$ *[J/K•kg]*

5) Mechanics II

A particle of mass m is attracted toward a given point by a force of magnitude , where k is a constant. Derive an expression for the Hamiltonian and Hamilton's equations of motion.

Use polar coordinates (r,θ).

6) Electromagnetism II

Consider a medium with nonzero conductivity σ (gives the current density) and no net charge ($\rho = 0$).

a) Write the set of Maxwell's equations appropriate for this medium.

b) Derive the wave equation for (or) in this medium,

) =

c) Consider a monochromatic wave moving in the +x direction with E_y (or E_z or B_y or B_z) given by

Show that this wave has an amplitude which decreases exponentially; find the attenuation length (skin depth).

7) Quantum Mechanics II

Consider a spin-½ particle which is bound in a three-dimensional harmonic oscillator with frequency ω. The ground state Hamiltonian H_0 and spin interaction are

Where λ is a constant and are the Pauli matrices.

Neglect the spin-orbit interaction. Use perturbation theory to calculate the change in the ground state energy to order $O(\lambda_2)$.

8) Thermodynamics II

You have two-state atoms in a thermal radiation field at temperature T. The following three processes take place:

a) Atoms can be promoted from state 1 to state 2 by absorption of a photon according to

b) Atoms can decay from state 2 to state 1 by spontaneous emission according to

c) Atoms can decay from state 2 to state 1 by stimulated emission according to

The populations N_1 and N_2 are in thermal equilibrium, and the radiation density is

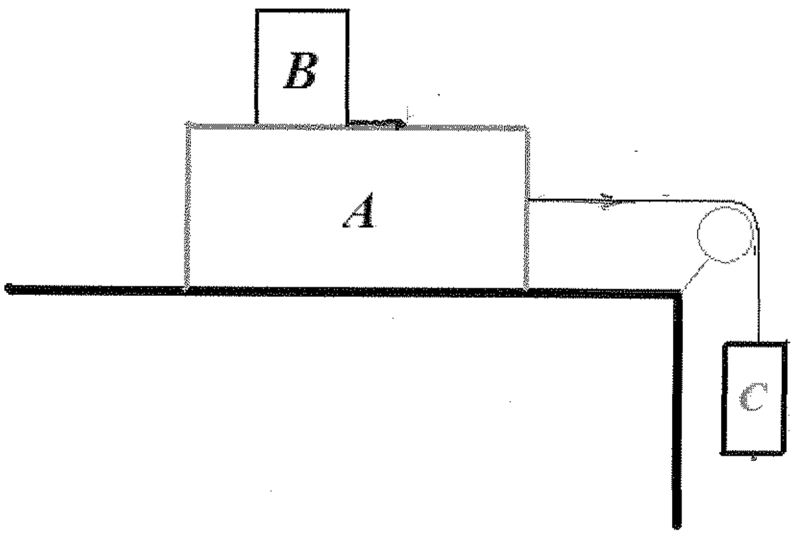
A) What is the ratio ?
B) Calculate the ratio of coefficients .

Qualifying exam, undergraduate material section

17 January 2009

1) Mechanics I

Block B rests on Block A, which in turn is on a horizontal tabletop. There is no friction between Block A and the tabletop, but the coefficient of static friction between Block A and Block B is μ_{AB}. A light string attached to Block A passes over a frictionless, massless pulley, and Block C is suspended from the other end of the string (see figure below).



What is the largest mass that Block C can have so that Blocks A and B still slide together when the system is released from rest?

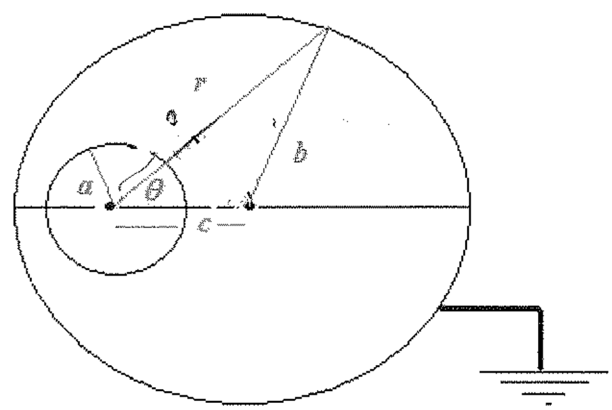
2) Electromagnetism I

As can be seen in the figure below, the inner conducting shell of radius a carries charge Q, and the outer shell of radius b is grounded. The distance between the centers of the spheres is c, which is a small quantity.

Show that to first order in c, the equation describing the outer sphere is

$$r(\theta) = b + c \cdot \cos\theta \, .$$

Hint: Use the center of the inner sphere as origin.



3) Quantum Mechanics I

A particle of mass m moves in one dimension where the only potential

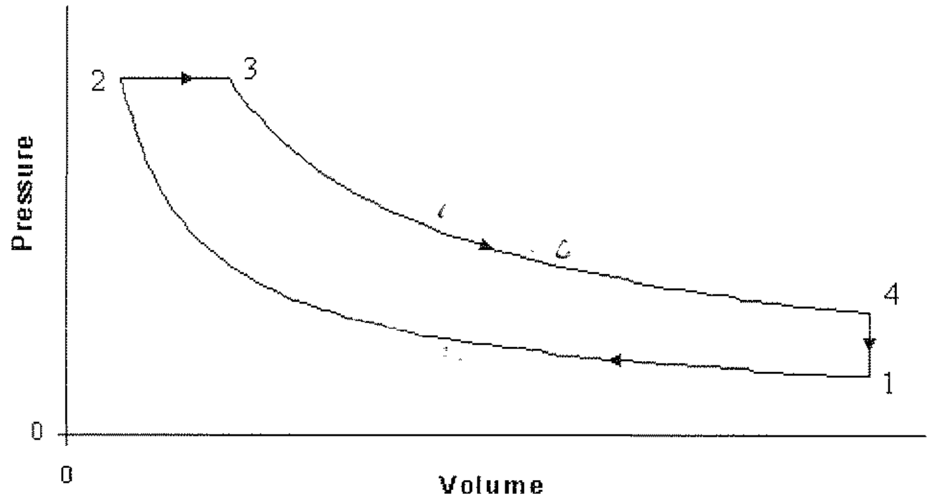
$$V(x) = C \cdot \delta(x)$$

is at the origin with C > 0. A free particle of wave vector k approaches the origin from the left.

Derive an expression for the amplitude T of the transmitted wave as a function of k, C, m, and * .

4) Thermodynamics I

One mole of an ideal monatomic gas undergoes the cycle shown. Process 1→2 is adiabatic, and process 2→3 takes place at constant pressure. Process 3→4 is also adiabatic, and process 4→1 takes place at constant volume.



The temperatures of the gas at points 1, 2, 3, and 4 are:
$T_1 = 300$ K, $T_2 = 1600$ K, $T_3 = 1800$ K, $T_4 = 365$ K

a) Find V_1/V_2, the ratio of the volume of the gas at point 1 to the volume at point 2.

b) Find the magnitude of the heat added to the gas in the process 2→3. Express the answer in terms of the gas constant R.

c) Find the magnitude of the work done by the gas in the process 2→3. Express the answer in terms of the gas constant R.

d) Find the magnitude of the heat removed from gas in the process 4→1. Express the answer in terms of the gas constant R.

e) How much work is done by the gas in one complete cycle? Express the answer in terms of the gas constant R.

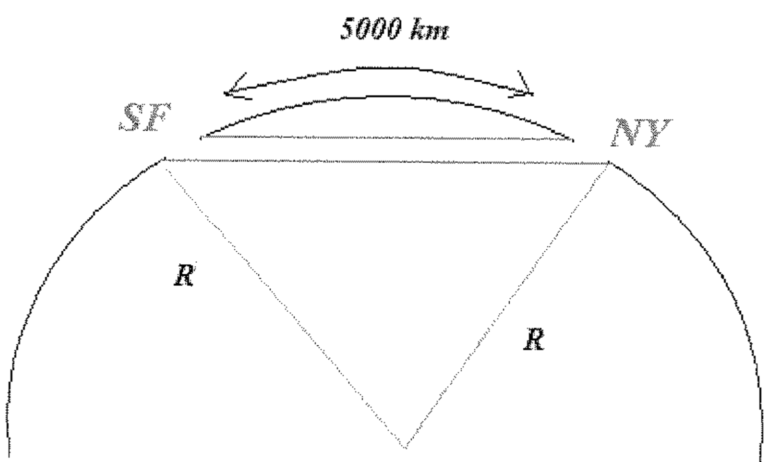
f) Find the efficiency of a heat engine operating with this cycle.

5) Mechanics II

A straight tunnel is dug from New York to San Francisco (SF), a distance of 5000 kilometers measured along the surface. A car rolling on steel rails is released from rest at NewYork (NY), and rolls through the tunnel to San Francisco (see figure below; note: tunnel not shown to scale in figure).

(a) Neglecting friction and also the rotation of the Earth, how long does it take to get there?
Hint: Take the gravitational acceleration to be $g = 980\ cm/s^2$ *and the radius of the Earth* $R = 6400\ km$.

(b) Suppose there is now friction proportional to the square of the velocity (but still ignoring the rotation of the Earth). What is the equation for the phase space trajectory?
Hint: Introduce suitable symbols for the constant of proportionality and for the mass of the car, and also draw a sketch.



6) Electromagnetism II

What is the attenuation distance for a plane wave propagating in a good conductor? Express your answer in terms of the conductivity σ, permeability μ, and frequency ω.

7) Quantum Mechanics II

Consider three particles of spin ½ which have no motion.

The raising ($s^+ = s_x + is_y$) and lowering ($s^- = s_x - is_y$) operators of the individual spins have the property

$$s^+|\downarrow\rangle = |\uparrow\rangle$$
$$s^-|\uparrow\rangle = |\downarrow\rangle,$$

Where the arrows indicate the spin orientation with regard to the z-direction.

(a) Write explicit wave functions for the four J = 3/2 states: M = 3/2,1/2,-1/2,-3/2.

(b) Using the definition that $J^\pm = \sum_i s_i^\pm$, construct the 4x4 matrices which represent the J_+ and J_- operators.

8) Thermodynamics II

Consider a system consisting of 4 spins, labeled 1, 2, 3, or 4. Each spin points up or down and has an energy $-\varepsilon$ if pointing up (parallel to the magnetic field) and an energy ε if pointing down (antiparallel to the magnetic field). Suppose the spin system is in contact with a heat reservoir which is at a constant temperature of T.

a.) What is the partition function Z for a single spin?

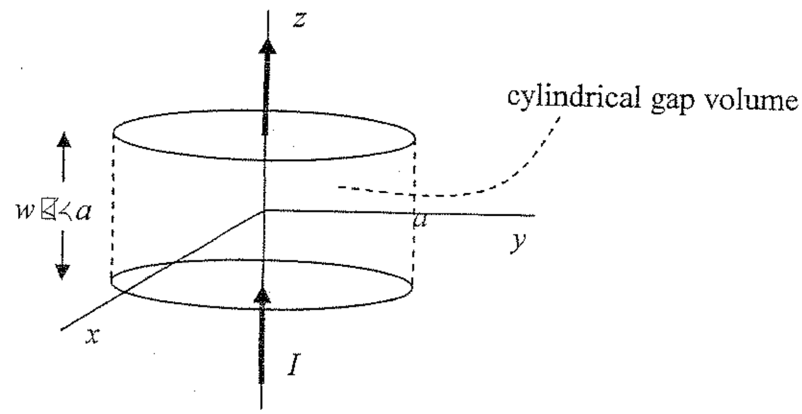
b.) What is the average energy of the 4-spin system at temperature T?

c) What is the energy of this 4-spin system in the limit of $T \to \infty$?

Undergraduate Physics
Comprehensive examination 2007
Part I

Problem 1.

Consider electric & magnetic fields, $\mathbf{E}$ & $\mathbf{B}$, in the cylindrical gap volume between circular capacitor plates parallel to the $x-y$ plane, of radius a, separated by a much smaller distance, $w \ll a$, as shown below. Assume a constant current, I, flows in the $\hat{\mathbf{z}}$ direction, starting at $t=0$. As functions of time, t: a) determine the electric, $\mathbf{E}$, and the magnetic, $\mathbf{B}$, fields between capacitor plates; and b) determine the associated Poynting vector, $\mathbf{S}$, on the surface of the volume at radius a.



Hint – consider cylindrical coordinates (s,ϕ,z), applied to the integral version of Maxwell's equations.

Problem 2.

A slab of uniform thickness –d/2 to d/2 along the x-axis, extends infinitely in the y and z directions. The slab has uniform volume charge ρ. The electric field is zero in the middle of the slab, at x = 0.

(a) What is a total enclosed charge within the surface?

(b) What is E at the surface?

(c) Sketch the magnitude of the electric field as function of x.

Problem 3

Derive continuity equation for the probability density

$$\frac{\partial \rho}{\partial t} + div\mathbf{j} = 0; \ \rho = |\psi(\mathbf{r},t)|^2$$

for a particle in a magnetic field. What is the expression for probability current j? Use the following Hamiltonian for this problem:

$$H = \frac{(-i\hbar\nabla + e\mathbf{A})^2}{2m} + V(\mathbf{r})$$

where **A** is a vector potential defined as follows:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Problem 4.

Imagine two noninteracting particles, each of mass *m*, in the infinite square well. If one is in the state *n* and the other is in the state *l* (l is not equal to n), calculate $<(x_1 - x_2)^2>$, assuming (a) (10 points) they are distinguishable, (b) (10 points) they are identical bosons, and (c) (10 points) they are identical fermions.

Undergraduate Physics
Comprehensive examination 2007
Part II

Problem 5.

A mass m_1, with initial velocity V_0, strikes a mass-spring system m_2, initially at rest but able to recoil. The spring is massless with the spring constant k. There is no friction.
(a) What is the maximum compression of the spring?
(b) If. long after the collision, both objects travel in the same direction, what are the final velocities V_1 and V_2 of m_1 and m_2, respectively.

Problem 6.

Determine the oscillations (trajectories) of a system with two degrees of freedom whose Lagrangian is

$$L = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}\omega_0^2(x^2 + y^2) + \alpha xy .$$

Find the normal coordinates.

Problem 7.

The three lowest energy levels of a certain molecule are $E_1 = 0$, $E_2 = \varepsilon$, and $E_3 = 10\varepsilon$. Show that at sufficiently low temperatures (how low?) only levels E_1 and E_2 are populated. (a) Find the average energy E of the molecule at temperature T. (b) Find the contribution of these levels to the specific heat per unit mole, C_v, and (c) sketch C_v as a function of T.

Problem 8.

The quantum energy levels of a rigid rotator are

$$\varepsilon_j = j(j+1)\hbar^2/8\pi^2 ma^2$$
$$j = 0,1,2...$$

The degeneracy of each level is

$$g_j = 2j + 1.$$

(a) Find the expression for the partition function, and show that at high temperatures it can be approximated as an integral.
(b) Evaluate the high temperature energy and local capacity.
(c) Find the low temperature approximations to Z, U, and C_V.

Undergraduate Physics -1 (2006)

Problem 1.
Consider the flyball governor for steam engine shown in Figure 1. Two balls, each of mass m, are attached by means of four hinged arms, each of length l, to sleeves on a vertical rod. The upper sleeve is fastened to the rod; the lower sleeve has mass M and is free to slide up and down the rod as the balls move from or in toward the rod. The rod-and-ball system rotates with constant velocity ω. Set up the equation of motion, neglecting the weight of the arms and rod. Use as variable the distance y between the sleeves.

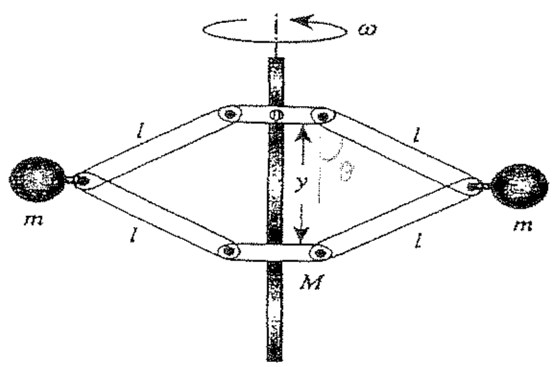


Figure 1.

Problem 2.
A person on Earth observes two rocket ships moving directly toward each other and colliding as shown in Fig 2a. At time $t = 0$ in the Earth frame, the Earth observer

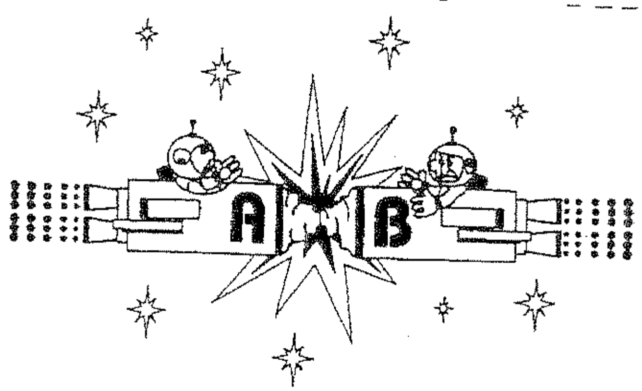


Figure 2a.

determines that rocket A, traveling to the right at $v_A = 0.8c$, is at point a, and rocket B is at point b, traveling to the left at $v_B = 0.6c$. They are separated by the distance $l = 4.2 \bullet 10^8 m$ (see Fig. 2b)

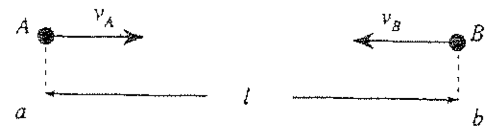


Figure 2.b.

a) In the Earth frame, how much time will pass before the rockets collide?
b) How fast is rocket B approaching in A's frame? How fast is rocket A approaching in B's frame?
c) How much time will elapse in A's frame form the time rocket A passes point a until collision? How much time will elapse in B's frame from the time rocket B passes point b until collision?

Problem 3.
An electric dipole of moment p is placed at a height h above a perfectly conducting plane and makes an angle of θ with respect to the normal to the plane (see Fig. 3).

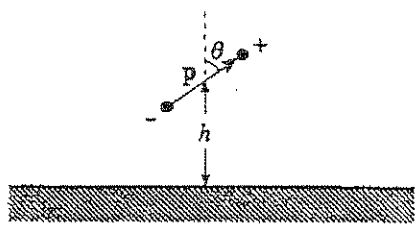


Figure 3.

a) Indicate the position and orientation of the image dipole and the direction of the force left by the dipole.
b) Calculate the work required to remove the dipole to infinity.

Problem 4.
A hollow cylinder or radius r and height h has a total charge q uniformly distributed over its surface. The axis of the cylinder coincides with the z-axis, and the cylinder is centered at the origin. (a) Calculate the electric potential at the origin. (b) Evaluate the potential in the limit of $h << r$.

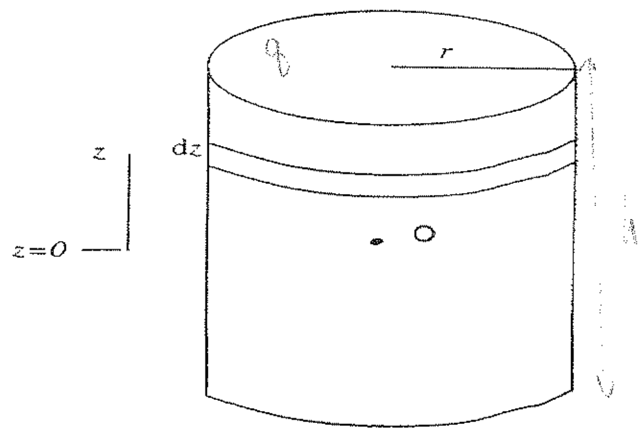


Figure 4.

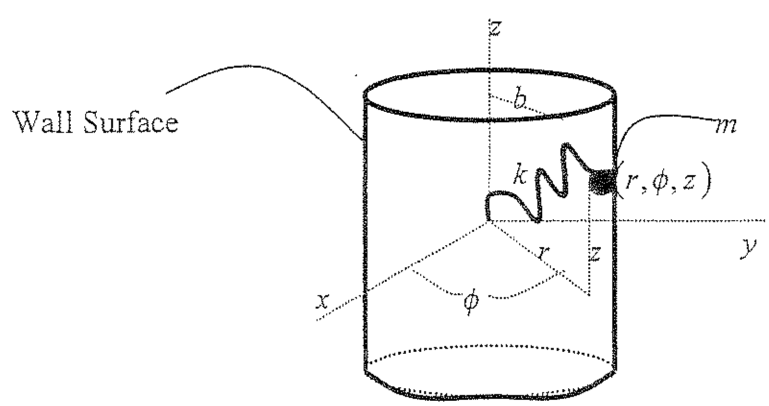
Useful formulas: $\int da\sqrt{a^2+b^2} = \ln(a/b+\sqrt{1+a^2/b^2})$; $\int da 1/\sqrt{a^2+b^2} = (1/b)\arctan(a/b)$

Physics (undergraduate level) -2 (2006)

Problem 1.
Construct the spin matrices (S_x, S_y, and S_z) for a particle of spin 1.
Hint: How many eigenstates are there? Determine the action of S_z, S_+, and S_- on each of these states.

Problem 2.
Calculate the maximal work obtained after the connection of two vessels with two ideal gases having the same temperature T_0 and the number of particles N but the different volumes V_1 and V_2.

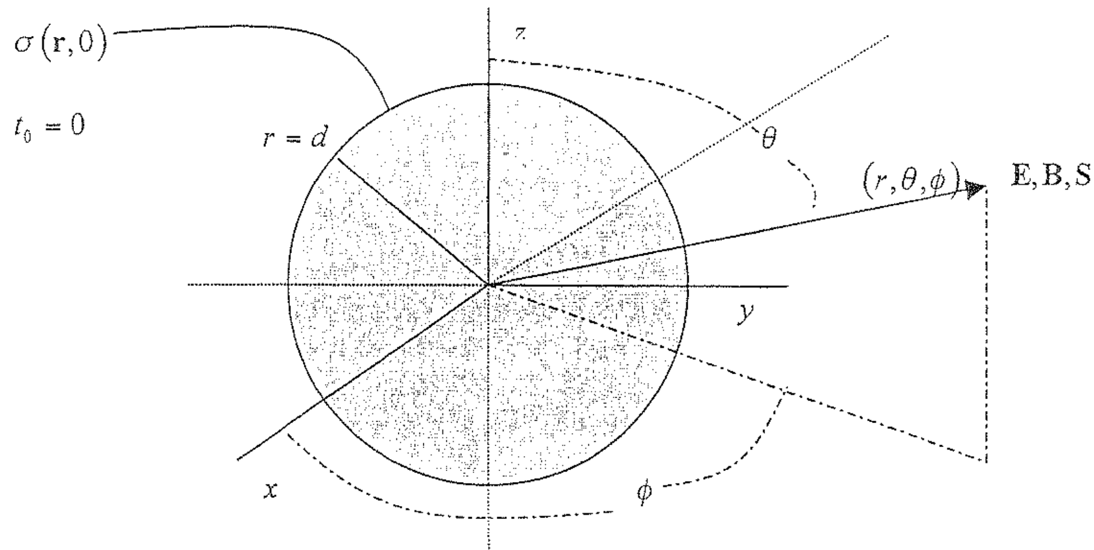
Problem 3.
Consider a mass, m, which moves in cylindrical coordinates, (r,ϕ,z), which is connected to a massless spring, of spring constant k, such that the restoring force, $\mathbf{F}_{\text{spring}} = -k(r\hat{\mathbf{r}} + z\hat{\mathbf{z}})$, is along the vector line going from the mass to the origin, $r = z = 0$. In addition, consider the situation that the mass moves without friction or gravity influences, on the inside wall surface of a long cylinder, of radius b, which encircles the z axis, as shown below. **a) Determine the equations of motion for each mass coordinate, r,ϕ,z, expressed using the given parameters. b) Finally, solve the equations for the mass coordinates, r,ϕ,z, as functions of time, t; and determine the force, Q, that the wall must exert, and describe its composition. Note all initial conditions which must be assumed.**



Hint: You might consider solving this problem using either 1) Force analysis, or 2) Lagrangian analysis, with a Lagrange multiplier for the constraint determination.

Problem 4.

Consider electric & magnetic fields, $\mathbf{E}(\mathbf{r},t)$ & $\mathbf{B}(\mathbf{r},t)$, associated with electric dipole, $\mathbf{p}$, radiation, in large, r, radius "radiation zone" domain. Here, a harmonically oscillating surface charge distribution produces the dipole radiation field; assume at the initial retarded time, $t_0 = 0$, the charge is distributed over a spherical shell of radius $r = d$, where $\sigma(\mathbf{r},0) = \dfrac{q_0 \cos\theta}{4\pi d^2}$. **a) Determine initial, $t_0 = 0$, total charge, $q(0)$, contained on the sphere; also, determine the initial electric dipole moment, $\mathbf{p}(0)$, expressed in the given parameters, q_0, d, using Cartesian unit vectors, $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$, and do not guess, do integrals. b) For the situation that the surface charge density oscillates harmonically, as $\sigma(\mathbf{r},t_0) = \sigma(\mathbf{r},0)\cos(\omega t_0)$, determine the electric dipole moment, $\mathbf{p}(t_0)$, as a function of the retarded time, t_0, and the electric, magnetic, & Poynting vector** fields, $\mathbf{E}(\mathbf{r},t)$, $\mathbf{B}(\mathbf{r},t)$, & $\mathbf{S}(\mathbf{r},t)$, as a function of a general position vector, $\mathbf{r}$, and time, t, expressed using the given parameters, ω, q_0, d, μ_0, c, spherical coordinates, (r,θ,ϕ), and unit vectors, $\hat{\mathbf{r}},\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\phi}}$.



Hints: 1) E&M dipole fields are $\mathbf{E}(\mathbf{r},t) = \frac{\mu_0}{4\pi r}\left\{\hat{\mathbf{r}} \times \left[\hat{\mathbf{r}} \times \ddot{\mathbf{p}}(t_0)\right]\right\}$& $\mathbf{B}(\mathbf{r},t) = -\frac{\mu_0}{4\pi c r}\left[\hat{\mathbf{r}} \times \ddot{\mathbf{p}}(t_0)\right]$; and 2) Retarded time is $t_0 = t - r/c$.

2005

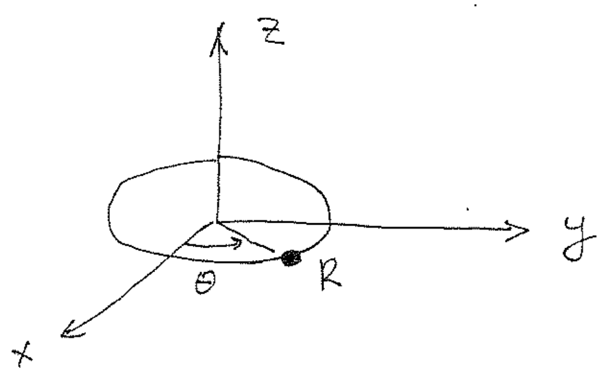
Day 1 Morning
Undergraduate Physics

Problem 1. Treating the Earth's atmosphere as an incompressible fluid that forms a thin layer above the surface of Earth (radius $\boldsymbol{R}$ and mass M), so that the gravitational acceleration, g, is a constant at any height within a cylinder of thickness, h. Write an expression for the pressure, P, at the surface of Earth.

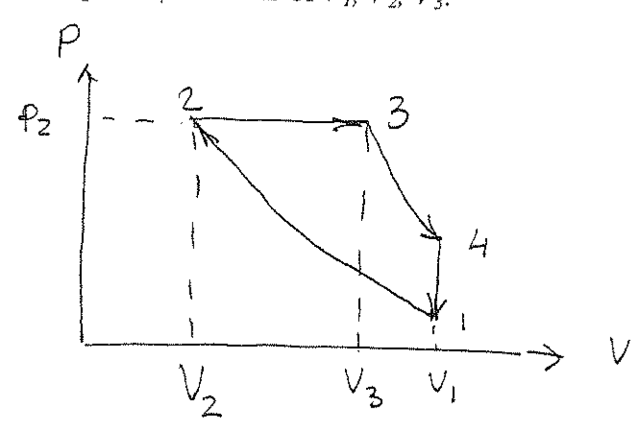
Problem 2. Consider a rocket launch from the surface of Earth. The rocket's initial mass is *2.8* million kg, *2.1* million of which is a full tank of fuel. Assume a constant thrust (force) of *37* million newtons, and an exhaust velocity of *2600* m/s. What is the final velocity at burnout when the fuel is consumed? What is the maximum vertical height? Assume that g is constant.

Quantum Mechanics 1

Problem 3. What are the energies and eigenstates for a particle of mass m moving on a ring of radius $\boldsymbol{R}$ as shown in *Fig. 1.*

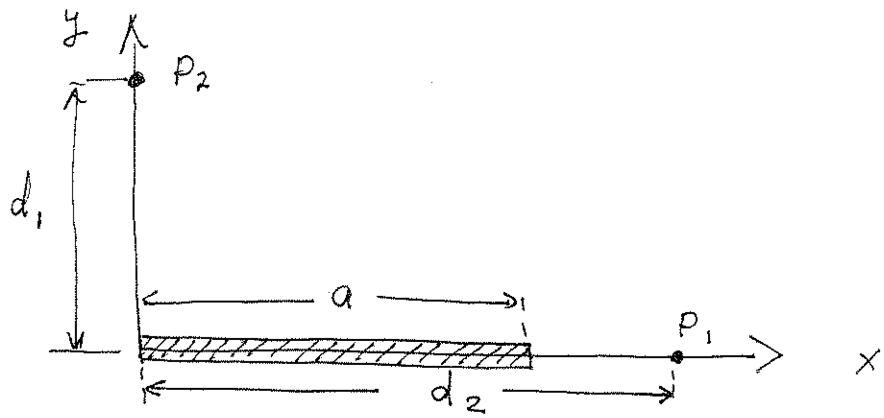


Problem 4. Calculate the efficiency of the Diesel cycle consisting of two adiabats, $1 \rightarrow 2, 3 \rightarrow 4$; and one isobar; $2 \rightarrow 3$ and are constant volume process $4 \rightarrow 1$. Assume C_v as C_p are constant. Express η in terms of V_1, V_2, V_3.



Day 1 Afternoon
Undergraduate Physics

Problem 5. A wire of length a and charge Q is lying on the x-axis as shown. What is the potential at points P_1 and P_2?



Problem 6. A cylindrical wire of permeability μ carries a steady current I. If the radius of the wire is $\boldsymbol{R}$, find $\boldsymbol{B}$ and $\boldsymbol{H}$ inside and outside the wire.

Problem 7. A ~~water~~ meter-stick moves parallel to its length with speed v=0.6c relative to you.

a) Find the length of the stick measured by you
b) How long does the stick take to pass you?
c) What is the ratio of the stick's rest mass energy to its kinetic energy?

Problem 8. Find the minimum energy a gamma ray must have to initiate the reaction $\gamma + p \to \pi^0 + p$ if the target proton is at rest. Note that $m_p c^2 = 940 MeV$ and $m_\pi c^2 = 140 MeV$.

Part I: Section A

Do each of the six (6) problems in Section A numbered 1 through 6 inclusive.

1. A particle of mass m is constrained to move on the surface of a right circular cylinder defined by $x^2 + y^2 = R^2$ and is subjected to a force proportional to the distance from the origin and directed toward the origin, $F_r = -k\sqrt{x^2+y^2+z^2}$. Select appropriate generalized coordinates, find the Lagrangian for the system, find the equation of motion, and solve these for the motion of the particle in time. Identify any constants of motion and describe how the spring constant k will affect the motion.

2. (a) Find an expansion for the Green function in spherical coordinates for the Laplace-Poisson operator outside a sphere of radius R which obeys the boundary conditions

$$\hat{n} \cdot \nabla\, G\,(\vec{r}, \vec{r}\,') = 0 \text{ with } \vec{r} \text{ on the sphere and}$$

$$\lim_{|\vec{r}| \to \infty} G(r, \vec{r}\,') = 0.$$

Reduce the problem to a one dimensional Green function equation in the radial coordinate.

(b) If the electrostatic potential at infinity is zero, if the charge density in space is zero, and if the normal component of the $\vec{E}$ field on the sphere is

$$E_r = \frac{1}{2} E_0 (3\cos^2\theta - 1),$$

find the electrostatic potential everywhere outside the sphere.

3. The electric fields for two monochromatic waves of circular frequency ω and vector amplitudes $\vec{A}_1$ and $\vec{A}_2$ are given by

$$\begin{aligned} \vec{E}_1 &= \vec{A}_1, \cos(\delta_1 - \omega t) \\ \vec{E}_2 &= \vec{A}_2 \cos(\delta_2 - \omega t) \end{aligned}$$

where δ_1 and δ_2 are phase factors. If these two electric fields are added, find a general expression for the irradiance , $I = < \vec{E}^2 >$, in terms of the phase difference $\delta_1 - \delta_2$. What is the value of I when $\vec{A}_1$ is orthoganal to $\vec{A}_2$? You may wish to recall that

$$< \cos(\delta_1 - \omega t)\ \cos(\delta_2 - \omega t) > = \cos(\delta_1 - \delta_2).$$

4. A solid contains a collection of N paramagnetic ions each having a total angular momentum j and a magnetic dipole moment given by

$$\vec{\mu} = \mu_o \vec{J}$$

where $\vec{J}$ is the angular momentum operator in units of $\hbar$. Find the partition function for this system in a uniform magnetic field $\vec{B}$, neglecting the dipole - dipole interaction, and carry out all indicated summations. From it, find the magnetic moment of the system.

5. A particle of mass m is governed by a one dimensional, time independent Schroedinger equation with an attractive potential, whose form is a Dirac δ-function, located at the origin,

$$V(x) = -V_0\,\delta(x)$$

(a) Give the Schroedinger equation in coordinate space. (b) Transform the equation and the wave function to momentum space. (c) Solve for the bound state wave function in momentum space. This solution implies that the total energy must be negative. (d) Transform the wave function to coordinate space. (e) Determine the eigenvalue of energy which will make the expression for the wave function consistent.

6. (a) You measure the current flowing through a resistor to be $I = 0.100 \pm 0.005 A$ and independently measure the resistance of that same resistor to be $R = 100 \pm 5\Omega$. What is the power flowing through that resistor and the error in that estimate if the errors in the measurements obey Gaussian statistics?

   (b) You are attempting to measure the amplitude of pulses roughly 1 μs long from a photomultiplier preamp through a long cable, but find the signal contaminated by a sizable 60 Hz background added most likely by unavoidable ground loops. Describe how you might salvage the experiment using some miscellaneous resistors and capacitors lying about the laboratory, and make the description as quantitative as possible.

Do any one (1) of the three (3) problems in Section B numbered 7 through 9 inclusive.

7. (a) Using the notation for $L-S$ coupling, give the complete electronic configuration of a sodium atom with a nucleus $^{23}Na^{11}$ in its ground state.

   (b) Give a simple sketch with appropriate spectroscopic labeling showing the lowest two excited, electronic levels for the sodium atom with the lowest allowed principal quantum number, identify the fine structure splitting, and show the transitions allowed via electric dipole coupling.

   (c) Give a second sketch showing the effect of an external magnetic field on these electronic levels.

   (d) Give a third sketch showing how the nuclear spin ($I = \frac{3}{2}$) splits and shifts the levels forming a hyperfine structure and how the hyperfine levels react to a small external magnetic field.

8. The probability for an electron to occupy a state at a finite temperature is given by the Fermi distribution

$$f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1}$$

where E_f is the Fermi energy. All electrons in an intrinsic semiconductor at $0°K$ are in the valence bands and completely fill these bands. At a finite temperature, a number of electrons are in the conduction band and hence an equal number of vacancies appear at the top of the highest valence band. Since the band gap is large compared to kT, the Fermi energy is approximately in the middle of the gap. An n-type semiconductor is doped with donor impurities that easily give electrons to the conduction band; hence the majority carriers are electrons. A p-type semiconductor is doped with acceptor impurities that easily absorb electrons from the valence band; hence the majority carriers are holes.

(a) How is the Fermi level affected by n-type doping? How is it affected by p-type doping?

(b) Draw a diagram indicating the location of the Fermi level, the bottom of the conduction band, and the top of the valence bands at the junction of an n-type semiconductor and a p-type semiconductor.

(c) Indicate the direction of the electric field generated at this junction.

9. (a) The total angular momenta, parities, and energies of the lowest few excited states of many nuclei whose neutron and proton numbers are both far from "magic numbers" are well described by rotational and vibrational collective motion. For a certain nucleus with $A = 180$ and $Z = 72$, ^{180}Hf, the ground state and lowest four (4) excited states are known to form a pure rotational band with no vibration. Give the total angular momentum and parity for each of these five (5) states. Why?

(b) The first excited state is 93.3 keV above the ground state. If the energy spectrum is similar to that for some states of a rigid rotor, give the lowest order formula for these levels and calculate the energies predicted for the other three (3) excited states.

(c) How and why would you expect the measured energies to deviate from the predictions? How might you "correct" the formula?

Do each of the four (4) problems in Section A numbered 1 through 4 inclusive.

1. The gamma function may be defined by the integral

$$\Gamma(z) = \int_0^\infty e^{-t}\ t^{z-1}\ dt\ ,\quad Re(z)\ > 0.$$

Use the Cauchy-Goursat theorem to prove that, for x real and $0 < x < 1$,

$$\int_0^\infty \cos t\ \ t^{x-1}\ dt \ = \ \Gamma(x)\cos(\frac{\pi}{2}x)$$

$$\int_0^\infty \sin t\ \ t^{x-1}\ dt \ = \ \Gamma(x)\sin(\frac{\pi}{2}x)$$

Select a contour integral and contour appropriate for the task, discuss any singularities in the integrand with care and indicate these on a sketch of the t-plane, and evaluate integrals on unwanted parts of the contour going to zero or infinity with great care.

2. A point mass m is suspended by a massless rod of length l in a uniform gravitational field to form a spherical pendulum which can move without friction in both the azimuthal coordinate ϕ and the angular deviation from verticle θ. (a) Select generalized coordinates and find the Lagrangian. Give a drawing which clearly shows the coordinates. (b) Find the canonical momenta and find the Hamiltonian for the system. (c) Find Hamilton's equations of motion for the system and identify any cyclic coordinates. (d) If the coordinate θ is constant, complete the solution.

3. (a) A point charge of magnitude Q moves on a trajectory $\vec{r}_\circ(t)$. Find expressions for the charge density and the associated current density.

   (b) The Green function for a retarded solution to the wave equation with a source at $\vec{r}\,'$ and t' is

$$G(\vec{r},t;\vec{r}\,',t') \ = \ \frac{1}{|\vec{r}-\vec{r}\,'|}\ \ \delta(t-t'-\frac{|\vec{r}-\vec{r}\,'|}{c}),$$

   where the δ denotes a Dirac δ - function. Find the scalar and vector potentials for the electromagnetic field in the Lorentz gauge as a function of $\vec{r}$ and t. These are the Lienard-Wiechart potentials. You may wish to use the notation

$$\vec{\beta}(\tau) = \frac{1}{c}\frac{d\,\vec{r}_\circ}{dt}\bigg|_{t=\tau}$$

$$\hat{n}(\tau) = \frac{\vec{r}-\vec{r}_\circ(\tau)}{|\vec{r}-\vec{r}_\circ(\tau)|}.$$

Indicate the retarded time at which quantities must be evaluated clearly.

4. (a) A spinless particle which obeys the time independent Schroedinger equation is scattered from a fixed center by a potential $V(\vec{r})$ under conditions for which the Born approximation is valid. Derive the first order Born approximation.

(b) Calculate the differential cross section in the first order Born approximation for scattering from the potential

$$V(r) = \frac{V_0}{r}\,e^{-\frac{r}{a}}$$

where $a > 0$. You need not bother determining the constant multiplicative factors.

Do any two (2) of the three (3) problems in Section B numbered 5 through 7 inclusive.

5. An astronomer measures the spectrum of a hydrogen emission region in a starburst galaxy which has been reddened by intervening dust. The following line fluxes are observed: $F_{obs}(H\alpha) = 1.0 \times 10^{-12}$ erg s^{-1} cm^{-2} and $F_{obs}(H\beta) = 2.5 \times 10^{-13}$ erg s^{-1} cm^{-2}. Observations of forbidden line ratios indicate that the electrons in the region have a temperature $T_e = 10^4 K$ and a density $n_e = 10^4$ cm^{-3}. Case B recombination theory predicts an intrinsic ratio F_{int} $(H\alpha)$ / $F_{int}(H\beta) = 2.85$ for this temperature and density. The extinction at $H\beta$ is 1.424 times greater than that at $H\alpha$. Assume that the dust exists only in an homogeneous, gas-free, plane-parallel slab lying between the emission region and the observer.
(a) Assuming that the source function is independent of optical depth in the dust slab, write the solution to the radiative transfer equation in the form of intensity emerging from the slab. How does this solution simplify when the dust slab contributes no emission or scattering to the emerging beam?
(b) Determine the optical depth through the dust slab for the $H\alpha$ line.

(c) Calculate the intrinsic $H\alpha$ flux, $F_{int}(H\alpha)$.

(d) Calculate the star formation rate (SFR) for the galaxy if SFR = 1.8 x $10^{11} F_{int}(H\alpha) M\odot$ yr^{-1}.

6. Using the equation of hydrostatic equilibrium, the ideal gas law, and the approximate scaling relations for mean density in terms of stellar mass M^*, radius R^*, and temperature, determine:

(a) The functional dependence of pressure on M^* and R^*,

(b) The functional dependence of temperature on M^* and R^*, and

(c) The functional dependence of luminosity on M^* (i.e., the main sequence mass-luminosity relationship).

7. A "wave front tilt sensor" for a telescope with adaptive optics images a star onto a "quad cell" as drawn below. If the telescope is not aligned with the star, the star is displaced from the center of the quad cell. For this problem, assume that the image of the star is a uniformly illuminated square whose sides are an equivalent 1.0 arcsecond across. Also assume that displacements of the star of Δ arcseconds are always small enough that some flux falls on each of the cells and that all of the flux is detected by the quad cell. The flux on each of the cells if f_1, f_2, f_3, and f_4 as indicated in the drawing; hence the signal for a displacement in x is $S_x = f_1 + f_2 - (f_3 + f_4)$.

(a) Assuming a flux of N photons per second from a star centered in the quad cell and an integration time of δt, calculate the four average fluxes $f_1, f_2, f_3,$ and f_4 and the root mean squared uncertainty in these numbers. (b) If the star is displaced a small distance Δ_x arcseconds, calculate S_x. Using the noise calculated in part (a) above which neglects changes in noise due to Δ_x, calculate the root mean squared uncertainly in S_x. (c) Assuming a telescope with a 2.3 m diameter and a 70% net transmission, a detector with a 1000 Angstrom bandpass at 5500 Angstroms, and the need to measure the tilt to better than 0.1 arcseconds more than 90% of the time within 10 milliseconds, how faint a star can be observed before the noise ruins the measurement? Recall that the flux for a 0^{th} magnitude star is 1000 photons/$(cm^2 \cdot$ sec $\cdot A)$

2004

Day 1

Afternoon

Quantum Mechanics

Problem 1.

Consider a particle beam approximated by a plane wave directed along the z-axis from the left and incident upon a potential $V(x) = \gamma\delta(x)$, where $\gamma > 0$. Calculate the probability of the transmission. Hint: (a) find a *w.f.* for $x < 0$, (b) find a *w.f.* for $x > 0$, and (c) give the condition on the *w.f.* at the boundary between the regions.

Statistical Mechanics

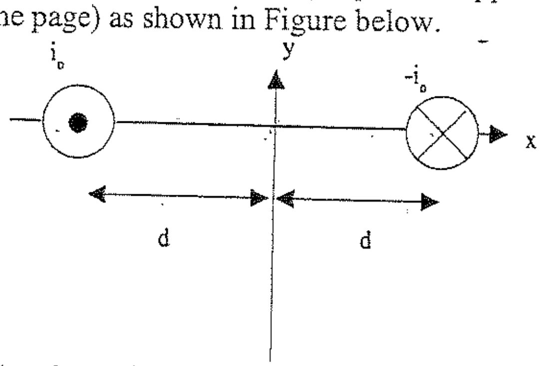
Problem 1.

Sometimes helium gas in a low-temperature physics lab is kept temporarily in a large rubber bag at essentially atmospheric pressure. A physicist left a 40-L bag filled with He floating near the ceiling before leaving on vacation. When she returned, all the helium was gone. Find the entropy change of the gas. Assume that the atmospheric helium concentration is approximately 5×10^{-4} %. What is the minimum work needed to collect the helium back into the bag?

Electricity and Magnetism

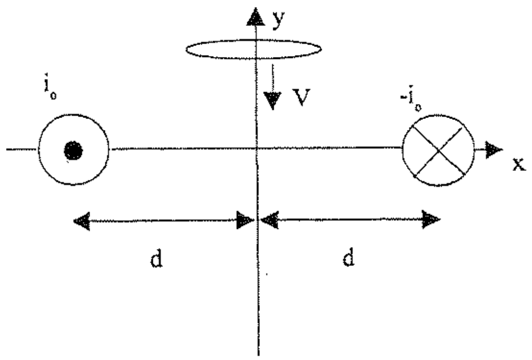
Problem 1.

Two long straight wires carry equal but opposite currents along the z-axis (perpendicular to the page) as shown in Figure below.



(a) At what point of the y-axis is the total magnetic field produced by the wires largest?
(b) Find B_{max}. The parameters are given in the figure.
(c) What is the direction of B at this point?

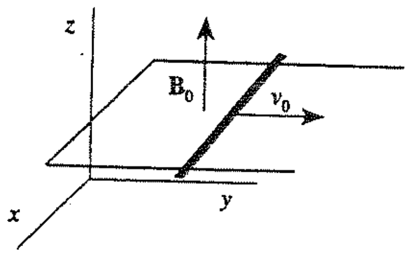
(d) A conducting loop with its axis along the y-axis is lowered at constant speed down the y-axis, as shown. Sketch the current I induced in the loop as a function of time for $-\infty < t < \infty$. Assume that the loop crosses y = 0 at t = 0, and that a positive current corresponds to counterclockwise current flow from the loop.



(e) Does the loop feel the force as it moves toward the wires from the above?

Problem 2.

A copper rod slides on frictionless rails in the presence of a constant magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. At t = 0, the rod is moving in the y direction velocity v_0 (see Figure below).



(a) What is subsequent velocity of the rod if σ is the conductivity and ρ_m is the mass density of copper?
(b) For copper $\sigma = 5\times10^{17}\ \mathrm{s}^{-1}$ and $\rho_m = 8.9\ \mathrm{g/cm}^3$. If $B_0 = 1$ gauss, estimate the time it takes the rod to stop ($v = 10^{-9}v_0$).

Comprehensive exam 2004

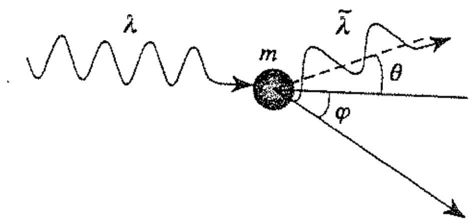
Day 1

Morning

Modern Physics

Problem 1

In the Compton effect, a γ-ray photon of wavelength λ strikes a free, but initially stationary, electron of mass m. The photon is scattered an angle θ, and its scattered wavelength is $\bar{\lambda}$. The electron recoils at an angle ϕ (see Figure below).



(a) What is the relativistic equation for the momentum and energy conservation?
(b) Find an expression for the change $\bar{\lambda} - \lambda$ in the photon wavelength for the special case $\theta = \pi/2$.

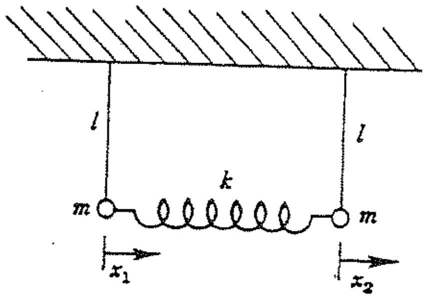
Problem 2

Consider an idealized Sun and Earth, both black bodies, in otherwise empty flat space. The Sun is at temperature $T_S = 6000\ K$, and heat transfer by oceans and atmosphere on the Earth is so effective as to keep the Earth's surface temperature uniform. The radius of the Earth is $R_E = 6.4 \times 10^8\ m$, and the Earth-Sun distance is $d = 1.5 \times 10^{11}\ m$. The mass of Sun $M_S = 2 \times 10^{30}\ kg$. (a) Find the temperature of the Earth. (b) Find the radiation force on the Earth. (c) Compare these results with those for an interplanetary "chrondule" in the form of a spherical, perfectly conducting blackbody with a radius $R = 0.1\ cm$, moving in a circular orbit around the Sun at a radius equal to the Earth-Sun distance d. (d) At what distance from the Sun would a metallic particle melt (melting temperature T_m = 1550 K)? (e) For what size particle would the radiation force calculated in (c) be equal to the gravitational force from the Sun at a distance d?

Classical Mechanics

Problem 1

Consider the coupled pendulums shown in Figure below. Assume the initial conditions: $x_1 = x_2 = 0,\ \dot{x}_1 = 0,\ \dot{x}_2 = 0$ at $t = 0$. Find the trajectories $x_1(t)$ and $x_2(t)$.



Problem 2

(a) Consider a test particle (let us say a star) within a thin disk of stars of radius R and uniform volume mass density ρ. Find a general expression for the gravitational field, g, on the axis of the disk as a function of height, z, above the midplane of this gravitational disk. Take $z < h$, where h is the half height of the disk.

(b) Simplify the expression for g(z) at $z \ll R$.

(c) Now imagine the star is released from the height H above the midplane of the disk, where $H \leq h$. Assume that the star is free to pass through the disc without collisions. Write the equation of motion for the star.

(d) Describe the motion of the star as a function of time and give an expression for the period.

109. $\int \frac{\sqrt{a+bx}}{x^m} = -\frac{1}{(m-1)a}\left[\frac{\sqrt{(a+bx)^3}}{x^{m-1}} + \frac{(2m-5)b}{2}\int \frac{\sqrt{a+bx}\,dx}{x^{m-1}}\right] m \neq 1.$

110. $\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b}.$

111. $\int \frac{xdx}{\sqrt{a+bx}} = -\frac{2(2a-bx)}{3b^2}\sqrt{a+bx}.$

112. $\int \frac{x^2dx}{\sqrt{a+bx}} = \frac{2(8a^2-4abx+3b^2x^2)}{15b^3}\sqrt{a+bx}.$

113. $\int \frac{x^mdx}{\sqrt{a+bx}} = \frac{2x^m\sqrt{a+bx}}{(2m+1)b} - \frac{2ma}{(2m+1)b}\int \frac{x^{m-1}dx}{\sqrt{a+bx}}.$

114. $\int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}}\log\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right).\quad a>0$ See #23 for $a<0$

115. $\int \frac{dx}{x\sqrt{a+bx}} = \frac{-2}{\sqrt{a}}\tanh^{-1}\sqrt{\frac{a+bx}{a}}.\quad a>0$

116. $\int \frac{dx}{x^2\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{a+bx}}.$

117. $\int \frac{dx}{x^n\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{(n-1)ax^{n-1}} - \frac{(2n-3)b}{(2n-2)a}\int \frac{dx}{x^{n-1}\sqrt{a+bx}}.$

118. $\int (a+bx)^{\pm n/2}dx = \frac{2(a+bx)^{\frac{2\pm n}{2}}}{b(2\pm n)}.$

119. $\int x(a+bx)^{\pm n/2}dx = \frac{2}{b^2}\left[\frac{(a+bx)^{\frac{4\pm n}{2}}}{4\pm n} - \frac{a(a+bx)^{\frac{2\pm n}{2}}}{2\pm n}\right].$

120. $\int \frac{dx}{x(a+bx)^{m/2}} = \frac{1}{a}\int \frac{dx}{x(a+bx)^{\frac{m-2}{2}}} - \frac{b}{a}\int \frac{dx}{(a+bx)^{m/2}}.$

121. $\int \frac{(a+bx)^{n/2}dx}{x} = b\int (a+bx)^{\frac{n-2}{2}}dx + a\int \frac{(a+bx)^{\frac{n-2}{2}}}{x}dx.$

122. $\int f(x, \sqrt{a+bx})\,dx = \frac{2}{b}\int f\left(\frac{z^2-a}{b}, z\right)zdz$

Forms Containing $\sqrt{x^2 \pm a^2}$

123. $\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2\log(x + \sqrt{x^2 \pm$

124. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}).$

125. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$

126. $\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\log\left(\frac{a+\sqrt{x^2+a^2}}{x}\right).$

127. $\int \frac{\sqrt{x^2+a^2}}{x}dx = \sqrt{x^2+a^2} - a\log\left(\frac{a+\sqrt{x^2+a^2}}{x}\right).$

128. $\int \frac{\sqrt{x^2-a^2}}{x}dx = \sqrt{x^2-a^2} - a\sec^{-1}\frac{a}{x}.$

129. $\int \frac{x\,dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}.$

130. $\int x\sqrt{x^2 \pm a^2}\,dx = \frac{1}{3}\sqrt{(x^2 \pm a^2)^3}.$

131. $\int \sqrt{(x^2 \pm a^2)^3}\,dx = \frac{1}{4}\left[x\sqrt{(x^2 \pm a^2)^3} \pm \frac{3a^2x}{2}\sqrt{x^2 \pm a^2} + \frac{3a^4}{2}\log(x + \sqrt{x^2 \pm\right.$

132. $\int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}.$

133. $\int \frac{x\,dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{-1}{\sqrt{x^2 \pm a^2}}.$

134. $\int x\sqrt{(x^2 \pm a^2)^3}\,dx = \frac{1}{5}\sqrt{(x^2 \pm a^2)^5}.$

135. $\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\log(x + \sqrt{x^2 \pm$

136. $\int x^3\sqrt{x^2 + a^2}\,dx = (\frac{1}{5}x^2 - \frac{2}{15}a^2)\sqrt{(a^2 + x^2)^3}.$

Comprehensive exam 2004

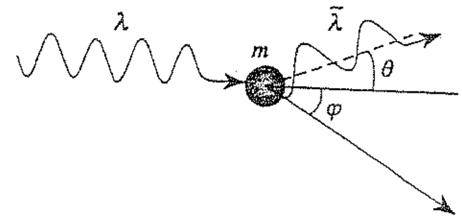
Day 1

Morning

Modern Physics

Problem 1

In the Compton effect, a γ-ray photon of wavelength λ strikes a free, but initially stationary, electron of mass m. The photon is scattered an angle θ, and its scattered wavelength is $\bar{\lambda}$. The electron recoils at an angle ϕ (see Figure below).



(a) What is the relativistic equation for the momentum and energy conservation?
(b) Find an expression for the change $\bar{\lambda} - \lambda$ in the photon wavelength for the special case $\theta = \pi/2$.

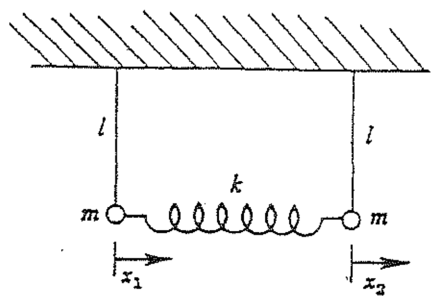
Problem 2

Consider an idealized Sun and Earth, both black bodies, in otherwise empty flat space. The Sun is at temperature $T_S = 6000\ K$, and heat transfer by oceans and atmosphere on the Earth is so effective as to keep the Earth's surface temperature uniform. The radius of the Earth is $R_E = 6.4\times10^8\ m$, and the Earth-Sun distance is $d = 1.5\times10^{11}\ m$. The mass of Sun $M_S = 2\times10^{30}\ kg$. (a) Find the temperature of the Earth. (b) Find the radiation force on the Earth. (c) Compare these results with those for an interplanetary "chrondule" in the form of a spherical, perfectly conducting blackbody with a radius $R = 0.1\ cm$, moving in a circular orbit around the Sun at a radius equal to the Earth-Sun distance d. (d) At what distance from the Sun would a metallic particle melt (melting temperature T_m = 1550 K)? (e) For what size particle would the radiation force calculated in (c) be equal to the gravitational force from the Sun at a distance d?

Classical Mechanics

Problem 1

Consider the coupled pendulums shown in Figure below. Assume the initial conditions: $x_1 = x_2 = 0,\ \dot{x}_1 = 0,\ \dot{x}_2 = 0$ at $t = 0$. Find the trajectories $x_1(t)$ and $x_2(t)$.



Problem 2

(a) Consider a test particle (let us say a star) within a thin disk of stars of radius R and uniform volume mass density ρ. Find a general expression for the gravitational field, g, on the axis of the disk as a function of height, z, above the midplane of this gravitational disk. Take z < h, where h is the half height of the disk.

(b) Simplify the expression for g(z) at z << R.

(c) Now imagine the star is released from the height H above the midplane of the disk, where H $\leq h$. Assume that the star is free to pass through the disc without collisions. Write the equation of motion for the star.

(d) Describe the motion of the star as a function of time and give an expression for the period.

109. $\int \frac{\sqrt{a+bx}}{x^m} = -\frac{1}{(m-1)a}\left[\frac{\sqrt{(a+bx)^3}}{x^{m-1}} + \frac{(2m-5)b}{2}\int \frac{\sqrt{a+bx}\,dx}{x^{m-1}}\right] m \neq 1.$

110. $\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b}.$

111. $\int \frac{xdx}{\sqrt{a+bx}} = -\frac{2(2a-bx)}{3b^2}\sqrt{a+bx}.$

112. $\int \frac{x^2dx}{\sqrt{a+bx}} = \frac{2(8a^2-4abx+3b^2x^2)}{15b^3}\sqrt{a+bx}.$

113. $\int \frac{x^m dx}{\sqrt{a+bx}} = \frac{2x^m\sqrt{a+bx}}{(2m+1)b} - \frac{2ma}{(2m+1)b}\int \frac{x^{m-1}dx}{\sqrt{a+bx}}.$

114. $\int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}}\log\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right). \quad a>0$ See #23 for $a<0$

115. $\int \frac{dx}{x\sqrt{a+bx}} = \frac{-2}{\sqrt{a}}\tanh^{-1}\sqrt{\frac{a+bx}{a}}. \quad a>0$

116. $\int \frac{dx}{x^2\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{ax} - \frac{b}{2a}\int \frac{dx}{x\sqrt{a+bx}}.$

117. $\int \frac{dx}{x^n\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{(n-1)ax^{n-1}} - \frac{(2n-3)b}{(2n-2)a}\int \frac{dx}{x^{n-1}\sqrt{a+bx}}.$

118. $\int (a+bx)^{\pm n/2}dx = \frac{2(a+bx)^{\frac{2\pm n}{2}}}{b(2\pm n)}.$

119. $\int x(a+bx)^{\pm n/2}dx = \frac{2}{b^2}\left[\frac{(a+bx)^{\frac{4\pm n}{2}}}{4\pm n} - \frac{a(a+bx)^{\frac{2\pm n}{2}}}{2\pm n}\right].$

120. $\int \frac{dx}{x(a+bx)^{m/2}} = \frac{1}{a}\int \frac{dx}{x(a+bx)^{\frac{m-2}{2}}} - \frac{b}{a}\int \frac{dx}{(a+bx)^{m/2}}.$

121. $\int \frac{(a+bx)^{n/2}dx}{x} = b\int (a+bx)^{\frac{n-2}{2}}dx + a\int \frac{(a+bx)^{\frac{n-2}{2}}}{x}dx.$

122. $\int f(x, \sqrt{a+bx})\,dx = \frac{2}{b}\int f\left(\frac{z^2-a}{b}, z\right) zdz$

Forms Containing $\sqrt{x^2 \pm a^2}$

123. $\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}[x\sqrt{x^2 \pm a^2} \pm a^2 \log(x + \sqrt{x^2 \pm$

124. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log(x + \sqrt{x^2 \pm a^2}).$

125. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$

126. $\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\log\left(\frac{a+\sqrt{x^2+a^2}}{x}\right).$

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131. $\int \sqrt{(x^2 \pm a^2)^3}\,dx = \frac{1}{4}\left[x\sqrt{(x^2 \pm a^2)^3} \pm \frac{3a^2x}{2}\sqrt{x^2 \pm a^2} + \frac{3a^4}{2}\log(x + \sqrt{x^2 \pm\right.$

132. $\int \frac{dx}{\sqrt{(x^2 \pm a^2)^3}} = \frac{\pm x}{a^2\sqrt{x^2 \pm a^2}}.$

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135. $\int x^2\sqrt{x^2 \pm a^2}\,dx = \frac{x}{4}\sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8}x\sqrt{x^2 \pm a^2} - \frac{a^4}{8}\log(x + \sqrt{x^2 \pm$

136. $\int x^3\sqrt{x^2 \pm a^2}\,dx = \dots$

Day 1

Afternoon

Quantum Mechanics

Problem 1.

Consider a particle beam approximated by a plane wave directed along the x-axis from the left and incident upon a potential $V(x) = \gamma\delta(x)$, where $\gamma > 0$. Calculate the probability of the transmission. Hint: (a) find a *w.f.* for $x < 0$, (b) find a *w.f.* for $x > 0$, and (c) give the condition on the *w.f.* at the boundary between the regions.

Statistical Mechanics

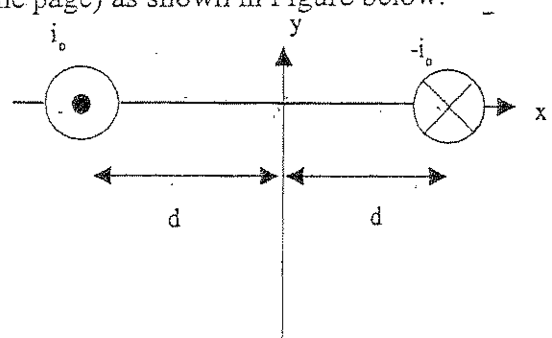
Problem 1.

Sometimes helium gas in a low-temperature physics lab is kept temporarily in a large rubber bag at essentially atmospheric pressure. A physicist left a 40-L bag filled with He floating near the ceiling before leaving on vacation. When she returned, all the helium was gone. Find the entropy change of the gas. Assume that the atmospheric helium concentration is approximately 5×10^{-4} %. What is the minimum work needed to collect the helium back into the bag?

Electricity and Magnetism

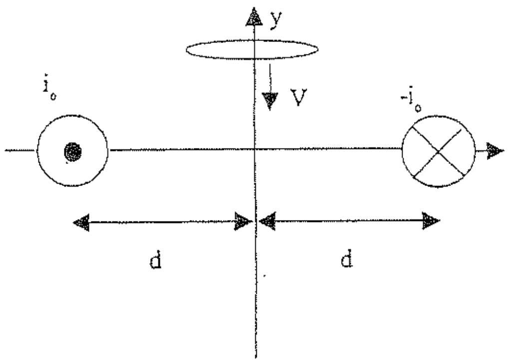
Problem 1.

Two long straight wires carry equal but opposite currents along the z-axis (perpendicular to the page) as shown in Figure below.



(a) At what point of the y-axis is the total magnetic field produced by the wires largest?
(b) Find B_{max}. The parameters are given in the figure.
(c) What is the direction of B at this point?

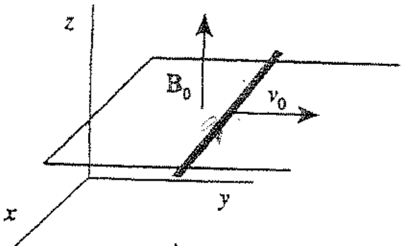
(d) A conducting loop with its axis along the y-axis is lowered at constant speed down the y-axis, as shown. Sketch the current I induced in the loop as a function of time for $-\infty < t < \infty$. Assume that the loop crosses y = 0 at t = 0, and that a positive current corresponds to counterclockwise current flow from the loop.



(e) Does the loop feel the force as it moves toward the wires from the above?

Problem 2.

A copper rod slides on frictionless rails in the presence of a constant magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$. At t = 0, the rod is moving in the y direction velocity v_0 (see Figure below).



(a) What is subsequent velocity of the rod if σ is the conductivity and ρ_m is the mass density of copper?
(b) For copper $\sigma = 5\times10^{17}$ s^{-1} and $\rho_m = 8.9$ g/cm^3. If $B_0 = 1$ gauss, estimate the time it takes the rod to stop ($v = 10^{-9}v_0$).

Day 2

Morning

Statistical Mechanics

Problem 1

An ideal gas of N spin-$1/2$ fermions is confined to a volume V. Calculate the zero-temperature limit of (a) the chemical potential, (b) the average energy per particle, (c) the pressure, (d) the Pauli spin susceptibility. Assume that each fermion interacts with an external field in the form: $2\mu_B H S_z$, where μ_B is the Bohr magneton.

Quantum Mechanics

Problem 1

A particle is placed into an infinite potential well of the width a ($0<x<a$) in the ground state. At some moment the wall of the well is moved to the point b ($b>a$) for a short period of time. Find the probability of excitations to different stationary states after the motion of the wall was stopped. Find the validity condition of obtained results. In particular case consider $b = 2a$.

Relativity

Problem 1

Proton with $\gamma = 1/\sqrt{1-(v/c)^2}$ collides elastically with a proton at rest. If two protons rebound with equal energies, what is the angle θ between them?

Electricity and Magnetism

Problem 1

Write down Maxwell's equations in a non-conducting medium with constant permeability and susceptibility ($\rho = \mathbf{j} = 0$). Show that **E** and **B** each satisfy the wave equation, and find an expression for the wave velocity. Write down the plane wave solutions for **E** and **B** and show how **E** and **B** are related.

Problem 2

Discuss the reflection and refraction of electromagnetic waves at a plane interface between the dielectrics and derive the relationships the angle of incidence, refraction, and reflection (see problem 1).

VECTOR IDENTITIES

Triple Products

(1) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$

(2) $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$

Product Rules

(3) $\nabla(fg) = f(\nabla g) + g(\nabla f)$

(4) $\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$

(5) $\nabla \cdot (f\mathbf{A}) = f(\nabla \cdot \mathbf{A}) + \mathbf{A} \cdot (\nabla f)$

(6) $\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$

(7) $\nabla \times (f\mathbf{A}) = f(\nabla \times \mathbf{A}) - \mathbf{A} \times (\nabla f)$

(8) $\nabla \times (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A})$

Second Derivatives

(9) $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

(10) $\nabla \times (\nabla f) = 0$

(11) $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$

FUNDAMENTAL THEOREMS

Gradient Theorem : $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla f) \cdot d\mathbf{l} = f(\mathbf{b}) - f(\mathbf{a})$

Divergence Theorem : $\int (\nabla \cdot \mathbf{A})\, d\tau = \oint \mathbf{A} \cdot d\mathbf{a}$

Curl Theorem : $\int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint \mathbf{A} \cdot d\mathbf{l}$

Astronomy Qualifying Exam Spring 2004

Interstellar Medium and Diffuse Matter (required)

1) Consider a hot star just formed within a typical cloud of neutral Hydrogen (HI region) of density $n_0 = 10^7\ m^{-3}$ which subsequently ionizes a spherical volume within the cloud (i.e., the Stromgren Sphere).

a) Roughly, how does the number density inside the HII region compare to the number density in the surrounding HI region? What is the physical reason for this particular ratio?

b) Roughly, how does the temperature inside the HII region compare to that in the surrounding HI region?

c) What is the ratio of gas pressures for the HII region relative to the HI region and what does the pressure imbalance imply will happen to the nascent (just-born) HII region?

d) Derive an expression for the initial radius of the Stromgren sphere in terms of the important properties of the HI region and the hot star.

e) Assume that after a very long time the Stromgren radius stabilizes. Compute the ratio of the final Stromgren radius to that of the initial Stromgren radius.

f) Compute the ratio of the mass of the final Stromgren sphere to that of the initial Stromgren sphere.

g) Now suppose that the star goes supernovae. What will happen to the radius of the ionized volume and why?

h) Suppose that a shock moves outward from the supernovae at 500 km/sec and that the shocked gas is at a temperature of 10^4 °K. What is the Mach number (M) of the shock?

i) Suppose that when the shock front has a radius R = 1 pc, it has swept up all the interstellar material within R, and that the ratio of the density within the swept-up shell ρ_2 to that in the surrounding ISM ρ_0 can be approximated by the square of the Mach number M. Derive an expression for the thickness ΔR of the swept-up region. Evaluate your expression.

j) Assuming no other external influences, the ionized gas will eventually cool down. What is the most likely mechanism by which it will cool (be specific)?

Qualifying Examination - Part I

Time: 9:30-11:30 a.m. **Saturday, September 25, 1999**

The examination is to be written on the single sheets that are provided; no more than one question is to be answered on a single sheet. Number each question. You are supposed to work all questions in Part I; however, if you must omit any questions, ***cross out those numbers on your title page***. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from your title page) on the back of each sheet of paper.

Part I counts one-third (1/3) of the final grade.
Part II is in this same room at 1:00 p.m.

PHYSICAL CONSTANTS

Planck's constant	$h = 6.63 \times 10^{-34}$ J•s $= 4.14 \times 10^{-15}$ eV•s
Vacuum speed of light	$c = 3.00 \times 10^{8}$ m/sec
	$\hbar c = 197$ MeV•fm $= 1.97 \times 10^{-5}$ eV•cm
Electron charge	$e = 1.60 \times 10^{-19}$ C $= 4.80 \times 10^{-10}$ esu
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Gas constant	$R = 8.31$ J/(mol•K)
Gravitational constant	$G = 6.67 \times 10^{-11}$ N•m^2/kg^2
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12}$ F/m
Electron mass	$m_e = 9.11 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
Proton mass	$m_p = 1.67 \times 10^{-27}$ kg $= 938$ MeV/c^2
Bohr radius of hydrogen	$a_B = 5.3 \times 10^{-11}$ m
Ionization energy of hydrogen	13.6 eV
Avogadro's number	6.02×10^{23} / mole

Conversion Factors

1 eV $= 1.6 \times 10^{-19}$ J $= 1.6 \times 10^{-12}$ erg

1 m $= 10^{10}$ Å $= 10^{15}$ fm $= 6.25 \times 10^{-4}$ mi

1 atm $= 1.01 \times 10^{5}$ N/m^2 $= 760$ Torr

1 cal $= 4.186$ J

Divergence and curl in

spherical coordinates

$$\nabla \cdot E = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\theta) + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial\phi}$$

$$\nabla \times E = r\frac{1}{r sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta E_\phi) - \frac{\partial E_\theta}{\partial\phi}\right] + \theta\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial E_r}{\partial\phi} - \frac{\partial(rE_\phi)}{\partial r}\right]$$
$$+ \phi\frac{1}{r}\left[\frac{\partial(rE_\theta)}{\partial r} - \frac{\partial E_r}{\partial\theta}\right]$$

where r, θ, ϕ are the unit vectors associated with the spherical coordinates r, θ, ϕ

Useful integrals

$$\int_0^\infty e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty x e^{-\alpha x^2}\,dx = \frac{1}{2\alpha}$$

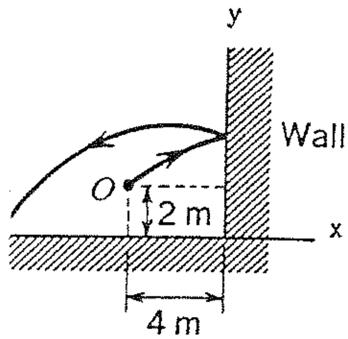
$$\int_0^\infty x^2 e^{-\alpha x^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\int \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}sin2x$$

Fall 1999 Qualifier

7-minute Questions

1.



A ball is thrown against a wall, making a perfectly elastic collision, and bounces over the head of the thrower as shown in the figure. When it leaves the thrower's hand, the ball is 2 m above the ground and 4 m from the wall, and has $V_x(0) = V_y(0) = 10$ m/sec. How far behind the thrower does the ball hit the ground? (Numerical answer required, let $g = 10$ m/s^2.)

2. One atmosphere is 1.0×10^5 N/m^2 and radius of the Earth is 6.4×10^6 m.

   a. Find an approximate value for the total mass of the atmosphere.

   b. If all the air were moving relative to the Earth at speed 6.7 m/sec (15 mph), what would be the total kinetic energy of the atmosphere?

   c. If this whole energy could be used up every month (and then re-established for the next month), what would be the total power available?

3. Consider a long uniform wire under tension T, along which a transverse wave is generated by a transverse driving force $F(t)$ which is turned on at $t = 0$ and acts at one end, $x = 0$. The other end of the wire is firmly attached to a wall at $x = 10$ m. For $t < 1.0$ s, the displacement is given by:

$$y(x,t) = 0.01 \sin(4x - 40t) \qquad x < 10\,t$$
$$= 0.0 \qquad x > 10\,t$$

where all quantities are in MKS units.

   a. What is the frequency of the driving force (in Hz)?

   b. What is the wavelength of the displacement wave (in meters), and what is its velocity?

4. A thin circular hoop is suspended in such a way that it can oscillate freely in the vertical plane of the hoop about the point of suspension at the top of the hoop. The mass of the hoop is M and the radius of the hoop is R.

   a. Calculate the period of small oscillations about the rest position.

   b. How does the answer for the period change if the oscillations were instead perpendicular to the plane of the hoop?

5. A particle with mass M and charge q>0 moves in the uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$ and also in the field of a charge Q<0 located at $x = y = z = 0$. At t = 0, the particle is at $x = z = 0$, $y = a$, and its velocity is $\mathbf{v} = v_0\,\hat{\mathbf{x}}$. For what B will the trajectory of the particle be a circle around $x = y = z = 0$ with radius a?

6. A thin disk of radius 23 cm carries a total charge of 1.5×10^{-7}C spread uniformly over the surface. Calculate the work (in Joules) done to bring a charge 2×10^{-8}C at rest from infinity to a point along the disk axis and 78 cm from its surface.

7. A limit is reached on the electrical current that can be carried by superconducting wire when the magnetic field at the surface reaches a critical value. For lead, the critical field is 0.08 Tesla. How large a direct current, in amperes, can be carried by a lead wire of 1 mm radius?

8. Consider a diatomic molecule at a temperature sufficiently high to excite rotational degrees of freedom, giving it a specific heat of $C_v = 7/2$. The rotational energy is $E_{rot} = \frac{1}{2} I_x \omega_x^2 + \frac{1}{2} I_y \omega_y^2$ where I_x and I_y are the moments of inertia about two internal axes of rotational and ω_x and ω_y are rotation rates about those axes.

   a. What is the joint probability distribution for ω_x and ω_y?

   b. What is the rms of the angular momentum $L_x = I_x\omega_x$?

9. A gas of nitrogen molecules (N_2) is at the ground in thermal equilibrium, at a temperature 20°C. Find the average height in km to which an upward going molecule will go, if collisions with other molecules after release are ignored. Is this a reasonable estimate of the thickness of the atmosphere?

10. The ground state of the carbon atom has two electrons in the 1s shell, two electrons in the 2s shell, and two electrons in the 2p shell.

   a. Write the electron configuration for a carbon atom in which one of the 2p electrons in the ground state is excited to the 3p shell.

   b. Apply the LS-coupling (Russell-Saunders coupling) to determine the values of L, S, and J for each energy level of the atom with electron configuration described in part a.

Qualifying Examination - Part II

Time: 1:00-5:00 p.m. **Saturday, September 25, 1999**

The examination is to be written on the single sheets that are provided; no more than one question is to be answered on a single sheet. Number each question. You are supposed to work 10 questions in Part II; however, when you omit questions, ***cross out those numbers on your title page***. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from your title page) on the back of each sheet of paper.

Part II counts two-thirds (2/3) of the final grade and is in this same room at 1:00 p.m.

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Vacuum speed of light	$c = 3.00 \times 10^{8}$ m/sec
	$\hbar c = 197$ MeV•fm $= 1.97 \times 10^{-5}$ eV•cm
Electron charge	$e = 1.60 \times 10^{-19}$ C $= 4.80 \times 10^{-10}$ esu
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Gas constant	$R = 8.31$ J/(mol•K)
Gravitational constant	$G = 6.67 \times 10^{-11}$ N•m^2/kg^2
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12}$ F/m
Electron mass	$m_e = 9.11 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
Proton mass	$m_p = 1.67 \times 10^{-27}$ kg $= 938$ MeV/c^2
Bohr radius of hydrogen	$a_B = 5.3 \times 10^{-11}$ m
Ionization energy of hydrogen	13.6 eV
Avogadro's number	6.02×10^{23} / mole

Conversion Factors

1 eV $= 1.6 \times 10^{-19}$ J $= 1.6 \times 10^{-12}$ erg

1 m $= 10^{10}$ Å $= 10^{15}$ fm $= 6.25 \times 10^{-4}$ mi

1 atm $= 1.01 \times 10^{5}$ N/m^2 $= 760$ Torr

1 cal $= 4.186$ J

Divergence and curl in

spherical coordinates

$$\nabla \cdot E = \frac{1}{r^2}\ \frac{\partial}{\partial r}\ (r^2 E_r) \ + \ \frac{1}{r\sin\theta}\ \frac{\partial}{\partial\theta}\ (\sin\theta E_\theta) \ + \ \frac{1}{r\sin\theta}\ \frac{\partial E_\phi}{\partial\phi}$$

$$\nabla \times E \ = \ r\ \frac{1}{r sin\theta}\ \left[\frac{\partial}{\partial\theta}(\sin\theta E_\phi) \ - \ \frac{\partial E_\theta}{\partial\phi}\right] \ + \ \theta\frac{1}{r}\ \left[\frac{1}{\sin\theta}\ \frac{\partial E_r}{\partial\phi} \ - \ \frac{\partial(rE_\phi)}{\partial r}\right]$$

$$+ \ \phi\frac{1}{r}\ \left[\frac{\partial(rE_\theta)}{\partial r} \ - \ \frac{\partial E_r}{\partial\theta}\right]$$

where r, θ, ϕ are the unit vectors associated with the spherical coordinates r, θ, ϕ

Useful integrals

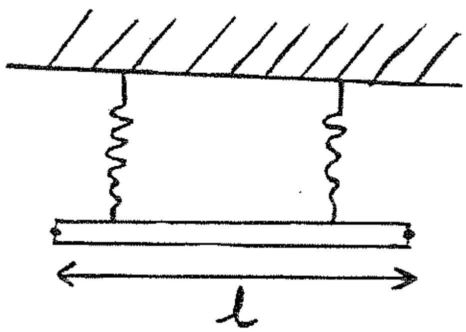
$$\int_0^\infty e^{-\alpha x^2}\ dx \ = \ \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty xe^{-\alpha x^2}\ dx \ = \ \frac{1}{2\alpha}$$

$$\int_0^\infty x^2 e^{-\alpha x^2}\ dx \ = \ \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\int \sin^2 x\ dx \ = \ \frac{x}{2} \ - \ \frac{1}{4}\ sin2x$$

11. A rigid bar of mass m and length l is supported by two springs, each with force constant k (see figure). The motion of the bar is in the vertical plane and the center of gravity is constrained to move along the vertical direction. Show that the bar has two modes of vibration (a symmetric and antisymmetric mode) and calculate the frequencies. (moment of inertia $I = ml^2/12$)



12. A particle in a central field moves in the spiral orbit $r = c\theta^2$ where c is a constant. For initial angular momentum L and mass m, determine:

    a. the form of the force law.
    b. how the angle θ varies with time t.

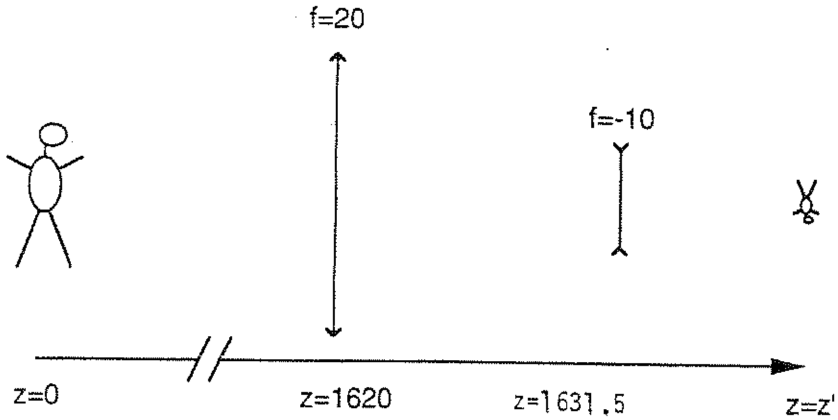
13. A conducting sphere of radius a is placed in a uniform external electric field $E = E_0\hat{z}$. Determine (a) the electric potential in the region surrounding the sphere, and (b) the induced surface charge density on the sphere.

14. The total power radiated by a non-relativistic charge q with acceleration a is

$$P = \frac{2}{3}\frac{1}{4\pi\epsilon_0}\frac{q^2a^2}{c^3}.$$

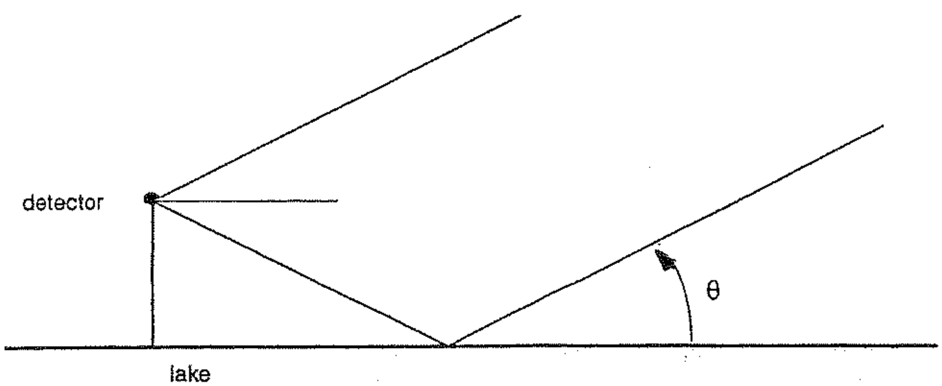
Find the power P in Watts scattered by an electron irradiated by an electromagnetic wave of intensity $I = 100$ Watts/m^2, and the total cross section for scattering $\sigma_T = P/I$ in m^2.

15. The telephoto lens pair shown below (all units are mm) images an object at $z = 0$ onto a screen at $z = z'$. Find the image position z' and magnification. What would be the magnification for a single lens at $z = 1620$ mm with the same object and image positions?



16. Using graphical methods, show how one locates the paraxial images for convex and concave spherical mirrors characterized by a radius of curvature R. Where are the images when (1) the source is infinitely far away, (2) the source is at the surface of the mirror? Can one have both real and virtual images with both types of mirrors? What object distance marks the boundary between real and virtual images?

17. A microwave detector is located on the shore of Lake Michigan 50 cm above the calm surface. As a radio star emitting monochromatic microwaves of 21 cm wavelength rises slowly above the horizon, the detector observes an increasing signal. It will observe the first maximum when the star is at what angle θ above the horizon? ($2\sin^2\theta = 1 - \cos 2\theta$)



22. One mole of monatomic ideal gas initially at a volume of 10 liter and a temperature of 300 K is heated at constant volume to a temperature of 600 K, allowed to expand isothermally to its initial pressure, and finally compressed at a constant pressure to its original volume, pressure and temperature.

  a. During the cycle, how much energy is absorbed by the gas?
  b. What is the net work done by the gas?
  c. What is the efficiency of the cycle?

23. a. Assume that the Sun and the Earth are perfect blackbodies. Using the observation that the average temperature of the surface of the Earth is about 273 K, determine the temperature of the Sun's "surface." The radius of the Sun is 7×10^8 m and the Earth-Sun distance is 1.5×10^{11} m.

  b. Imagine a space probe that orbits the Sun at the same radius as the Earth. Suppose this probe is made of a material with emissivity $\epsilon = 0.1$ at all wavelengths. What is the temperature of the probe? (Neglect the effects of the Earth on the probe.)

  c. Now suppose the emissivity of the space probe is 0.01 for $\lambda > 5\mu m$ and 1.0 for $\lambda > 5\mu m$. Roughly, what is the equilibrium temperature of the space probe? (Wien Displacement Law: $\lambda_{max} = 3$ mm • K)

24. Consider the pair production reaction $\gamma + e^- \rightarrow e^+ + e^- + e^-$ initiated by bombarding electrons at rest with photons of energy E. Find the threshold energy for this reaction.

25. Choose two of the following measurements. For each of these, sketch an experimental setup and describe the equipment required in as much detail as you can in six minutes.

  a. Zeeman splitting of mercury lines
  b. Magnetic moment of potassium atoms
  c. Lifetime of cosmic ray muons
  d. Planck's constant from photoelectric effect
  e. The angular diameter of a star
  f. Temperatures of 0.1 K, 300 K, 600 K, 6000 K
  g. Gas pressures of 10^{-8}, 10^{-2}, 10, 10^9 Torr (1 Torr = 10^2 Pa)

Qualifying Examination - Part I

Time: 9:30-11:30 a.m. **Saturday, February 20, 1999**

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$$\int_0^\infty x e^{-\alpha x^2}\, dx = \frac{1}{2\alpha}$$

$$\int_0^\infty x^2 e^{-\alpha x^2}\, dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

1. Two particles of mass m and M are initially at rest and infinitely separated from each other. Determine their relative velocity of approach attributable to gravitational attraction, given in terms of their separation d.

2. An elevator motor has a torque versus rotation rate curve as show in Figure A. At time zero, the elevator starts from $\omega = 0$ and begins to lift the mass as shown in Figure B. The cable, spool, and pulleys are massless and the cable does not stretch. The spool has radius R.

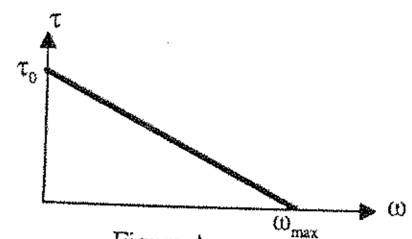


Figure A

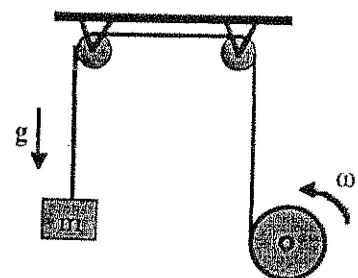


Figure B

spool radius R

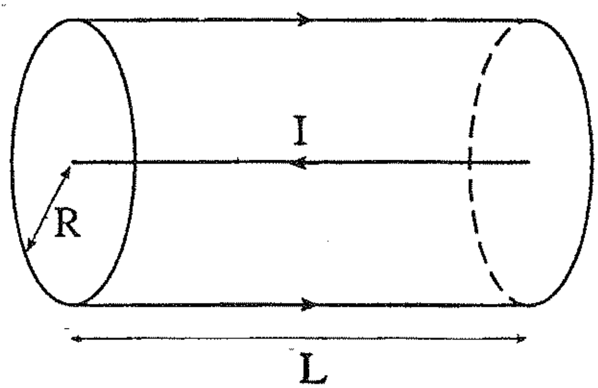
   a. At t = 0 when $\omega = 0$, find the tension in the cable, τ_0, and the upward acceleration of the mass, a_0.

   b. After a short period of acceleration, the mass reaches a constant upward speed. What is that "terminal velocity," v_T, if $\tau_0 = 1.5$ mgR?

   c. What power is supplied by the motor when raising the mass at v_T?

3. A car is driving at 20 m/s in a region where the Earth's magnetic field is 5×10^{-5}T and tilted at 45° to the horizontal. It is equipped with a vertical whip antenna 1.5 m long. Find the induced emf in the antenna.

4. A central wire carries a current I, which is returned by a conducting can as shown. What is the magnetic field everywhere? Assume the conductors have negligible thicknesses.



5 The relation between the frequency ω and wave vector k in a plasma is $\omega^2 = \omega_p^R + c^2k^2$ for an electromagnetic wave. Here, ω_p is a constant and c is the speed of light.

a. What is the group velocity?

b. What is the phase velocity?

c. What happens to the wave if $\omega < \omega_p$?

6. a. Use dimensional analysis to derive the speed of propagation of a pulse on a stretched string of mass M, length L, and tension T.

b. Use your result to find the frequency of the fundamental mode of a violin string of length 50 cm, whose mass is .4 g if the tension on the string is 30N.

c. If the string is plucked, the amplitude decays to half the original value in 2 s How large will be the amplitude after 6 s?

7. Use energy equipartition to estimate the molar heat capacity at constant volume for a diatomic gas, such as hydrogen, in various temperature ranges. Express your result in terms of the universal gas constant, R. Sketch the heat capacity as a function of temperature, indicating heat capacity values. [You do not need to be quantitative about the temperature except to indicate a very low temperature (20K or so) and roughly room temperature (300K).]

8. Calculate the change in entropy of the universe if 2 kg of water at 100°C is mixed with 3 kg of water at 20°C and comes to thermal equilibrium in a thermally insulated container.

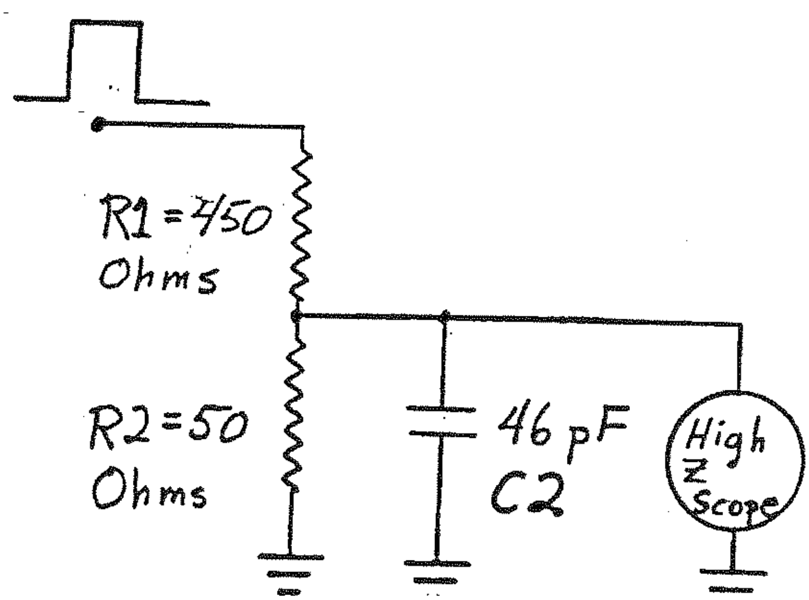
9. Electrons in semiconductors can be bound to positively charged impurity ions analogous to the binding of an electron by a proton. In solids, the "ionization" energy corresponds to the energy needed to promote an electron from the lowest bound state to the bottom of the conduction band. The Hamiltonian for the bound electron can be written as $p^2/2m^* - e^2/(4\pi\varepsilon r)$, where ε is the dielectric constant of the semiconductor and m^* is the effective mass of the electron in the conduction band. Calculate the ionization energy and the radius of the first Bohr orbit for a material where $\varepsilon = 10\varepsilon_0$ and $m^*/m = 0.5$. Express your answers in terms of the corresponding energy and radius for the H atom (assume the proton is infinitely massive).

10. A rectangular pulse is applied to a voltage divider to reduce the pulse height suitably for input to an oscilloscope. An input capicitance for the scope causes the pulse recorded by the scope to appear distorted.

a. Explain why this happens

b. In the circuit sketched here, what value capacitor across R1 would fully correct the distortion?

c. Explain in words why or how an extra capacitor corrects the pulse shape.



Qualifying Examination - Part II

Time: 1:00-5:00 p.m. **Saturday, February 20, 1999**

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Part II counts two-thirds (2/3) of the final grade and is in this same room at 1:00 p.m.

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Divergence and curl in spherical coordinates

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$$\nabla \times E = r\,\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta E_\phi) \quad \frac{\partial E_\theta}{\partial\phi}\right] + \theta\frac{1}{r}\left[\frac{1}{\sin\theta}\quad \frac{\partial E_r}{} - \frac{\partial(rE_\phi)}{\partial r}\right]$$
$$+ \phi\frac{1}{r}\left[\frac{\partial(rE_\theta)}{\partial r}\right.$$

where r, θ, ϕ are the unit vectors associated with the spherical coordinates r, θ, ϕ

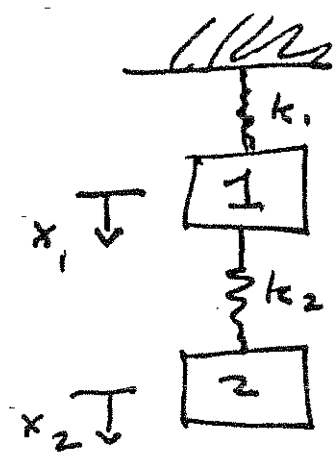
Useful integrals

$$\int_0^\infty e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{}$$

$$\int_0^\infty x e^{-\alpha x^2}\,dx = \frac{1}{}$$

$$\int_0^\infty x^2 e^{-\alpha x^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

11 A mass m_2 is hung by a spring (spring constant k_2) below a mass m_1, which is hung by a spring (spring constant k_1) below a fixed support. Find the normal mode frequencies of vertical oscillations of the masses about equilibrium.



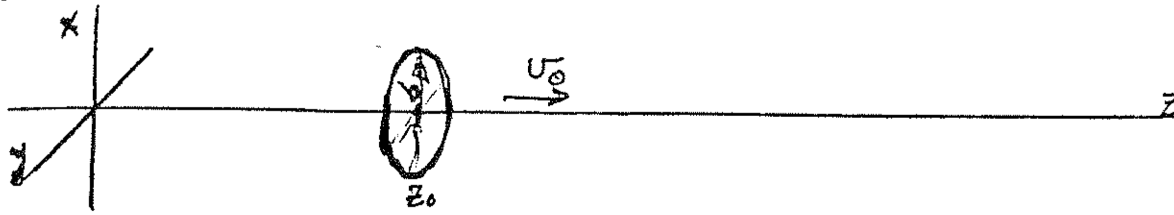
12. Compare Mount Vinson in Antarctica and Mount Cotopaxi in Ecuador (both summits are about 5 km above sea level).

a. Which of the two summits is farther from the center of the earth? By how much? Explain the result qualitatively.

b. Estimate the difference δr by calculating the potential energy due to the centrifugal force and gravity. Assume that Cotopaxi lies on the equator and Vinson at the south pole. Approximate the gravitational force by that of a spherical earth with radius $R = 6400$ km. Use $g = 9.8\ m/s^2$ for the acceleration on the surface of the earth.

c. The actual shape of the earth is not spherical. Does this make δr increase or decrease?

13 Consider a conducting loop with resistance R and radius b which is in a plane parallel to the x-y plane. There is a magnetic field with cylindrical symmetry given, in cylindrical coordinates, by: $\boldsymbol{B} = \alpha(-(r/2)\hat{r} + z\hat{z})$. ($\hat{r}$ is a unit vector pointing outward from the z-axis, and r is the distance measured from that axis.) At an instant in time the loop is located with its center on the z-axis at $z = z_0$.



What is the magnetic flux through the loop?

b If the loop were to be moved with a constant velocity v_0 along the z-axis, what emf would be generated around the loop?

c. What is the direction and the magnitude of the current induced in the loop for part b above?

d. What force is necessary to maintain the motion at a constant velocity?

You may neglect the magnetic field produced by the induced current.

14 The interatomic potential for the H_2 molecule may be represented fairly accurately by the Morse potential

$$V = V_0 \left[-2e^{-\beta(r-r_e)} + e^{-2\beta(r-r_e)}\right]$$

where $r_e = 0.74 \times 10^{-10}$ m is the equilibrium separation of the hydrogen atoms, $V_0 = 4.75$ eV, and $\beta = 1.8 \times 10^{10}\ \text{m}^{-1}$.

a. Determine the ground state energy of H_2 assuming the oscillations around equilibrium are small. What is the dissociation energy D of the molecule? (A sketch of the potential may help.) Give numerical results as well as analytic expressions.

b. Estimate the number of excited states of H_2 below dissociation Explain your argument.

c. Do you expect the exact solution to give more or fewer excited states than your estimate? Explain your argument in terms of the form of the potential and sketch.

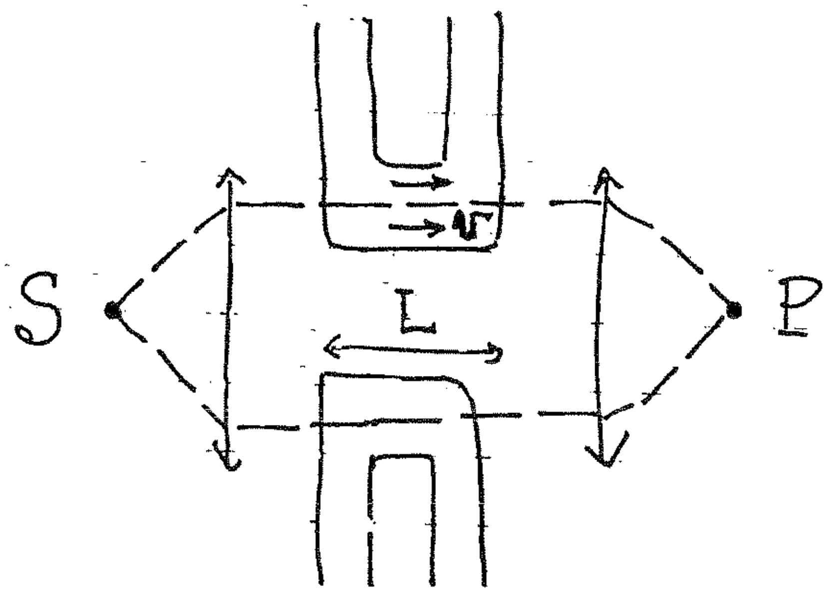
5 A gas leaks from a constant temperature chamber of volume V out into a vacuum through a small hole of area A. Calculate the time required for the pressure to decrease to $1/e$ of its original value.

16. A source emits sound waves at a frequency f_0 in its own reference frame at rest in the fluid. It is moving toward an observer at a constant velocity, V_s. The observer is moving toward the source at a constant velocity V_0. The speed of sound is c.

a. What is the frequency f seen by the observer?

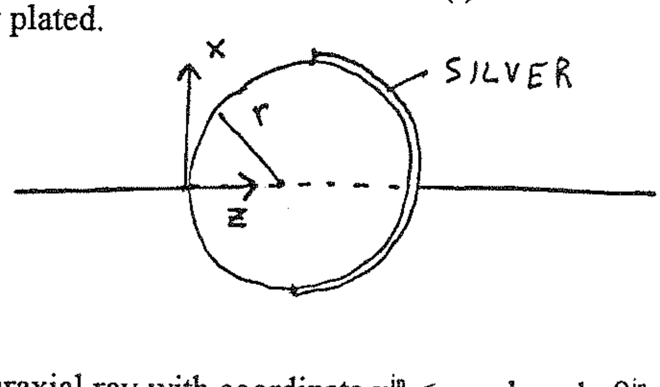
b. Is this formula correct for light? Explain.

17. Light (frequency f) emitted by a source S is directed through the optical system shown in the figure. A liquid (index of refraction n) flows through the upper conduit with velocity v, and is at rest in the lower (otherwise identical) one. What flow speed v results in destructive interference at P?



18. A plane wave with wavelength $\lambda = 500$ nm and with an intensity of 1W/cm^2 is incident normally on a circular aperture. A screen is placed 200 cm behind the circular aperture. If the aperture is equal to one Fresnel zone, what is the radius of the hole? What is the intensity on axis at the screen?

19. Consider a transparent sphere in air of radius (r) and index of refraction (n), with one hemisphere silver plated.



For an incident paraxial ray with coordinate $x^{in} < r$ and angle $\theta^{in} \cong dx/dz$, find x^{out} and θ^{out}.

20. The excited state of a 2-state atom has a radiative lifetime τ, and its ground state is stable. A second identical atom is placed in close proximity to the first, far enough apart that any overlap of the atoms' electron clouds can be neglected, but well within one wavelength. Find the lifetimes of each of the 4 states of the atom pair in leading order perturbation theory.

21. Let $\hat{A}_1 = \dfrac{a^\dagger a^\dagger + aa}{4}$ $\hat{A}_2 = \dfrac{a^\dagger a + aa^\dagger}{4}$ $\hat{A}_3 = i\,\dfrac{a^\dagger a^\dagger - aa}{4}$ where $[a,a^\dagger] = 1$ Find $[\hat{A}_i, \hat{A}_j]$.

A photon with energy $E_0 = 1$ MeV collides with an electron that is at rest. Starting from the fundamental conservation laws, find the kinetic energy of the recoiling electron, assuming that the photon scatters directly backwards.

23 The equations for ideal MHD are

$$\rho\frac{dv}{dt} = \mathbf{J} \times \mathbf{B}$$

$$\mathbf{E} + v \times \mathbf{B} = 0,$$

where ρ is the mass density, v is the flow velocity, **J** is the current, **E** is the electric field, and **B** is the magnetic field. Using dimensional arguments, with an equilibrium field B_0 show that there are waves that propagate in a plasma with a velocity $V_A = (B_0^2/4\pi\rho)^{1/2}$.

The equation of state of a classical gas of "hard" particles has the form $pV/RT = f(V/V_0)$ where p, V, T are the pressure, molar volume, and temperature of the gas. R and V_0 are constants.

a. What condition is imposed on f by the requirement of positive bulk modulus (mechanical stability)?

b. Find the sign of the coefficient of thermal expansion at constant pressure

c. A theory of liquids based on such a model is called a "geometrical theory" because it emphasizes the pressure arising from packing constructs. How applicable is it to water near freezing?

25 W bosons, of mass $m_w = 80.4$ GeV/c^2, couple with equal strength to all known quarks and leptons.

a. List the allowed decay modes of a W^- to two quarks or leptons.

b. Determine the approximate branching fraction of the decay $W^- \rightarrow e^-\bar{\nu}_e$.

26. Design a bandpass filter to pass signals between approximately 1.5 kHz and 8kHz (take these as 3dB frequencies). Assume that the next stage driven by the bandpass filter has an input impedance of 1MΩ. HINT: Choose resistances to keep the output impedance of each stage less than 1/10 the input impedance of the next stage.

27. In a left-right scattering experiment to determine the polarization of an elementary particle, 670 particles scattered right (N_R) and 330 particles scattered left (N_L).

a. What is the statistical uncertainty in the N_R and N_L counts?

b. The asymmetry parameter is defined as: $A = \dfrac{N_R - N_L}{N_R + N_L}$

What is the statistical uncertainty in A?

c. Suppose the asymmetry has been predicted to be 0.43. Assuming that this is the correct value, what is the approximate statistical likelihood that a single measurement will deviate from the correct value by as much as or more than the measurement in part b.

Qualifying Examination - Part I

Time: 9:30-11:30 a.m. **Saturday, February 22, 1997**

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	$\hbar c = 197$ MeV fm $= 1.97 \times 10^{-5}$ eV cm
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Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Gas constant	$R = 8.31 J/(mole K)$
Gravitational constant	$G = 6.67 \times 10^{-11}\ N\,\mathrm{m}^2/\mathrm{kg}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12}$ F/m
Electron mass	$m_e = 9.11 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
Proton mass	$m_p =$ ..67 $\times 10^{-27}$ kg $= 938$ MeV/c^2
Bohr radius of hydrogen	$a_B = 5.3 \times 10^{-11}$ m
Ionization energy of hydrogen	13.6 eV
Avogadro's number	6.02×10^{23} / mole

Conversion Factors

1 eV $= 1.6 \times 10^{-19}$ Joule $= 1.6 \times 10^{-12}$ erg

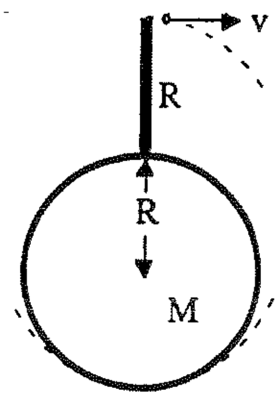
1 m $= 10^{10}$ Å $= 10^{15}$ fm $= 6.25 \times 10^{-4}$ mi

1 atm $= 1.01 \times 10^{5}$ N/m^2 $= 760$ Torr

1 cal $= 4.186$ J

1 A rope of uniform density and length L is suspended with its top end fixed, and its lower end free. Calculate the time required for a small (both in length and amplitude) transverse perturbation to propagate from the top of the rope to the bottom.

2. You are sitting on the top of a flagpole of height R, on an asteroid of mass M and radius R. What is the minimum horizontal speed v_1 that you can throw a baseball and have it return to you?



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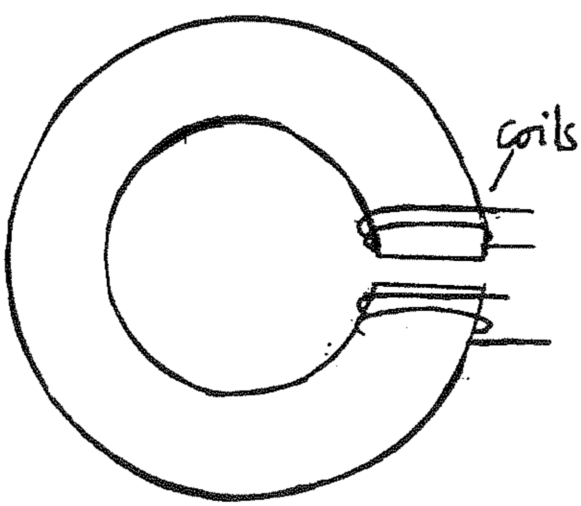


A 1.0 m long bar of iron of cross section 10 cm x 10 cm is bent into a circle forming a donut with a uniform air gap of 2 cm. Two coils of 20 turns each are wound around the iron near the gap. The magnetic field in the gap is B=0.3 T (=3,000 G), and the relative permeability of the iron is 500.

a) Find H in the gap and in the iron

b) Write Ampere's line integral law, and the find the required current in each of the coils.

4. In 1897, J.J. Thomson identified the electron as a constituent of all matter, and found its charge-to-mass ratio e/m, which was about 2000 times the charge-to-mass ratio of the lightest ion, H^+. He used a cathode ray tube, in which the electron beam was accelerated to speed v by an applied potential V. He could apply external E and B fields to the beam. He found that v was about c/10, a very large speed in those days!

a) Find an expression for v in terms of e/m and V.

b) A magnetic field B bent the beam into a circle of radius r. Find an expression for e/m in terms of measurable quantities.

c) Balanced E and B fields left the beam undeflected. Find the ratio E/B in terms of measurable quantities.

d) From the measured e/m and v, what was a reasonable guess for the charge of the electron compared to the H^+ ion?

e) What other evidence did Thomson use to support his guess (you have to know this--you can't get it from what is given)?

5) Find the maximum work which can be obtained by mixing two ideal gases with the same temperature, T_0, and same number of particles, N, but with different volumes, V_1 and V_2. The specific heat per particle, C_v, is the same for both gases.

Hints: a) maximum work is obtained when entropy does not change
b) energy of an ideal gas $E = NC_vT$
c) entropy of an ideal gas

$$S = N \log \frac{V}{N} + NC_v \log T$$

6. The lowest three configurations of the He atom are $1s^2$, 1s2s, and 1s2p.

a) What energy levels arise from these configurations?

b) Describe the energies of the levels that arise from the 1s2s and 1s2p configurations (i.e., which level is lowest, next lowest . . . to the highest energy level).

c) What are the primary interactions that cause these levels to have different energies? Which of the interactions have a classical analog, and which are uniquely quantum mechanical?

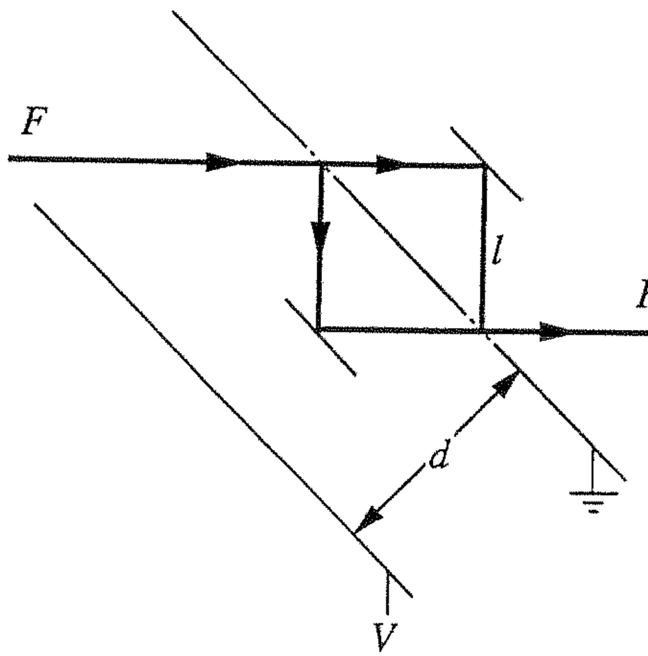
7. In an ideal gas of molecules of mass M at temperature T, the probability for a molecule to have velocity v is given by the Maxwell-Boltzmann distribution:

$$P(v) = Ce^{-Mv^2/2K_BT}$$

C is a normalization constant.

What is the root-mean-square speed? From the velocity distribution, derive the probable speed. What is the average speed?

8. A beam of atoms (mass M, speed v, polarizability α) enters a two-beam interferometer. Each leg of the interferometer is of length l. One half of the interferometer is in between two capacitor plates which are separated by a distance d and have a voltage V applied across them. If the beam flux is F entering the interferometer, find the transmitted flux F^1 assuming the beamsplitters are 50/50 and lossless. Assume $\alpha V^2/d^2 \ll Mv^2$.



9. A beaker contains two immiscible (non-mixing) liquids of different densities. The speed of sound in liquid A is V_A; in liquid B, sound propates with speed V_B. A sound wave in A propagates toward the interface with a propagation vector that makes an angle $\varphi_{A\ in}$ with respect to the normal to the interface between the two liquids. What is the angle, $\varphi_{B\ tr}$, between the direction of propagation of the transmitted sound wave and the interface normal? What is the angle, $\varphi_{A\ re}$, between the direction of propagation of the reflected wave and the interface normal? Under what conditions is there total reflection? Justify your answers.

10. Fourier's theorem says that it is possible to construct a symphonic movement from a superposition of suitably chosen sine and cosine wave oscillators. Assuming that the ear cannot hear above 20 kHz and the duration of the piece of music is 1000 seconds:

a) What is the period of the lowest frequency oscillator (not counting the constant term)?

b) How many oscillators of this frequency are necessary?

c) How many oscillators are necessary altogether?

d) Shannon's theorem from information theory states that to reproduce a wave whose highest Fourier component is f_{max}, one must specify the amplitude every $1/(2f_{max})$ seconds. Compare this result with your answer in c).

Qualifying Examination - Part II

Time: 1:00-5:00 p.m. **Saturday, February 22, 1997**

The examination is to be written on the single sheets that are provided; no more than one question is to be answered on a single sheet. Number each question. You are supposed to work 10 questions in Part II; however, when you omit questions, ***cross out those numbers on your title page***. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from your title page) on the back of each sheet of paper.

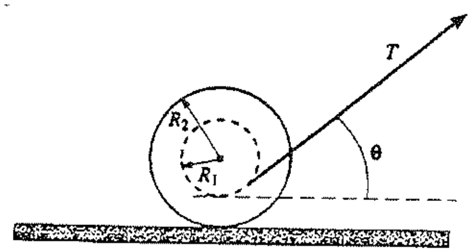
Part II counts two-thirds (2/3) of the final grade and is in this same room at 1:00 p.m.

PHYSICAL CONSTANTS

Planck's constant	$h = 6.63 \times 10^{-34}$ J sec $= 4.14 \times 10^{-15}$ eV sec
Vacuum speed of light	c 3.00×10^{8} m/sec
	$\hbar c = 197$ MeV fm 1.97×10^{-5} eV cm
Electron charge	$e = 1.60 \times 10^{-19}$ C $= 4.80 \times 10^{-10}$ esu
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Gas constant	$R = 8.31 J/(mole K)$
Gravitational constant	G 6.67×10^{-11} N m^2/kg^2
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	ϵ_0 8.85×10^{-12} F/m
Electron mass	$m_e = 9.11 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
Proton mass	$m_p = 1.67 \times 10^{-27}$ kg $= 938$ MeV/c^2
Bohr radius of hydrogen	$a_B = 5.3 \times 10^{-11}$ m
Ionization energy of hydrogen	13.6 eV
Avogadro's number	6.02×10^{23} / mole

Conversion Factors

1 eV	1.6×10^{-19} Joule	$.6 \times 10^{-12}$ erg
1 m	10^{10} Å $= 10^{15}$ fm	6.25×10^{-4} mi
1 atm	1.01×10^{5} N/m^2	760 Torr
1 cal	4.186 J	



11 A spool rests on a rough table as shown. A thread wound on the spool is pulled with force T at angle θ.

a) Show that there is an angle θ for which the spool remains at rest.

b) At this critical angle find the maximum T for equilibrium to be maintained. Assume a coefficient of friction μ.

12. A point particle of mass m and charge q is moving in a constant gravitational field (of acceleration $\mathbf{g}$ = (0,0,-g)) and a uniform magnetic field given by the vector potential $\mathbf{A}(\mathbf{r})=(0,Bz,0)$.

a) For which initial velocity $\mathbf{v} = (v_x,v_y,v_z)$ does the particle travel uniformly, in a straight line?

b) Give the energy of the particle in terms of its velocity and position in space.

c) Give the relation between the canonical momentum $\mathbf{p}$ and the mechanical momentum $m\mathbf{v}$ of the particle.

d) Give the Hamiltonian of the particle in the external magnetic and gravitational fields.

e) Derive the Lagrangian describing the motion of this particle.

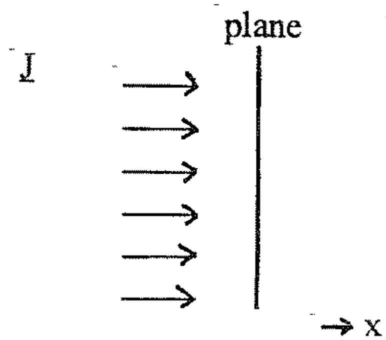
13. Find the electric field $\mathbf{E}(\mathbf{r})$ and displacement field $\mathbf{D}(\mathbf{t})$ due to a distribution of polarization $\mathbf{P}(\mathbf{r})$ which is constant inside the region $-\infty < x < \infty$, $-\infty < y < \infty$, $-a < z < a$ (i.e., an infinite flat slab), and vanishes outside.

14. A non-conducting disk of mass M and radius R has a constant charge density σ. The disk freely rotates about its axis as shown with angular velocity ω. A spatially uniform magnetic field B(t) is turned on such that $B(0) = \omega(0) \equiv 0$. Assuming the magnetic field of the disk can be neglected (back emf), find $\omega(t)$. If $B(\infty)$ is positive, which way does the disk rotate as viewed from above?

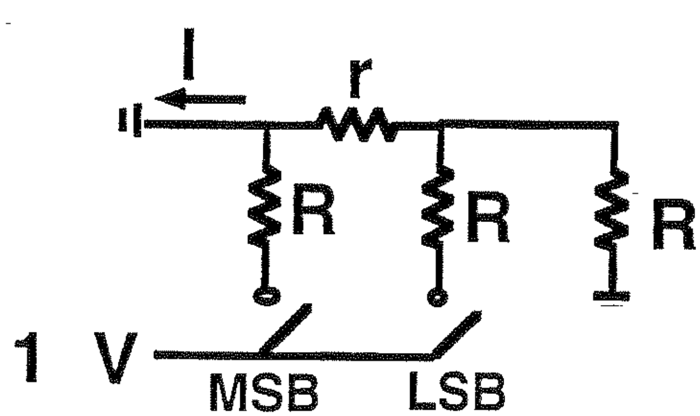
15 An initially uncharged, infinite, perfectly conducting plane is situated at x=0. A uniform current (of current density J=Jx) impinges on the plane from the left side.

a) Find the electric field everywhere for t>0.

b) Find the magnetic field at x>0, assuming B=0 at t=0.



16 Find resistance values r and R so that the 2 bit binary digital to analog converter produces current I=1mA/bit.



17. A one dimensional harmonic oscillator is started at time t=0 in a quantum state with wave function:

$$\Psi(x,0) = N[\Psi_0(x) + a\Psi_1(x)],$$

where a is real, and Ψ_0 and Ψ_1 are the normalized wave functions:

$$\Psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}; \qquad \Psi_1 = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\left(\frac{m\omega}{\hbar}\right)^{1/2} x\, e^{-m\omega x^2/2\hbar}$$

a) Give an argument from the form of the Ψ's which shows (knowing that they correspond to energy eigenstates) that Ψ_0 and Ψ_1 are the wave functions for the ground state and first excited state of the oscillator.

b) Determine the normalization constant N, and the wave function $\Psi(x,t)$ at time t.

c) Calculate the expectation value of x at time t. You do not need to calculate the final integral, but must show which terms are zero or nonzero. How does the result reflect the motion of a classical oscillator?

18. The Hamiltonion for the outer electrons of some atom has a term for the spin-orbit interaction

$$V_{LS} = f(r)\,\mathbf{L} \bullet \mathbf{S},$$

where **L** is the operator for total orbital angular momentum and **S** is the operator for total spin angular momentum. Show that if ℓ and s are the eigenvalues of total orbital angular momentum and spin respectively, then the expectation value for the spin-orbit interaction energy must be proportional to

$$\langle V_{LS}\rangle \propto (\hbar^2/2)[j(j+1) - \ell(\ell+1) - s(s+1)],$$

where j is the eigenvalue for the *total* angular momentum operator **J**.

9. Metastable atoms are produced by pulsed electron beam excitation between two parallel metal surfaces separated by 1 cm. The metal surfaces quench the metastable atoms. The spatial distribution of the metastable atoms is described by $M(z) = M_0 \sin(\pi z/1\text{ cm})$ where z is the distance normal to one of the metal surfaces. The diffusion coefficient of the metastable atoms in their parent gas at S.T.P. is $D = 1\ \text{cm}^2/\text{s}$. The gas is actually at 273 °C and 7.6 Torr. What fraction of the metastable atoms survive for 10^{-3} sec?

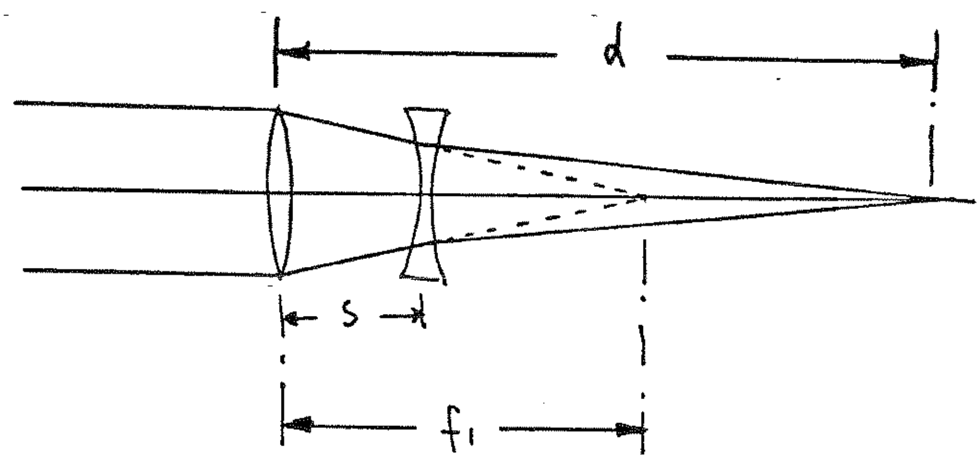
20. The equation of state of a classical hard sphere gas has the form, $pV = Tf(V/V_0)$, where f is a complicated function of the ratio of the molar volume V to the intrinsic volume of spheres of diameter D, $V_0 = \pi D^3/6$. Find the specific heat at constant volume C_V of this gas.

21 A telephoto lens can be made shorter than its focal length by combining a converging lens with a diverging lens, as shown in the figure. Let f_1 be the focal length of the converging lens and consider the case where the diverging lens has equal (but negative) focal length $f_2 = -|f_1|$ and is located at a distance $s = 3/4\ f_1$ from the converging lens.

a) Calculate the distance d between the first lens and combined focus, the focal length f of the combined lenses, and their ratio.

b) How can such an arrangement of lenses be implemented by mirrors only? Provide a sketch.

(Use the approximations for thin lenses and rays close to the optical axis.)

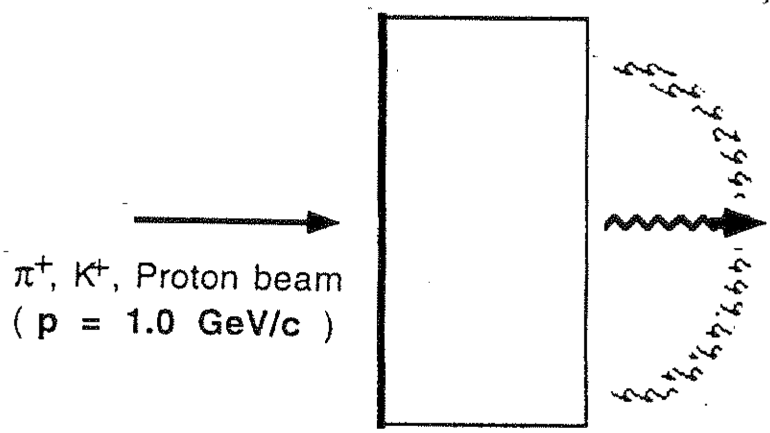


22. a) A free neutron is not stable. Give the products of its decay. Which life-time is closest to that of the neutron: 1 sec, 10 min, 1 month? What important quantity is not conserved in the decay?

b) The lightest nucleus is the deuteron, made of a proton and neutron. Explain why the neutron is stable.

c) Give the quantum numbers: L,S,J, and Isospin, for the dominant component of the deuteron wave function. Can a nucleus of 2 neutrons have these quantum numbers? Explain. No such nucleus is stable in nature. Why not? (Hint: On what quantities must the nuclear force depend?)

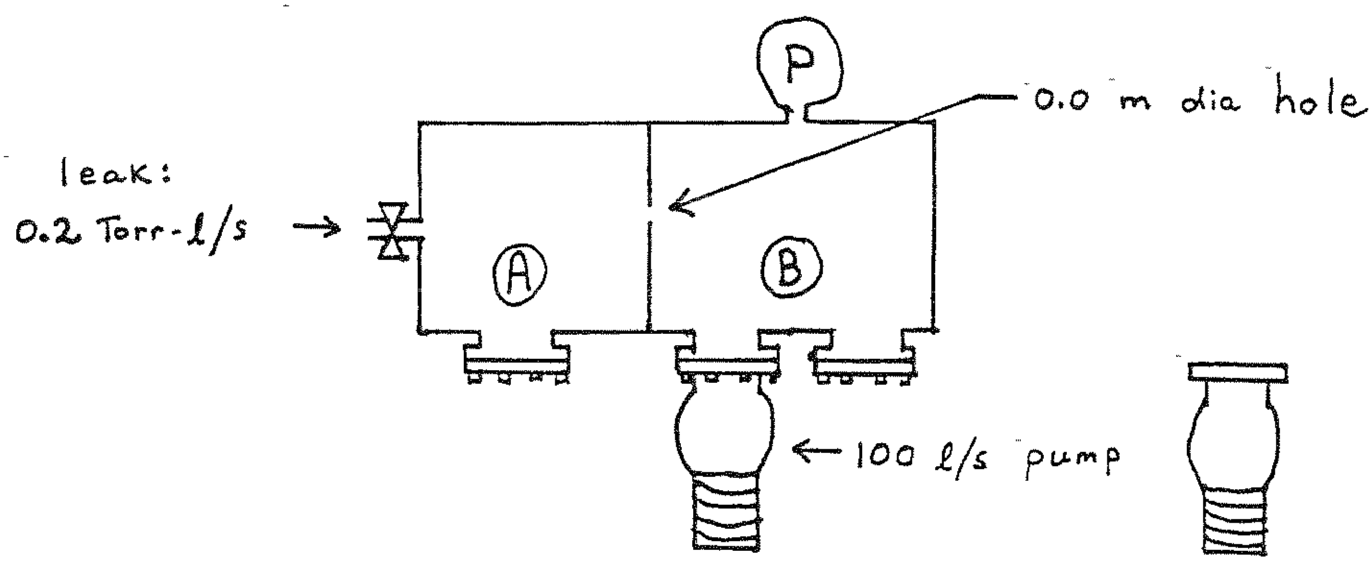
23. A block of transparent material (n=1.5) is used as a cerenkov counter. (Assume all radiation that is backscattered into the front face is absorbed.)



a) Particles with a range of velocities will produce light that emerges from the front face. What is this range of velocities?

b) A beam of mixed (pi mesons (m = 0.139 GeV), K mesons (m = 0.494 GeV) and protons (m = 0.9382 GeV) all with momentum 1.0 GeV/c are incident on the counter. Which particles can be identified by their emergent radiation? *Show your calculations.*

24.



Two chambers are connected by a hole of diameter D = .01 m. Air leaks into chamber A at a steady rate of 0.2 Torr − ℓ/s, and a 100 ℓ/s pump is attached to chamber B. The conductance of a hole for air at room temperature is

$$S = 9 \times 10^4 \left(\frac{D}{1m}\right)^2 \ \ell/s$$

a) Find the equilibrium pressure in chamber B

b) A second identical pump is available to further reduce the pressure in chamber B. Should it be attached to chamber A or chamber B?

c) With the second pump in its best position, what is the new equilibrium pressure in chamber B?

University of Wyoming
Department of Physics & Astronomy
Written Preliminary Examination - Part I
Friday, January 24, 1997
9:00 am - 2:00 pm

Instructions for Part I

1. Do seven (7) problems as instructed on the following pages.

2. Answer each problem on a separate, numbered set of papers.

3. Enter the identification number assigned to you on the upper right hand corner of <u>each</u> page. Do <u>not</u> write your name on these pages.

4. Some integrals, numerical constants, and mathematical expressions which might be useful are provided on the data pages at the end of the examination.

5. Each problem will contribute equally to your grade on this part of the examination.

6. Partial credit will be given for <u>orderly</u> progress towards a correct solution of a problem; however <u>no</u> credit will be given for an illogical, disorganized presentation or for irrelevant material.

Part I: Section A

Do each of the six (6) problems in Section A numbered 1 through 6 inclusive.

1. A particle of mass m is constrained to move on the surface of a right circular cylinder defined by $x^2 + y^2 = R^2$ and is subjected to a force proportional to the distance from the origin and directed toward the origin, $F_r = -k\sqrt{x^2+y^2+z^2}$. Select appropriate generalized coordinates, find the Lagrangian for the system, find the equation of motion, and solve these for the motion of the particle in time. Identify any constants of motion and describe how the spring constant k will affect the motion.

2. (a) Find an expansion for the Green function in spherical coordinates for the Laplace-Poisson operator outside a sphere of radius R which obeys the boundary conditions

$$\hat{n} \cdot \nabla\, G\,(\vec{r}, \vec{r}\,') = 0 \text{ with } \vec{r} \text{ on the sphere and}$$

$$\lim_{|\vec{r}| \to \infty} G(r, \vec{r}\,') = 0.$$

Reduce the problem to a one dimensional Green function equation in the radial coordinate.

(b) If the electrostatic potential at infinity is zero, if the charge density in space is zero, and if the normal component of the $\vec{E}$ field on the sphere is

$$E_r = \frac{1}{2} E_0 (3\cos^2\theta - 1),$$

find the electrostatic potential everywhere outside the sphere.

3. The electric fields for two monochromatic waves of circular frequency ω and vector amplitudes $\vec{A}_1$ and $\vec{A}_2$ are given by

$$\begin{aligned} \vec{E}_1 &= \vec{A}_1, \cos(\delta_1 - \omega t) \\ \vec{E}_2 &= \vec{A}_2 \cos(\delta_2 - \omega t) \end{aligned}$$

where δ_1 and δ_2 are phase factors. If these two electric fields are added, find a general expression for the irradiance , $I = <\vec{E}^2>$, in terms of the phase difference $\delta_1 - \delta_2$. What is the value of I when $\vec{A}_1$ is orthoganal to $\vec{A}_2$? You may wish to recall that

$$< \cos(\delta_1 - \omega t)\ \cos(\delta_2 - \omega t) > = \cos(\delta_1 - \delta_2).$$

4. A solid contains a collection of N paramagnetic ions each having a total angular momentum j and a magnetic dipole moment given by

$$\vec{\mu} = \mu_o \vec{J}$$

where $\vec{J}$ is the angular momentum operator in units of $\hbar$. Find the partition function for this system in a uniform magnetic field $\vec{B}$, neglecting the dipole - dipole interaction, and carry out all indicated summations. From it, find the magnetic moment of the system.

5. A particle of mass m is governed by a one dimensional, time independent Schroedinger equation with an attractive potential, whose form is a Dirac δ-function, located at the origin,

$$V(x) = -V_0\, \delta(x)$$

(a) Give the Schroedinger equation in coordinate space. (b) Transform the equation and the wave function to momentum space. (c) Solve for the bound state wave function in momentum space. This solution implies that the total energy must be negative. (d) Transform the wave function to coordinate space. (e) Determine the eigenvalue of energy which will make the expression for the wave function consistent.

6. (a) You measure the current flowing through a resistor to be $I = 0.100 \pm 0.005A$ and independently measure the resistance of that same resistor to be $R = 100 \pm 5\Omega$. What is the power flowing through that resistor and the error in that estimate if the errors in the measurements obey Gaussian statistics?

   (b) You are attempting to measure the amplitude of pulses roughly 1 μs long from a photomultiplier preamp through a long cable, but find the signal contaminated by a sizable 60 Hz background added most likely by unavoidable ground loops. Describe how you might salvage the experiment using some miscellaneous resistors and capacitors lying about the laboratory, and make the description as quantitative as possible.

Part I: Section B

Do any one (1) of the three (3) problems in Section B numbered 7 through 9 inclusive.

7. (a) Using the notation for $L - S$ coupling, give the complete electronic configuration of a sodium atom with a nucleus $^{23}Na^{11}$ in its ground state.

   (b) Give a simple sketch with appropriate spectroscopic labeling showing the lowest two excited, electronic levels for the sodium atom with the lowest allowed principal quantum number, identify the fine structure splitting, and show the transitions allowed via electric dipole coupling.

   (c) Give a second sketch showing the effect of an external magnetic field on these electronic levels.

   (d) Give a third sketch showing how the nuclear spin ($I = \frac{3}{2}$) splits and shifts the levels forming a hyperfine structure and how the hyperfine levels react to a small external magnetic field.

8. The probability for an electron to occupy a state at a finite temperature is given by the Fermi distribution

$$f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1}$$

where E_f is the Fermi energy. All electrons in an intrinsic semiconductor at $0°K$ are in the valence bands and completely fill these bands. At a finite temperature, a number of electrons are in the conduction band and hence an equal number of vacancies appear at the top of the highest valence band. Since the band gap is large compared to kT, the Fermi energy is approximately in the middle of the gap. An n-type semiconductor is doped with donor impurities that easily give electrons to the conduction band; hence the majority carriers are electrons. A p-type semiconductor is doped with acceptor impurities that easily absorb electrons from the valence band; hence the majority carriers are holes.

(a) How is the Fermi level affected by n-type doping? How is it affected by p-type doping?

(b) Draw a diagram indicating the location of the Fermi level, the bottom of the conduction band, and the top of the valence bands at the junction of an n-type semiconductor and a p-type semiconductor.

(c) Indicate the direction of the electric field generated at this junction.

9. (a) The total angular momenta, parities, and energies of the lowest few excited states of many nuclei whose neutron and proton numbers are both far from "magic numbers" are well described by rotational and vibrational collective motion. For a certain nucleus with $A = 180$ and $Z = 72$, ^{180}Hf, the ground state and lowest four (4) excited states are known to form a pure rotational band with no vibration. Give the total angular momentum and parity for each of these five (5) states. Why?

(b) The first excited state is 93.3 keV above the ground state. If the energy spectrum is similar to that for some states of a rigid rotor, give the lowest order formula for these levels and calculate the energies predicted for the other three (3) excited states.

(c) How and why would you expect the measured energies to deviate from the predictions? How might you "correct" the formula?

PHYSICS

University of Wyoming
Department of Physics & Astronomy
Written Preliminary Examination - Part II
For PHYSICS students.
Saturday, January 25, 1997
9:00 am - 2:00 pm

Instructions for Part II

1. Do six (6) problems as instructed on the following pages.

2. Answer each problem on a separate, numbered set of papers.

3. Enter the identification number assigned to you on the upper right hand corner of <u>each</u> page. Do <u>not</u> write your name on these pages.

4. Some integrals, numerical constants, and mathematical expressions which might be useful are provided on the data pages at the end of the examination.

5. Each problem will contribute equally to your grade on this part of the examination.

6. Partial credit will be given for <u>orderly</u> progress towards a correct solution of a problem; however <u>no</u> credit will be given for an illogical, disorganized presentation or for irrelevant material.

Do each of the four (4) problems in Section A numbered 1 through 4 inclusive.

1. The gamma function may be defined by the integral

$$\Gamma(z) = \int_0^\infty e^{-t}\ t^{z-1}\, dt\ , \quad Re(z) > 0.$$

Use the Cauchy-Goursat theorem to prove that, for x real and $0 < x < 1$,

$$\int_0^\infty \cos t\ \ t^{x-1}\, dt = \Gamma(x)\cos(\frac{\pi}{2}x)$$

$$\int_0^\infty \sin t\ \ t^{x-1}\, dt = \Gamma(x)\sin(\frac{\pi}{2}x)$$

Select a contour integral and contour appropriate for the task, discuss any singularities in the integrand with care and indicate these on a sketch of the t-plane, and evaluate integrals on unwanted parts of the contour going to zero or infinity with great care.

2. A point mass m is suspended by a massless rod of length l in a uniform gravitational field to form a spherical pendulum which can move without friction in both the azimuthal coordinate ϕ and the angular deviation from verticle θ. (a) Select generalized coordinates and find the Lagrangian. Give a drawing which clearly shows the coordinates. (b) Find the canonical momenta and find the Hamiltonian for the system. (c) Find Hamilton's equations of motion for the system and identify any cyclic coordinates. (d) If the coordinate θ is constant, complete the solution.

3. (a) A point charge of magnitude Q moves on a trajectory $\vec{r}_\circ(t)$. Find expressions for the charge density and the associated current density.

   (b) The Green function for a retarded solution to the wave equation with a source at $\vec{r}\,'$ and t' is

$$G(\vec{r}, t; \vec{r}\,', t') = \frac{1}{|\vec{r} - \vec{r}\,'|}\ \delta(t - t' - \frac{|\vec{r} - \vec{r}\,'|}{c}),$$

   where the δ denotes a Dirac δ - function. Find the scalar and vector potentials for the electromagnetic field in the Lorentz gauge as a function of $\vec{r}$ and t. These are the Lienard-Wiechart potentials. You may wish to use the notation

$$\vec{\beta}(\tau) = \frac{1}{c}\frac{d\,\vec{r}_\circ}{dt}\bigg|_{t\,=\,\tau}$$

$$\hat{n}(\tau) = \frac{\vec{r}-\vec{r}_\circ(\tau)}{|\vec{r}-\vec{r}_\circ(\tau)|}.$$

Indicate the retarded time at which quantities must be evaluated clearly.

4. (a) A spinless particle which obeys the time independent Schroedinger equation is scattered from a fixed center by a potential $V(\vec{r})$ under conditions for which the Born approximation is valid. Derive the first order Born approximation.

   (b) Calculate the differential cross section in the first order Born approximation for scattering from the potential

$$V(r) = \frac{V_0}{r}\, e^{-\frac{r}{a}}$$

   where $a > 0$. You need not bother determining the constant multiplicative factors.

Do any two (2) of the three (3) problems in Section B numbered 5 through 7 inclusive.

5. An astronomer measures the spectrum of a hydrogen emission region in a starburst galaxy which has been reddened by intervening dust. The following line fluxes are observed: $F_{obs}(H\alpha) = 1.0 \times 10^{-12}$ erg s^{-1} cm^{-2} and $F_{obs}(H\beta) = 2.5 \times 10^{-13}$ erg s^{-1} cm^{-2}. Observations of forbidden line ratios indicate that the electrons in the region have a temperature $T_e = 10^4 K$ and a density $n_e = 10^4$ cm^{-3}. Case B recombination theory predicts an intrinsic ratio F_{int} $(H\alpha)$ / $F_{int}(H\beta) = 2.85$ for this temperature and density. The extinction at $H\beta$ is 1.424 times greater than that at $H\alpha$. Assume that the dust exists only in an homogeneous, gas-free, plane-parallel slab lying between the emission region and the observer.
(a) Assuming that the source function is independent of optical depth in the dust slab, write the solution to the radiative transfer equation in the form of intensity emerging from the slab. How does this solution simplify when the dust slab contributes no emission or scattering to the emerging beam?
(b) Determine the optical depth through the dust slab for the $H\alpha$ line.

(c) Calculate the intrinsic $H\alpha$ flux, $F_{int}(H\alpha)$.

(d) Calculate the star formation rate (SFR) for the galaxy if SFR = 1.8 x $10^{11} F_{int}(H\alpha) M_\odot$ yr^{-1}.

6. Using the equation of hydrostatic equilibrium, the ideal gas law, and the approximate scaling relations for mean density in terms of stellar mass M^*, radius R^*, and temperature, determine:

(a) The functional dependence of pressure on M^* and R^*,

(b) The functional dependence of temperature on M^* and R^*, and

(c) The functional dependence of luminosity on M^* (i.e., the main sequence mass-luminosity relationship).

7. A "wave front tilt sensor" for a telescope with adaptive optics images a star onto a "quad cell" as drawn below. If the telescope is not aligned with the star, the star is displaced from the center of the quad cell. For this problem, assume that the image of the star is a uniformly illuminated square whose sides are an equivalent 1.0 arcsecond across. Also assume that displacements of the star of Δ arcseconds are always small enough that some flux falls on each of the cells and that all of the flux is detected by the quad cell. The flux on each of the cells if f_1, f_2, f_3, and f_4 as indicated in the drawing; hence the signal for a displacement in x is $S_x = f_1 + f_2 - (f_3 + f_4)$.

(a) Assuming a flux of N photons per second from a star centered in the quad cell and an integration time of δt, calculate the four average fluxes f_1, f_2, f_3, and f_4 and the root mean squared uncertainty in these numbers. (b) If the star is displaced a small distance Δ_x arcseconds, calculate S_x. Using the noise calculated in part (a) above which neglects changes in noise due to Δ_x, calculate the root mean squared uncertainly in S_x. (c) Assuming a telescope with a 2.3 m diameter and a 70% net transmission, a detector with a 1000 Angstrom bandpass at 5500 Angstroms, and the need to measure the tilt to better than 0.1 arcseconds more than 90% of the time within 10 milliseconds, how faint a star can be observed before the noise ruins the measurement? Recall that the flux for a 0^{th} magnitude star is 1000 photons/$(cm^2 \cdot \text{ sec } \cdot A)$

Qualifying Examination - Part I

Time: 9:30-11:30 a.m. **Saturday, February 17, 1996**

The examination is to be written on the single sheets that are provided; no more than one question is to be answered on a single sheet. Number each question. You are supposed to work all questions in Part I; however, if you must omit any questions, *cross out those numbers on your title page*. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from your title page) on the back of each sheet of paper.

Part I counts one-third (1/3) of the final grade.
Part II is in this same room at 1:00 p.m.

PHYSICAL CONSTANTS

Planck's constant	$h = 6.63 \times 10^{-34}$ J sec $= 4.14 \times 10^{-15}$ eV sec
Vacuum speed of light	$c = 3.00 \times 10^{8}$ m/sec
	hc 197 MeV fm $= 1.97 \times 10^{-5}$ eV cm
Electron charge	e 1.60×10^{-19} C $= 4.80 \times 10^{-10}$ esu
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Gas constant	$R = 8.31 J/(moleK)$
Gravitational constant	$G = 6.67 \times 10^{-11}$ N m^2/kg^2
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	ϵ_0 8.85×10^{-12} F/m
Electron mass	$m_e = 9.11 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
Proton mass	$m_p = 1.67 \times 10^{-27}$ kg $= 938$ MeV/c^2
Bohr radius of hydrogen	$a_B = 5.3 \times 10^{-11}$ m
Ionization potential of hydrogen	13.6 eV
Avogadro's number	6.02×10^{23} / mole

Conversion Factors

1 eV $= 1.6 \times 10^{-19}$ Joule $= 1.6 \times 10^{-12}$ erg

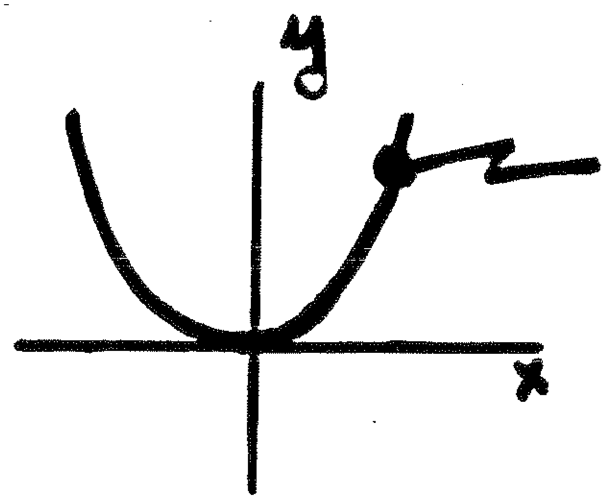
1 m $= 10^{10}$ Å $= 10^{15}$ fm $= 6.25 \times 10^{-4}$ mi

1 atm $= 1.01 \times 10^{5}$ N/m^2 $= 760$ Torr

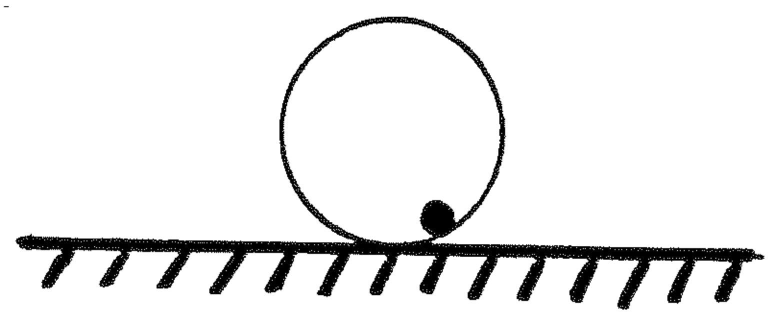
cal $= 4.186$ J

1. A certain near-sighted eye can correctly focus objects at a maximum distance of 25 cm. What focal length corrective lens placed 2.5 cm in front of this eye will allow it to see very distant objects clearly?

2. An incompressible fluid is placed in a cylindrical vessel which rotates with constant angular velocity ω. Show that the fluid's surface is a paraboloid. (Hint: The free surface of a liquid is one of constant pressure.)
The centrifugal potential energy density is: $\frac{1}{2}\rho\omega^2 x^2$



3. A point mass m is attached to the rim of an otherwise uniform hoop of mass M and radius R. Calculate the frequency of small-amplitude rolling oscillations.



4. Consider a system of N coupled 2-D oscillators, each of which has a spatial potential

$$V = \frac{1}{2} k_x x^2 + \frac{1}{2} k_y y^2$$

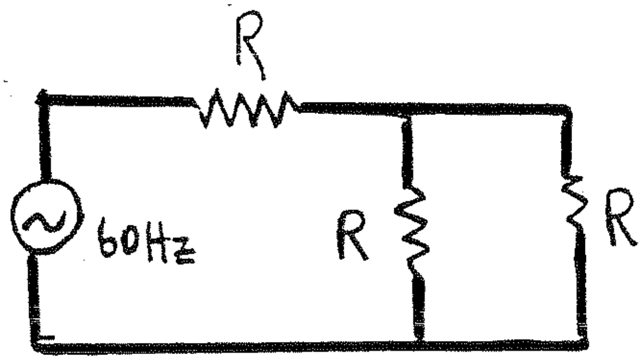
and mass m.

Initially, each oscillator has zero kinetic energy, zero displacement in the y direction, and a random displacement in the x direction. The mean x displacement is zero and the rms x displacement is x_o. After a time the system reaches equilibrium.

a) What is the temperature of the system?

b) What is the rms y displacement?

5.

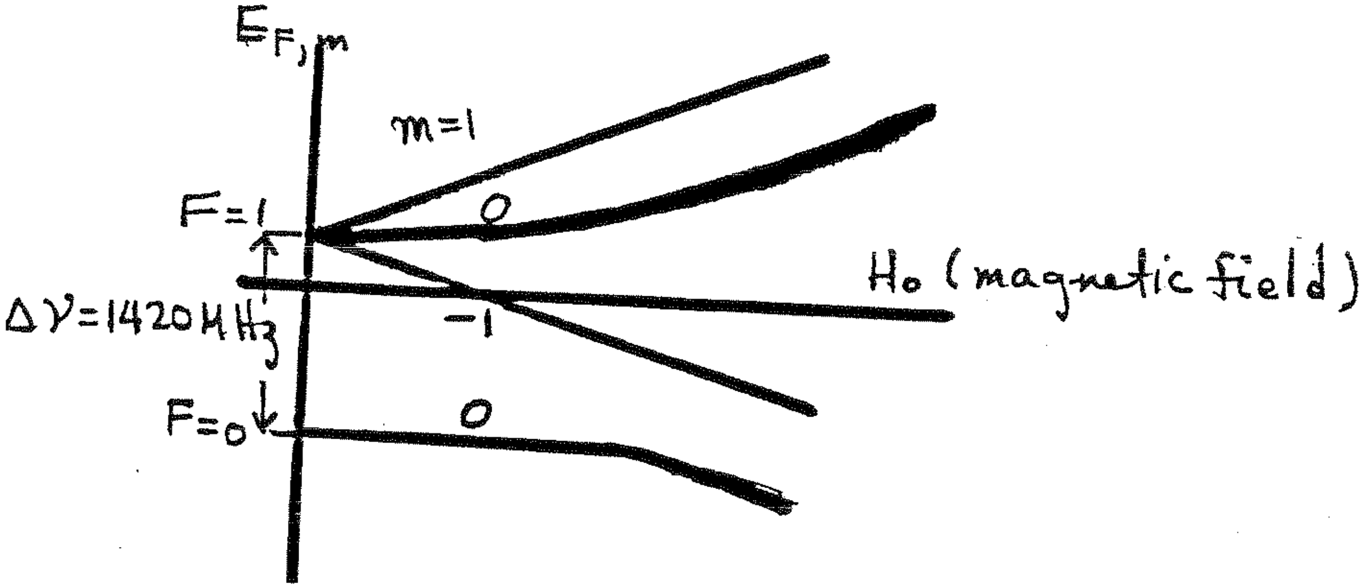


Suppose that the resistors in the circuit shown are actually "60 watt" lightbulbs, meaning that the power dissipated is 60 W when the bulb is hooked directly across the voltage source. How much power is dissipated in each lightbulb in the circuit given? Assume that the resistance value is independent of the power level.

6. a) Derive the minimum rotation period of a uniform spherical object held together by self-gravity, with mass M and radius R by considering the balance of forces on its surface.

b) If we observe an object of one solar mass ($M = 2 \times 10^{30}$ kg) to rotate with a period of 1 second, can we differentiate if the object is a white dwarf with a radius $R = 10^7$ m, or a neutron star with $R = 10^4$ m? Explain your answer.

7. The ground level of H has an electron spin of s = ½ and a nuclear spin of I = ½. The total angular momentum can be 1 or 0. The hyperfine energy separation between the F=1 and F=0 levels in zero magnetic field is 1420 MHz. The electron g factor is 2 and $g\mu_B/h$ = 2.8 MHz/gauss where μ_B is the Bohr magneton. The Zeeman-hyperfine energy levels are shown.



a) What type of transitions are possible among the 4 energy levels (electric dipole, magnetic dipole, electric quadupole, etc.)?

b) Sketch all the allowed transitions for the type of transition from (a)

c) What is the critical field for decoupling of S and I?

8. Atomic Na is excited to its resonance level using a short laser pulse at 589 nm. The fluorescence decays in time as F(t) = F_0 exp (-t/16 ns) with a buffer gas pressure of 10^{-2} Torr at room temperature. The fluorescence decays as F(t) = F_0 exp (-t/8 ns) with a buffer gas pressure of 10 Torr. Assume the buffer gas average thermal speed is 3 x 10^4 cm/s, find the cross section for quenching the resonance level of Na.

9. A point source S located at the origin of a coordinate system emits a spherical sinusoidal wave in which the optical disturbance E_1 is given by

$$E_1 = A \frac{D}{r} \cos\left(\omega t - \frac{2\pi r}{\lambda}\right), \quad \text{where r is the distance from S}$$

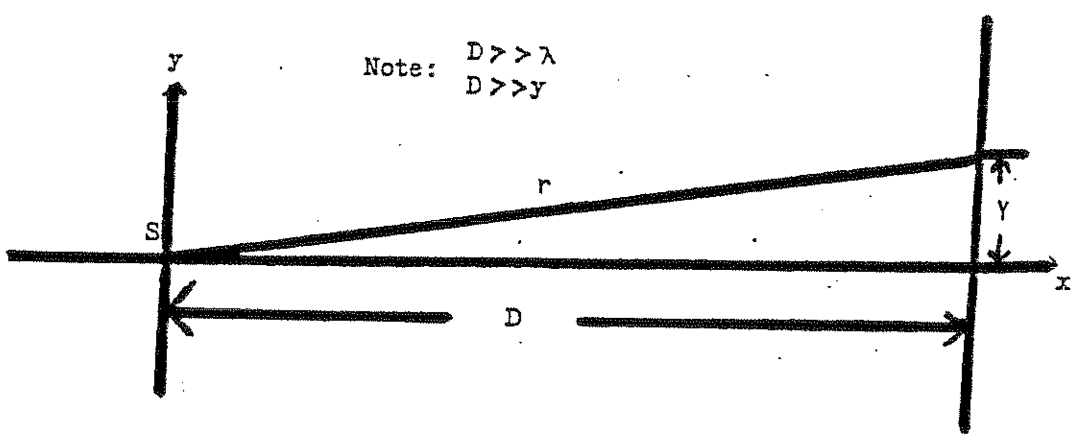
In addition, there is a plane wave propagating along the x-axis. This wave is given by

$$E_2 = A \cos\left(\omega t - \frac{2\pi x}{\lambda}\right).$$

(Note that we treat both E_1 and E_2 as scalar waves in this problem.)

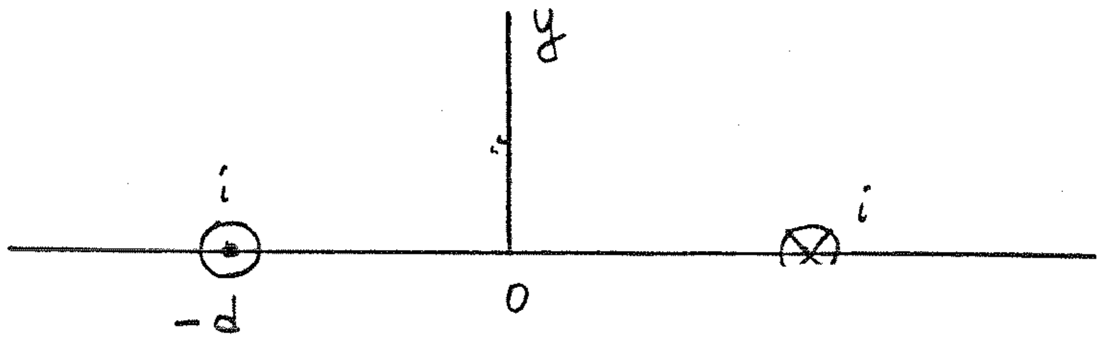
Both waves are incident on a flat screen perpendicular to the x-axis and at a distance D from the origin, as shown in the figure.

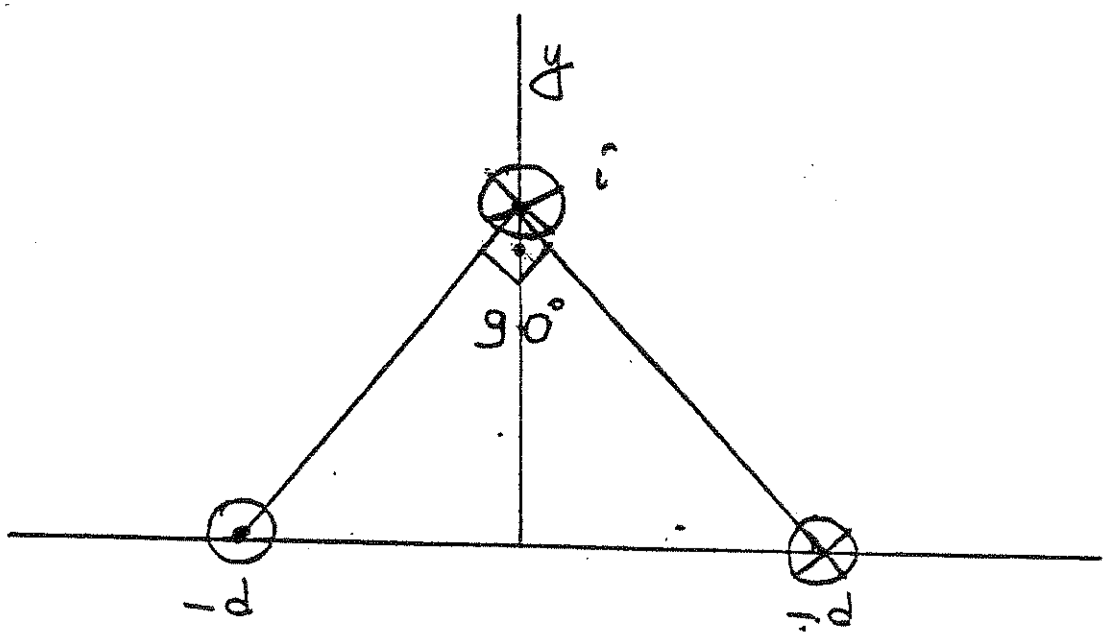
Compute the resultant intensity I at the screen as a function of distance Y from the x-axis for values of Y small compared with D. Express I in terms of Y, D, λ and the intensity I_0 at Y=0.



10. a) Write the Biot-Savart Law and Ampere's Law.

b) Find the force on a current i in to the page on the y-axis as shown, a distance R from the two currents, with a 90^0 angle as shown.





Qualifying Examination - Part II

Time: 1:00-5:00 p.m. **Saturday, February 17, 1996**

The examination is to be written on the single sheets that are provided; no more than one question is to be answered on a single sheet. Number each question. You are supposed to work all questions in Part I; however, if you must omit any questions, ***cross out those numbers on your title page***. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from your title page) on the back of each sheet of paper.

Part II counts two-thirds (2/3) of the final grade and is in this same room at 1:00 p.m.

PHYSICAL CONSTANTS

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Ionization potential of hydrogen	13.6 eV
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1 atm 1.01 x 10^5 N/m^2 = 760 Torr

1 cal = 4.186 J

Spring 1996 Qualifier **12-minute Questions**

1. An unstable particle has a velocity $\beta = \frac{4}{5}$

the particle decays 7.2 meters from its point of origin. Calculate:

a) the *Lorentz factor* γ;

b) its *time of flight* in the laboratory; and

c) the *elapsed proper time* in its rest frame.

12. For nine photon energies, $h\nu = 2 \times 10^{3n}$ eV($n = -3, -2, \ldots +5$)

a) Give a conventional name used for radiation at each photon energy (1 point)

b) Tell what type of detection could be used for each energy. A single example for each energy is sufficient (4 points).

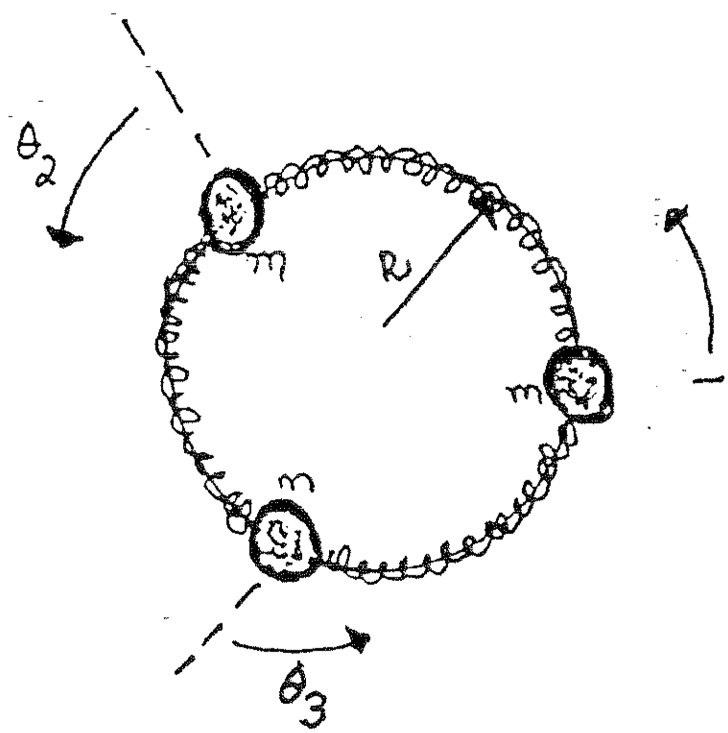
c) Choose <u>two</u> of the detectors named in part (b) and write a <u>very brief</u> description of how they work.

13. A model of a ring molecule consists of three equal masses m which slide without friction on a fixed circular wire of radius R. The masses are connected by identical springs of spring constant $m\omega_0^2$. The angular positions of the three masses, θ_1, θ_2, and θ_3, are measured from a rest position.

a) Write down the Lagrangian and derive the equations of motion.

b) Show that the mode in which $\theta_1 \quad \theta_2 \quad \theta_3$ corresponds to constant total angular momentum.

c) Assume the total angular momentum is zero and that $\theta_1 + \theta_2 + \theta_3 = 0$. Find two degenerate oscillatory modes and their frequency.



14. A 30 m cable in an inaccessible conduit has been damaged at some point along its length. You have access only to one end. You have a pulse generator, a scope, and a 10 m length of the same type of cable.

a) Describe how you use the pulse generator and scope to locate the distance to the point of damage.

b) What pulse type, pulse height, pulse width, and repetition rate do you set on the pulse generator to give the brightest oscilloscope trace to locate the damage within 1 m.

c) Draw a picture of the scope trace with labeled time axis if the cable is cut at 25 m.

d) Draw a picture of the scope trace with labeled time axis if the cable is shorted at 15 m.

15. A plane electromagnetic wave travels through a medium in which the current density, **J**, and electric field, **E**, are related by the constitutive relation $\mathbf{E} = A\, d\mathbf{J}/dt$ where A is a constant.

a) Find the wave energy dissipated in the medium over one wave period.

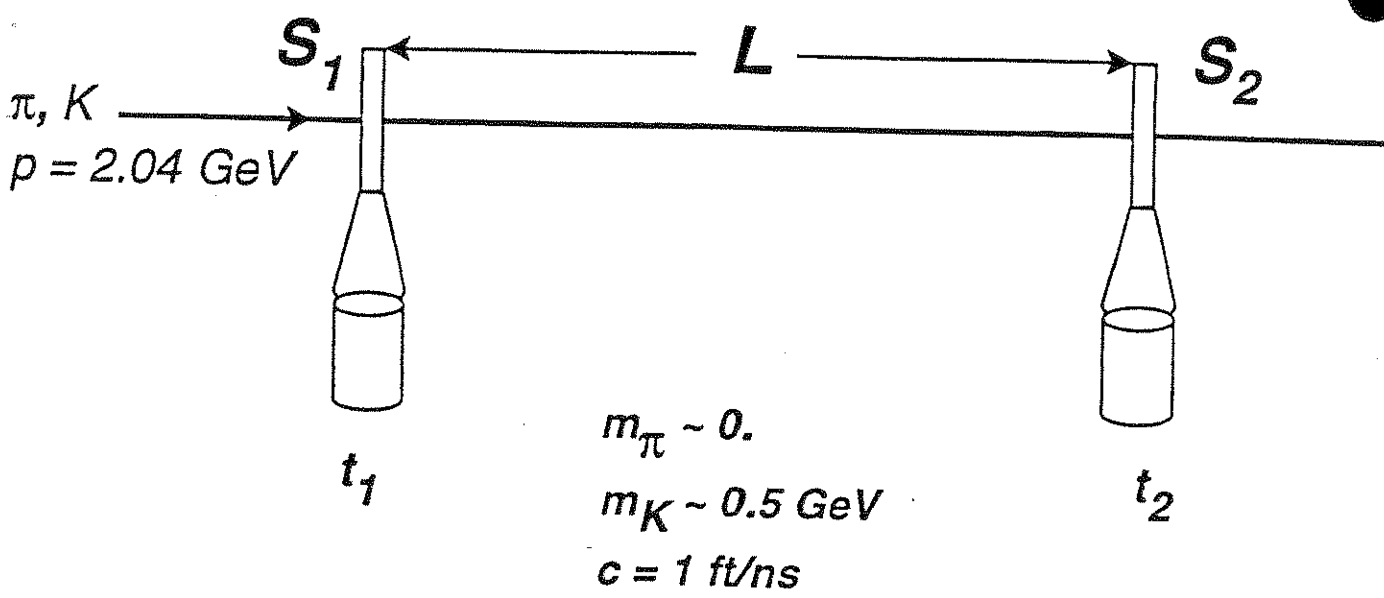
b) Find $\omega(k)$, the frequency as a function of wave number. (Assume vacuum permeability and permittivity.)

16. a) Describe qualitatively with a sentence or two what the nature of "normal" Zeeman effect is.

b) Describe precisely what properties of the "normal" Zeeman effect would allow one to determine some information about the direction of the magnetic field in a distant astronomical light source.

c) What is the relation between the magnitude of the magnetic field and characteristics of the "normal" Zeeman spectrum?

7.



a) Two identical scintillation counters, S_1 and S_2, are used to define a particle beam composed of pions and kaons. Each of the counters has a time resolution of $\sigma_0 = 1$ns. It is desired to use the difference in time of flight to separate the particles $|t_1 - t_2| = 3\sigma_T$, where σ_T is the time resolution of the measurement. How far apart (L ft.) must the counters be?

b) Describe one other way to identify the Kaons.

18. In nuclear physics, isospin is an approximate symmetry. In the isospin formalism the proton and neutron are considered two states of a particle called a "nucleon," with $T_3 = 1/2$ for the proton and $T_3 = -1/2$ for the neutron. The value of the isopin, T, for the nucleon in 1/2, such that $T^2 = 1/2(1/2 + 1)$. Isospin adds in the same fashion as angular momentum.

a) For ^{36}Ca, with Z 20 and N=16, what is the value of the projection of the total Isospin T_3?

b) What value of T can the eigenstates of ^{36}Ca have? Explain.

c) If in a nucleus the isospin conserving interaction between particle i and j is $V_{ij} = |V_0|t_i \bullet t_j$, show that the potential energy, i.e., $1/2\sum_{i\neq j} V_{ij}$, for ^{36}Ca is given by $.5|V_0|(T^2 - 24)$.

d) From the results of (c), what value of T would one expect for the ground state of ^{36}Ca? At what excitation energy would different values of T be expected to appear?

e) The lowest order Coulomb interaction $V_{ij}^c = e_i e_j / r_{ij}$ breaks the isospin symmetry (which would have required all states with the same value of T but different values of T_3 to be degenerate). Express e_i in terms of t_{3i}, and then show that the energy differences due to V^c are given by:

$$\Delta E(T_3) \quad c_1 + c_2 T_3 + c_3 T_3^2$$

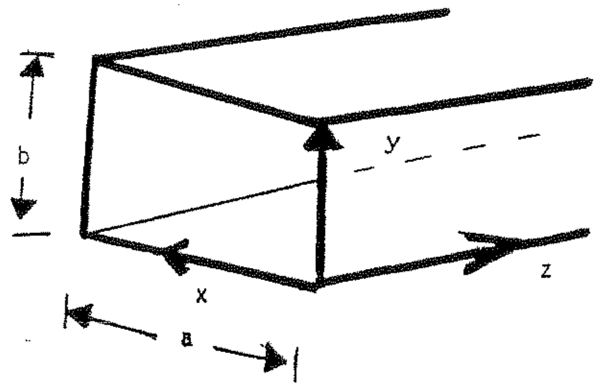
where the c's are constants.

19. A neutrino gas (N particles in Volume V) of spin ½ particles has the energy - momentum relation E=cp. The gas is, in good approximation, a non-interacting quantum gas and has the following relation between its pressure P, volume V, and internal energy E: $PV = E/3$.

a) Consider a reversible adiabatic expansion from initial state P_1, V_1, to final state P_2, V_2, with $V_2 = 8V_1$. Find P_2/P_1.

b) At low temperatures T, this gas has specific heat at constant volume $C_v = aT$. Find C_p/C_v as $T \to 0$.

20. Calculate the cut-off wavelength for propagation in the z-direction of an EM wave down the rectangular perfectly conducting wave guide shown. Assume a TE mod[e] with $E_x = E_z = 0$.



21. A transmitter on a beach sends radio waves out to sea with a spectrum sharply peaked at $\nu = 10^8\ s^{-1}$. Describe the spectrum of the back-scattered radiation. (Hin[t:] The relation between angular frequency and wave number of water waves is

$$\omega = \sqrt{gk}\ .)$$

22. The deuteron is a weakly bound state of a neutron and a proton with L=0 (S state). Suppose the np potential is a square well potential with radius a, $V(r) = -V_0$ for $0 \le r < a$, and $V(r) = 0$ for $r > a$. Take $m_n \approx m_p = m$.

a) Write the time-independent radial Schrodinger equation for the deuteron.

b) Give the form of the radical wave function $u(r) = r\psi(r)$ for $0 \le r < a$ and $r > a$, the boundary conditions satisfied by u(r), and the matching condition at r=a. What determines the bound state energy E?

c) Suppose the binding is very weak (it is), so that $E \approx 0$. Determine V_0 in terms of a. Estimate V_0 for $a \approx 1$ fm.

23. An X-ray beam with a continuous spectrum is diffracted by a Si (111) crystal with a lattice plane spacing $d=0.315$ nm. What is the exit angle θ_{out} with respect to the (111) planes if the entrance angle is $\theta_{in}=30^0$? What are the wavelengths of the diffracted X-rays? If N lattice planes contribute to the diffraction, estimate the energy widths $\delta E/E$ of the outcoming X-rays using the uncertainty relation.

University of Wyoming
Department of Physics & Astronomy
Written Preliminary Examination - Part I
Friday, January 26, 1996
9:00 am - 2:00 pm

Instructions for Part I

1. Do seven (7) problems as instructed on the following pages.

2. Answer each problem on a separate, numbered set of papers.

3. Enter the identification number assigned to you on the upper right hand corner of _each_ page. Do _not_ write your name on these pages.

4. Some integrals, numerical constants, and mathematical expressions which might be useful are provided on the data pages at the end of the examination.

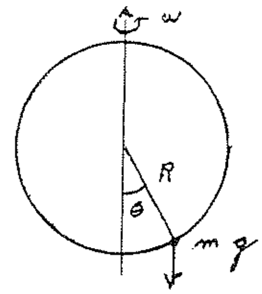
5. Each problem will contribute equally to your grade on this part of the examination.

6. Partial credit will be given for _orderly_ progress towards a correct solution of a problem; however _no_ credit will be given for an illogical, disorganized presentation or for irrelevant material.

Part I: Section A

Do each of the six (6) problems in Section A numbered 1 through 6 inclusive.

1. (a) A bead of mass m slides without friction on a circular loop of radius R which lies with a diameter along a vertical axis relative to a uniform gravitational field and which rotates about this diameter with a constant angular velocity ω. Find the Lagrangian for this system using θ as the generalized coordinate and find the equation of motion.



(b) For angular velocities greater than some critical angular velocity ω_c, the bead performs small oscillations about some point of stable equilibrium θ_0. Find ω_c *and* $\theta_0(\omega)$.

(c) Obtain an approximate equation of motion for small oscillations about $\theta_0(\omega)$ and find the period of the oscillations.

2. (a) A semi-infinite right circular cylindrical region of space has a radius R, has an axis coinciding with the z axis from 0 to ∞, and contains no materials. Find an expansion in cylindrical coordinates for the Green function for the Laplace-Poisson operator inside this region which obeys the boundary conditions

$$\begin{aligned} G(\vec{r},\vec{r}\,') &= 0 \text{ at } r = R, \\ \hat{n}\cdot\nabla G(\vec{r},\vec{r}\,') &= 0 \text{ at } z = 0, \text{and} \\ \lim_{z\to\infty} G(\vec{r},\vec{r}\,') &= 0 \end{aligned}$$

by reducing the problem to a one dimensional Green function equation in the z coordinate. Recall that

$$\int_0^R [J_m(\frac{\alpha_{mp}}{R} r)]^2 \; r\,dr = \frac{R^2}{2}[J_{m+1}(\alpha_{mp})]^2$$

where α_{mp} is the p^{th} zero of the Bessel function of index m.

(b) If the region described in (a) contains no volume or surface charge density and if

$$\begin{aligned} \phi - \phi_\infty &= 0 \text{ at } r = R \text{ and} \\ E_z &= E_0 \text{ at } z = 0 \end{aligned}$$

where E_0 is a constant, find the electrostatic potential in the cylindrical region. Recall that

$$\frac{d}{d\tau}(\tau J_1(\tau)) = \tau J_0(\tau)$$

and carry out as many integrations and sums as feasible.

3. A monochromatic plane wave is incident from the left and is normal to an opaque screen having two identical, narrow slits separated by distance d. For radiation diffracted at an angle β and observed at a large distance to the right of the screen, derive the necessary condition for maximum constructive interference.

4. An incident photon of wavelength λ is scattered at an angle θ in the laboratory from an electron initially at rest in the laboratory. Calculate the change in wavelength $\Delta\lambda$ between the incident and the scattered photon, the Compton effect.

5. (a) Let the vectors Z_n, $1 \le n \le N$ defined over the real numbers be a set of linearly independent vectors in an N dimensional vector space and let C_n, $2 \le n \le N$, be a set of real numbers. Show that vectors $Z_n - C_n Z_1$, $2 \le n \le N$, are also linearly independent.

   (b) Use the procedure of Schmidt orthogonalization to convert the functions

   $$1,\ x,\ \text{and } x^2$$

   to an orthonormal set on the interval $-1 \le x \le 1$ with weight function 1.

6. Give a schematic drawing for a single stage, inverting, DC coupled amplifier with purely resistive feedback using an operational amplifier which drives a single low-pass RC filter on the output. Calculate the voltage gain as a function of frequency at the output of the filter. If R = 10 kΩ in the RC filter, what C is needed to have a 3 dB frequency of 1 kHz?

Part I: Section B

Do any one (1) of the three (3) problems in section B numbered 7 through 9 inclusive.

7. (a) A spinless particle in a box $a \times b \times c$ obeys the time independent Schroedinger equation with periodic boundary conditions on the walls. Find the density of states per unit volume in terms of a volume element in $\vec{k}$ (wave vector) space.

   (b) Modify the density of states found in (a) for an electron with spin $\frac{1}{2}$.

   (c) If the system is isotropic, give the density of states per unit volume for an electron in terms of an increment of energy.

   (d) Since the Pauli exclusion principle allows only one electron to occupy a state, the lowest available states are filled at $T = 0$. Find the maximum energy an electron can have at this temperature which is the Fermi energy at $T = 0$.

   (e) Find the average energy of an electron at $T = 0$ in terms of the Fermi energy.

8. (a) The doubly excited state $(2p^2)\ ^3P$ for helium has an excitation energy well above the single ionization energy of 24.6 eV, but does not autoionize. Autoionization would typically occur in about $10^{-14}s$. It prefers to decay in two steps first to $(1s2p)\ ^3P + \gamma_1$ and then to $(1s^2)\ ^1S + \gamma_2$ which are typically much slower processes. If the wavelength $\lambda_1 = 32.0nm$ and the wavelength $\lambda_2 = 59.1nm$, calculate the total excitation energy of the $(2p^2)\ ^3P$ state and compare this energy to the single ionization energy.

   (b) Calculate the parity $(-1)^{\sum_i \ell_i}$ for all multielectron states mentioned in (a).

   (c) Demonstrate why the single photon transition $(2p^2)\ ^3P \rightarrow (1s^2)\ ^1S + \gamma_3$ is forbidden.

9. (a) The total angular momentum, parities, and energies of the lowest few excited states of many nuclei whose neutron and proton numbers are both far from "magic numbers" are well described by rotational and vibrational collective motion. For a

certain nucleus with $A = 180$ and $Z = 72$, ${}^{180}Hf$, the ground state and lowest four (4) excited states are known to form a pure rotational band with no vibration. Give the total angular momentum and parity for each of these five (5) states. Why?

(b) The first excited state is 93.3 keV above the ground state. If the energy spectrum for rotational levels is similar to that for some states of a rigid rotor, give the lowest order formula for these levels and calculate the energies predicted for the other three (3) excited states.

(c) How and why would you expect measured energies to deviate from these predictions? How would you "correct" the formula?

Mathematical Relations

$$\int_0^\infty e^{-ax^2} x^{2n} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{\Gamma(n+\frac{1}{2})}{a^n\Gamma(\frac{1}{2})},\ (a>0;\ n \text{ an integer} \geq 0)$$

$$\int_0^\infty e^{-ax} x^n dx = n!/a^{n+1},\ (a>0;\ n \text{ an integer } \geq 0)$$

$$n! \approx \sqrt{2\pi} \quad n^{n+\frac{1}{2}} e^{-n},\ for \quad n >>>> 1$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

$$\int e^{ax} \sin(bx)\ dx = (a^2+b^2)^{-1}\ e^{ax}(a\ \sin(bx) - b\ \cos(bx))$$

$$\int x\ \sin^2(ax)\ dx = \frac{x^2}{4} - \frac{x\ \sin(2ax)}{4a} - \frac{cos(2ax)}{8a^2}$$

$$\int \sin^3 x\ dx = -\frac{1}{3}\cos(x)(\sin^2 x + 2)$$

$$\int \sin^4 x\ dx = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32}$$

$$\int_0^\infty e^{-ax^2} \cos(bx) \quad dx = \frac{1}{2}(\pi/a)^{\frac{1}{2}} e^{-b^2/4a}$$

Cylindrical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z}$$

$$\nabla \times \vec{V} = \hat{r}(\frac{1}{r}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}) + \hat{\theta}(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}) + \hat{z}\frac{1}{r}(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial \theta})$$

$$\nabla \Phi = \hat{r}\ \frac{\partial \Phi}{\partial r} + \hat{\theta}\ \frac{1}{r}\ \frac{\partial \Phi}{\partial \theta} + \ \hat{z}\ \frac{\partial \Phi}{\partial z}$$

$$\nabla^2 \Phi = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

Spherical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial\phi}$$

$$\nabla \times \vec{V} = \hat{r}\frac{1}{r\sin\theta}(\frac{\partial}{\partial\theta}(\sin\theta V_\phi) - \frac{\partial V_\theta}{\partial\phi}) + \hat{\theta}(\frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rV_\phi)) + \hat{\phi}\frac{1}{r}(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial\theta})$$

$$\nabla\Phi = \hat{r}\frac{\partial\Phi}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial\Phi}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial\Phi}{\partial\phi}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\Phi}{\partial r}) + \frac{1}{r^2}(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\Phi}{\partial\theta}) + \frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2})$$

The Dirac matrices are given by

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

where σ is one of the Pauli spin matrices and I is the identity matrix.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

PHYSICAL CONSTANTS

$$1 \ \mathring{A} = 10^{-8}\text{cm}$$

$$1 \text{ fermi } = 10^{-13}\text{cm}$$

$$1 \text{ parsec } = 3.087 \text{ x } 10^{18}\text{cm}$$

$$G = 6.67 \text{ x } 10^{-11}\text{nt} - \text{m}^2/\text{kg}^2$$

$$k_B = 1.38 \text{ x } 10^{-23} \text{ joule/K } = 1.38 \text{ x } 10^{-16} \text{ erg/K}$$

$$N_A = 6.02 \text{ x } 10^{23} \text{ particles/mole}$$

$$e = 1.60 \text{ x } 10^{-19} \text{ coulomb}$$

$$c = 2.998 \text{ x } 10^{10}\text{cm/sec}$$

$$\sigma = 5.67 \text{ x } 10^{-8} \text{ joule/(deg}^4 - \text{m}^2 - \text{sec)}$$

$$1eV = 1.60 \text{ x } 10^{-19} \text{ joule}$$

$$h = 6.626 \text{ x } 10^{-34} \text{ joule} - \text{sec } = 6.626 \ 10^{-27}\text{erg} - \text{sec}$$

$$\hbar c = 197MeV - \text{fm} = 197 \ eV - \text{nm}$$

$$m_e = 0.511MeV/c^2 = 9.11 \text{ x } 10^{-31}\text{kg}$$

$$m_\pi = 139.6MeV/c^2$$

$$m_\mu = 105.7MeV/c^2$$

$$m_{proton} = 938MeV/c^2 = 1.67 \text{ x } 10^{-24}\text{gm}$$

$$\tau(\pi) = 2.60 \text{ x } 10^{-8}\text{sec}$$

ASTRONOMY

University of Wyoming
Department of Physics & Astronomy
Written Preliminary Examination - Part II
For ASTRONOMY students.
Saturday, January 27, 1996
9:00 am - 2:00 pm

Instructions for Part II

1. Do six (6) problems as instructed on the following pages.

2. Answer each problem on a separate, numbered set of papers.

3. Enter the identification number assigned to you on the upper right hand corner of each page. Do not write your name on these pages.

4. Some integrals, numerical constants, and mathematical expressions which might be useful are provided on the data pages at the end of the examination.

5. Each problem will contribute equally to your grade on this part of the examination.

6. Partial credit will be given for orderly progress towards a correct solution of a problem; however no credit will be given for an illogical, disorganized presentation or for irrelevant material.

Part II : Section A

Do each of the four (4) problems in Section A numbered 1 through 4 inclusive.

1. Prove the following theorem. If a complex function of a complex variable $f(z)$ is analytic in a simply connected domain D except at z_0 and if for every real $M > M_0$ a real $\delta > 0$ exists such that

$$|f(z)| > M \qquad \text{for } 0 < |z - z_0| < \delta$$

then $f(z)$ may be expressed as

$$f(z) = \frac{g(z)}{(z - z_0)^m}$$

for some integer $m > 0$ where $g(z)$ is analytic in some domain about z_0 and $g(z_0) \neq 0$. You may assume (1) a theorem about the zeros of an analytic function and (2) a theorem about removable singularities.

2. (a) A classical projectile of mass m moves in the $x - y$ plane under the influence of a uniform gravitational field, $-mg\hat{y}$. Initially at time $t = 0$, it is located at $x = y = 0$ and has a velocity $\dot{x} = \dot{y} = v_0$. Solve for the trajectory using Newton's equations of motion.

   (b) Find (1) the Lagrangian, (2) the canonical momenta, and (3) the Hamiltonian for the system. Solve for the trajectory using Hamilton's equations and compare the result to that in (a).

   (c) Find the Hamilton-Jacobi equation for the system, solve it using the separation of variables, obtain the trajectory, and compare the result to those of (a) and (b).

3. The Fourier transform by time of the vector potential in the Lorentz gauge for free space due to a current density in this space may be written as an integral of the transform by time of the current density times the scalar Green function for the Helmholtz operator in closed form. When approximated for very large distances from the current and retaining only terms in $\frac{1}{r}$, the approximate transform of the vector potential is given by

$$\vec{A}(\vec{r}) = \frac{1}{c}\frac{e^{ikr}}{r}\int_v e^{-i\vec{k}\cdot\vec{r}\,'}\,\vec{J}(\vec{r}\,')d\tau'$$

where $\vec{k} = k\hat{r}$. To the same accuracy, the transforms by time of the electric and magnetic fields and the flux of spectral density of energy radiated are given by

$$\vec{\mathcal{E}} = -ik\,(\hat{r} \times (\hat{r} \times \vec{\mathcal{A}}))$$
$$\vec{\mathcal{B}} = ik\,(\hat{r} \times \vec{\mathcal{A}})$$
$$\vec{\mathcal{S}} = \frac{c}{(2\pi)^2}\,k^2\,\hat{r}\,|\hat{r} \times \vec{\mathcal{A}}|^2$$

(a) A circular loop of current with radius R and a sinusoidal variation in time lies in the $x-y$ plane with its center at the origin. Its Fourier transform by time is given in spherical coordinates by

$$\vec{\mathcal{J}}(\vec{r}) = \frac{I}{R}\,\delta(r-R)\,\delta(\theta - \frac{\pi}{2})\,\hat{\phi}\,\{\pi(\delta(\omega-\Omega)e^{i\gamma} + \delta(\omega+\Omega)\,e^{-i\gamma})\}$$

where Ω is the frequency of the variation and γ a real phase shift. Calculate the approximation to the transform of the vector potential given above. Perform all integrals.

(b) From the result of (a), calculate transforms of the electric and magnetic fields and the spectral density of the flux of average power.

4. (a) Find the time independent Schroedinger equation for a quantum mechanical rigid rotor constrained to rotate about the z axis with a moment of inertia Θ.

(b) Solve the equation in (a) to find the eigenvalues of energy and their associated eigenfunctions. What degeneracies occur?

(c) The behavior of this rotor is perturbed by an electric dipole $\vec{P}$ in the x-y plane coupled to an electric field $\vec{E}$ in the x direction which introduces a potential

$$V = -\vec{P} \cdot \vec{E}$$

Find the shifts in energy caused by this potential to second order in perturbation theory. Why do the degeneracies not create a difficulty?

Part II : Section B (Astronomy)

Do any two (2) of the three (3) problems in Section B numbered 5 through 7 inclusive.

5. The observed photon flux rate from a star and the surrounding sky is 100 photons/second. A nearby piece of sky, assumed to be identical to that surrounding the star and hence to be satisfactory for subtracting the sky background contamination, has an observed photon flux rate of only 10 photons/second with the same instrumental field and integration time. If the only source of error is the statistical fluctuation in the photon flux, derive the integration time required to achieve a signal-to-noise ratio S/N = 100 in the sky corrected star flux.

6. If a spherical star is in hydrostatic equilibrium, is composed of an ideal gas, and has a temperature distribution making the density uniform, derive the relationships for core pressure and temperature in terms of the total mass and the radius of the star (that is, eliminate dependence on density).

7. Define the emergent flux $F_{\lambda,emg}$ from a star in terms of the specific intensity I_λ. Find the emergent flux for a black body radiator $F_{\lambda,emg}$ in the terms of the Planck function B_λ. What is the observed flux $F_{\lambda,obs}$ from a black body in terms of its emergent flux if it subtends a solid angle Ω as seen by the observer?

Mathematical Relations

$$\int_0^\infty e^{-ax^2} x^{2n} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \frac{\Gamma(n+\frac{1}{2})}{a^n \Gamma(\frac{1}{2})}, \; (a > 0; \; n \text{ an integer} \geq 0)$$

$$\int_0^\infty e^{-ax} x^n dx = n!/a^{n+1}, \; (a > 0; \; n \text{ an integer } \geq 0)$$

$$n! \approx \sqrt{2\pi} \quad n^{n+\frac{1}{2}} e^{-n}, \; for \quad n >>>> 1$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

$$\int e^{ax} \sin(bx) \; dx = (a^2 + b^2)^{-1} \; e^{ax}(a \; \sin(bx) - b \; \cos(bx))$$

$$\int x \; \sin^2(ax) \; dx = \frac{x^2}{4} - \frac{x \; \sin(2ax)}{4a} - \frac{cos(2ax)}{8a^2}$$

$$\int \sin^3 x \; dx = -\frac{1}{3} \cos(x)(\sin^2 x + 2)$$

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Cylindrical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z}$$

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$$\nabla \Phi = \hat{r} \; \frac{\partial \Phi}{\partial r} + \hat{\theta} \; \frac{1}{r} \; \frac{\partial \Phi}{\partial \theta} + \; \hat{z} \; \frac{\partial \Phi}{\partial z}$$

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Spherical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial\phi}$$

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$$\nabla\Phi \;=\; \hat{r}\,\frac{\partial\Phi}{\partial r} \;+\hat{\theta}\,\frac{1}{r}\,\frac{\partial\Phi}{\partial\theta} \;+\; \hat{\phi}\,\frac{1}{r\sin\theta}\,\frac{\partial\Phi}{\partial\phi}$$

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The Dirac matrices are given by

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

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PHYSICAL CONSTANTS

$$1\ \mathring{A} = 10^{-8}\text{cm}$$

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PHYSICS

University of Wyoming
Department of Physics & Astronomy
Written Preliminary Examination - Part II
For PHYSICS students.
Saturday, January 27, 1996
9:00 am - 2:00 pm

Instructions for Part II

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Part II : Section A

Do each of the four (4) problems in Section A numbered 1 through 4 inclusive.

1. Prove the following theorem. If a complex function of a complex variable $f(z)$ is analytic in a simply connected domain D except at z_0 and if for every real $M > M_0$ a real $\delta > 0$ exists such that

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then $f(z)$ may be expressed as

$$f(z) = \frac{g(z)}{(z - z_0)^m}$$

for some integer $m > 0$ where $g(z)$ is analytic in some domain about z_0 and $g(z_0) \neq 0$. You may assume (1) a theorem about the zeros of an analytic function and (2) a theorem about removable singularities.

2. (a) A classical projectile of mass m moves in the $x - y$ plane under the influence of a uniform gravitational field, $-mg\hat{y}$. Initially at time $t = 0$, it is located at $x = y = 0$ and has a velocity $\dot{x} = \dot{y} = v_0$. Solve for the trajectory using Newton's equations of motion.

   (b) Find (1) the Lagrangian, (2) the canonical momenta, and (3) the Hamiltonian for the system. Solve for the trajectory using Hamilton's equations and compare the result to that in (a).

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3. The Fourier transform by time of the vector potential in the Lorentz gauge for free space due to a current density in this space may be written as an integral of the transform by time of the current density times the scalar Green function for the Helmholtz operator in closed form. When approximated for very large distances from the current and retaining only terms in $\frac{1}{r}$, the approximate transform of the vector potential is given by

$$\vec{A}(\vec{r}) = \frac{1}{c}\frac{e^{ikr}}{r}\int_v e^{-i\vec{k}\cdot\vec{r}\,'}\,\vec{J}(\vec{r}\,')d\tau'$$

where $\vec{k} = k\hat{r}$. To the same accuracy, the transforms by time of the electric and magnetic fields and the flux of spectral density of energy radiated are given by

$$\begin{aligned}\vec{\mathcal{E}} &= -\,ik\,(\hat{r}\times(\hat{r}\times\vec{\mathcal{A}}))\\ \vec{\mathcal{B}} &= ik\,(\hat{r}\times\vec{\mathcal{A}})\\ \vec{\mathcal{S}} &= \frac{c}{(2\pi)^2}\,k^2\;\hat{r}\,|\hat{r}\times\vec{\mathcal{A}}|^2\end{aligned}$$

(a) A circular loop of current with radius R and a sinusoidal variation in time lies in the $x-y$ plane with its center at the origin. Its Fourier transform by time is given in spherical coordinates by

$$\vec{\mathcal{J}}(\vec{r}) = \frac{I}{R}\,\delta(r-R)\,\delta(\theta-\frac{\pi}{2})\,\hat{\phi}\,\{\pi(\delta(\omega-\Omega)e^{i\gamma}+\delta(\omega+\Omega)\,e^{-i\gamma})\}$$

where Ω is the frequency of the variation and γ a real phase shift. Calculate the approximation to the transform of the vector potential given above. Perform all integrals.

(b) From the result of (a), calculate transforms of the electric and magnetic fields and the spectral density of the flux of average power.

4. (a) Find the time independent Schroedinger equation for a quantum mechanical rigid rotor constrained to rotate about the z axis with a moment of inertia Θ.

(b) Solve the equation in (a) to find the eigenvalues of energy and their associated eigenfunctions. What degeneracies occur?

(c) The behavior of this rotor is perturbed by an electric dipole $\vec{P}$ in the x-y plane coupled to an electric field $\vec{E}$ in the x direction which introduces a potential

$$V = -\vec{P}\cdot\vec{E}$$

Find the shifts in energy caused by this potential to second order in perturbation theory. Why do the degeneracies not create a difficulty?

Part II: Section B (Physics)

Do any two (2) of the three (3) problems in Section B numbered 5 through 7 inclusive.

5. (a) A solid contains N mutually noninteracting nuclei having spin 1. They do however interact with an internal electric field in the solid through their quadrupole moments, so that nuclei with $m = 1$ or -1 have the same interaction energy $\mathcal{E} > 0$ while those with $m = 0$ have zero interaction energy. <u>Derive</u> an expression for the partition function.

   (b) Using the partition function from (a), <u>find</u> (1) the Helmholtz free energy, (2) the entropy, and (3) the internal energy all as a function of temperature.

6. (a) A quantum mechanical particle scattered by a potential has a total Hamiltonian $H = H_0 + V$ where H_0 is the Hamiltonian of the "free" particle and V is the scattering potential. The state $|\phi_n>$ is an eigenstate of the "free" particle Hamiltonian $H_0|\phi_n>= E_n|\phi_n>$. The state $|\Psi_n^{(+)}>$ is an eigenstate of the total Hamiltonian $H|\Psi_n^{(+)}> = E_n|\Psi_n^{(+)}>$ which describes a scattering from the initial state $|\phi_n>$ by the potential. It therefore satisfies the Lippman-Schwinger integral equation

$$|\Psi_n^{(+)}> \;=\; |\phi_n> \;+\; G_0^{(+)}(E_n)V|\Psi_n^{(+)}>$$

   where $G_0^{(+)}(E)$ is the noninteracting Green's operator for scattering at energy E,

$$G_0^{(+)}(E) \;=\; [E - H_0 \;+\; i\epsilon]^{-1}.$$

   An appropriate quantum mechanical solution is found in the limit $\epsilon \to 0$. If the T operator is defined by the equation

$$T(E_n)|\phi_n> \;=\; V|\Psi_n^{(+)}>,$$

   <u>derive</u> an equation satisfied by $T(E_n)|\phi_n>$ and <u>abstract</u> from this an operator equation for $T(E)$.

   (b) If the interacting Green's operator $G^{(+)}(E)$ is given by

$$G^{(+)}(E) \;=\; [E - H + i\epsilon]^{-1},$$

   <u>derive</u> an operator equation for $T(E)$ in terms of $G^{(+)}$ implied by the result of (a).

Mathematical Relations

$$\int_0^\infty e^{-ax^2}x^{2n}dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{\Gamma(n+\frac{1}{2})}{a^n\Gamma(\frac{1}{2})},\ (a>0;\ n \text{ an integer} \geq 0)$$

$$\int_0^\infty e^{-ax}x^n dx = n!/a^{n+1},\ (a>0;\ n \text{ an integer } \geq 0)$$

$$n! \approx \sqrt{2\pi}\quad n^{n+\frac{1}{2}}e^{-n},\ for \quad n >>>> 1$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

$$\int e^{ax}\sin(bx)\ dx = (a^2+b^2)^{-1}\ e^{ax}(a\ \sin(bx) - b\ \cos(bx))$$

$$\int x\ \sin^2(ax)\ dx = \frac{x^2}{4} - \frac{x\ \sin(2ax)}{4a} - \frac{cos(2ax)}{8a^2}$$

$$\int \sin^3 x\ dx = -\frac{1}{3}\cos(x)(\sin^2 x + 2)$$

$$\int \sin^4 x\ dx = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32}$$

$$\int_0^\infty e^{-ax^2}\cos(bx)\quad dx = \frac{1}{2}(\pi/a)^{\frac{1}{2}}e^{-b^2/4a}$$

Cylindrical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z}$$

$$\nabla \times \vec{V} = \hat{r}(\frac{1}{r}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}) + \hat{\theta}(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}) + \hat{z}\frac{1}{r}(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial \theta})$$

$$\nabla \Phi = \hat{r}\ \frac{\partial \Phi}{\partial r} + \hat{\theta}\ \frac{1}{r}\ \frac{\partial \Phi}{\partial \theta} + \ \hat{z}\ \frac{\partial \Phi}{\partial z}$$

$$\nabla^2 \Phi = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

Spherical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial\phi}$$

$$\nabla \times \vec{V} = \hat{r}\frac{1}{r\sin\theta}(\frac{\partial}{\partial\theta}(\sin\theta V_\phi) - \frac{\partial V_\theta}{\partial\phi}) + \hat{\theta}(\frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rV_\phi)) + \hat{\phi}\frac{1}{r}(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial\theta})$$

$$\nabla\Phi \;=\; \hat{r}\;\frac{\partial\Phi}{\partial r} \;+\hat{\theta}\;\frac{1}{r}\;\frac{\partial\Phi}{\partial\theta} \;+\; \hat{\phi}\;\frac{1}{r\sin\theta}\;\frac{\partial\Phi}{\partial\phi}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\Phi}{\partial r}) + \frac{1}{r^2}(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\;\theta\frac{\partial\Phi}{\partial\theta}) + \frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2})$$

The Dirac matrices are given by

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

where σ is one of the Pauli spin matrices and I is the identity matrix.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

PHYSICAL CONSTANTS

$$1 \ \mathring{A} = 10^{-8}\text{cm}$$

$$1 \text{ fermi} = 10^{-13}\text{cm}$$

$$1 \text{ parsec} = 3.087 \times 10^{18}\text{cm}$$

$$G = 6.67 \times 10^{-11}\text{nt} - \text{m}^2/\text{kg}^2$$

$$k_B = 1.38 \times 10^{-23} \text{ joule/K} = 1.38 \times 10^{-16} \text{ erg/K}$$

$$N_A = 6.02 \times 10^{23} \text{ particles/mole}$$

$$e = 1.60 \times 10^{-19} \text{ coulomb}$$

$$c = 2.998 \times 10^{10}\text{cm/sec}$$

$$\sigma = 5.67 \times 10^{-8} \text{ joule/(deg}^4 - \text{m}^2 - \text{sec)}$$

$$1eV = 1.60 \times 10^{-19} \text{ joule}$$

$$h = 6.626 \times 10^{-34} \text{ joule} - \text{sec} = 6.626 \ 10^{-27}\text{erg} - \text{sec}$$

$$\hbar c = 197 MeV - \text{fm} = 197 \ eV - \text{nm}$$

$$m_e = 0.511 MeV/c^2 = 9.11 \times 10^{-31}\text{kg}$$

$$m_\pi = 139.6 MeV/c^2$$

$$m_\mu = 105.7 MeV/c^2$$

$$m_{proton} = 938 MeV/c^2 = 1.67 \times 10^{-24}\text{gm}$$

$$\tau(\pi) = 2.60 \times 10^{-8}\text{sec}$$

University of Wyoming
Department of Physics & Astronomy
Written Preliminary Examination - Part I. Friday, January 27, 1995
9:00 am - 2:00 pm

Instructions for Part I

1. Do seven (7) problems as instructed on the following pages.
2. Answer each problem on a separate set of papers.
3. Enter the identification number assigned to you on the upper right hand corner of each answer page. Do not write your name on these pages.
4. Some integrals, numerical constants, and mathematical expressions which might be useful are provided on the data pages at the end of the examination.
5. Each problem will contribute equally to your grade on this part of the examination.
6. Partial credit will be given for orderly progress towards a correct solution of a problem; however no credit will be given for an illogical, disorganized presentation or for irrelevant material.

Part I : Section A

Do each of the five (5) problems in Section A, numbers 1 through 5 inclusive.

1. (a) If the sun were surrounded by a uniformly dense, concentric, spherical "cloud" of massive particles and if a planet were moving about the sun within this sphere, then the gravitational force due to the "cloud" would add to the usual gravitational force between the sun and the planet. Show that this background "cloud" of particles would produce a force of the form

$$\vec{F} = -b\,\vec{r}$$

where $\vec{r}$ is the position vector from the sun and center of the "cloud" to the planet.

(b) If the motion of the sun is neglected, then the force on the planet will be given by

$$\vec{F} = -(\frac{k}{r^2} + b\,r)\,\hat{r}$$

where $\frac{k}{r^2} \gg b\,r$ everywhere over the orbit. Find the time required for the perihelion of an almost circular orbit to precess 360° about the sun by considering the period of radial oscillations in a rotating frame. Express this period in terms of k, b, and r_0, the radius of the orbit.

2. The region between two long, coaxial cylindrical shells of conductor with radii r_1 and r_2, where $r_1 < r_2$, is filled with a linear, homogenous, isotropic dielectric with permitivity ϵ. If the shells are maintained at potentials ϕ_1 and ϕ_2 respectively, find the electric field and displacement field, and from these calculate the energy per unit length stored in the system. Also note that the energy per unit length stored may be expressed as $\frac{1}{2}C(\phi_1 - \phi_2)^2$ and identify C.

3. (a) A quantum mechanical plane rotor is a system of particles rigidly connected and free to rotate only about a fixed axis with moment of inertia I. Write the time independent Schroedinger equation for this system and solve it to find the eigenvalues and normalized eigenfunctions of energy under the constraint that an eigenfunction and its derivative must be continuous (periodicity).

(b) At $t = 0$, this rotor has the wavefunction $\psi(\phi, 0) = \frac{2}{\sqrt{3\pi}} \sin^2\phi$. Using the time dependent Schroedinger equations for the system and the results of (a) above, find the wave function at later times t.

4. (a) A monochromatic, plane electromagnetic wave is obliquely incident on a plane interface between vacuum and a semi - infinite, homogeneous, isotropic dielectric with index of refraction n and may be represented by the real part of the complex expression

$$\vec{E}_{inc} = \vec{E}_{0inc} \quad e^{i(\vec{k}_{inc}\cdot\vec{r}-\omega t)}$$

where $\vec{E}_{0inc}$ is constant and $\vec{E}_{0inc} \cdot \vec{k}_{inc} = 0$

One may attempt to satisfy boundary conditions at this interface by assuming plane, reflected and transmitted waves which are represented by the real parts of

$$\vec{E}_{ref} = \vec{E}_{0ref} \; e^{i(\vec{k}_{ref}\cdot\vec{r}-\omega t)}$$

$$\vec{E}_{trans} = \vec{E}_{0trans} \; e^{i(\vec{k}_{trans}\cdot\vec{r}-\omega t)}$$

and which are also transverse. Prove that $\vec{k}_{inc}$, $\vec{k}_{ref}$, and $\vec{k}_{trans}$ must lie in a common plane for the boundary conditions to be met over the entire plane of discontinuity.

(b) If the polarization of the electric field is in the plane of incidence and reflection, show how the angle of incidence for maximum transmission is related to the index of refraction n

(c) If the semi-infinite dielectric is replaced by a metal and if the incident plane wave is polarized 45° with respect to the plane of incidence and reflection, give qualitatively the type of the polarization of the reflected wave with simple arguments but no derivation.

5. In a "molecular sieve" pump, nitrogen gas (N_2 molecules) is exposed to a cold surface onto which N_2 molecules can be adsorbed. A simple model of the system assumes that the surface provides n sites which can bind molecules and that work ϕ is required to liberate a molecule from a site into the gaseous phase. If the system has a temperature T, has a volume V, and contains m moles of gas, calculate the number of N_2 molecules adsorbed on the surface as a function of the above parameters and fundamental constants such as k and $\hbar$.

Hint: One approach to this problem is calculating the partition function for an extremely dilute gas and its subsequent chemical potential, calculating the partition function for molecules bound to the surface and their chemical potential, and imposing requirements for equilibrium. However any correct solution is acceptable.

Part I : Section B

Do any two (2) of the five (5) problems in Section B, numbers 6 through 10 inclusive.

6. (a) Consider a quantum mechanical treatment of electrons in a solid. Give the definition of the Fermi level for the distribution of electrons at finite temperature in terms of the probability for a state being occupied.

   (b) Calculate the density of states as a function of energy for electrons in the conduction band of a metal.

   (c) Where is the Fermi level for an intrinsic semiconductor? (You may answer with an appropriate energy level diagram.)

   (d) Derive an expression for the density of carriers with energy $\mathcal{E}$ in the conduction band of an intrinsic semiconductor.

7. A chart of known nuclear isotopes plotted as Z versus A displays a region of stability with a very definite pattern. Also the binding energies of stable nuclei as a function of Z follow roughly an orderly trend. The semiempirical mass formula originally suggested by von Weizsäcker explains these rough regularities. Give the form of this formula as it appears today and explain the meaning of each of the seven (7) terms.

8. (a) Using one resistor, one capacitor, and one ideal operational amplifier, give (1) a schematic drawing in which these form an integrator and (2) a schematic drawing in which these form a differentiator. Ignore overall reversals of sign.

   (b) The overly simple circuit found in (a) will be unstable because of extremely high, unfedback gain at certain frequencies in each case. Eliminate this problem in each case without overly compromising the performance desired by adding a second resistor to each schematic.

   (c) Give practical values for R_1, R_2, and C for each of the two schematic drawings in (b) which would allow this circuit to perform adequately from 1 Hz to 1000 Hz.

   (d) Compute the complex voltage gain $G(\omega)$ for each of the two schematic drawings in (b).

9. A charged pi meson with mass M is at rest and decays to a muon with mass m_1, and a neutrino with mass m_2. Note that the neutrino mass m_2 may or may not be zero. Find expressions for the kinetic energies of the muon and the neutrino in terms of these three masses.

Hint: Manipulating the energy - momentum four vectors solves this problem quickly. However any correct solution is acceptable.

10. (a) The energy levels of the outermost atomic electron in a sodium atom ($Na, Z = 11$) are split by an external magnetic field due to the Zeeman coupling,

$$H_{Zeeman} = -\mu_B(\vec{L} + g_S\vec{S}) \cdot \vec{B},$$

where μ_B is the Bohr magnetron and g_S is the g-factor for the electron. Derive the first order, weak field splitting of these levels $\Delta E = -g_J\, \mu_B\, B$ in the Russel Saunders (LS) approximation using the vector model and express the Lande´ g - factor in terms of L, S, and J. Assume that $g_S = 2$.

(b) Calculate the Lande´ g-factor for the ground state of Na and for the $5^2p_{\frac{1}{2}}$ state.

(c) Draw an energy level diagram indicating the splittings found in (b) and the possible transitions between the $5^2p_{\frac{1}{2}}$ state and the ground state and indicate which yield σ and which yield π radiations.

Mathematical Relations

$$\int_0^\infty e^{-ax^2} x^{2n} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\frac{\Gamma(n+\frac{1}{2})}{a^n\Gamma(\frac{1}{2})},\ (a>0;\ n \text{ an integer} \geq 0)$$

$$\int_0^\infty e^{-ax} x^n dx = n!/a^{n+1},\ (a>0;\ n \text{ an integer } \geq 0)$$

$$n! \approx \sqrt{2\pi} \quad n^{n+\frac{1}{2}} e^{-n},\ for \quad n >>>> 1$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

$$\int e^{ax}\sin(bx)\ dx = (a^2+b^2)^{-1}\ e^{ax}(a\ \sin(bx) - b\ \cos(bx))$$

$$\int x\ \sin^2(ax)\ dx = \frac{x^2}{4} - \frac{x\ \sin(2ax)}{4a} - \frac{cos(2ax)}{8a^2}$$

$$\int \sin^3 x\ dx = -\frac{1}{3}\cos(x)(\sin^2 x + 2)$$

$$\int \sin^4 x\ dx = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32}$$

$$\int_0^\infty e^{-ax^2}\cos(bx)\ \ dx = \frac{1}{2}(\pi/a)^{\frac{1}{2}} e^{-b^2/4a}$$

Cylindrical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z}$$

$$\nabla \times \vec{V} = \hat{r}(\frac{1}{r}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}) + \hat{\theta}(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}) + \hat{z}\frac{1}{r}(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial \theta})$$

$$\nabla \Phi = \hat{r}\ \frac{\partial \Phi}{\partial r} + \hat{\theta}\ \frac{1}{r}\ \frac{\partial \Phi}{\partial \theta} + \ \hat{z}\ \frac{\partial \Phi}{\partial z}$$

$$\nabla^2 \Phi = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

Spherical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial \phi}$$

$$\nabla \times \vec{V} = \hat{r}\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta V_\phi) - \frac{\partial V_\theta}{\partial \phi}\right) + \hat{\theta}\left(\frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rV_\phi)\right) + \hat{\phi}\frac{1}{r}\left(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial \theta}\right)$$

$$\nabla\Phi = \hat{r}\,\frac{\partial \Phi}{\partial r} + \hat{\theta}\,\frac{1}{r}\,\frac{\partial\Phi}{\partial\theta} + \hat{\phi}\,\frac{1}{r\sin\theta}\,\frac{\partial\Phi}{\partial\phi}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\ \theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}\right)$$

The Dirac matrices are given by

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

where σ is one of the Pauli spin matrices and I is the identity matrix.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad .$$

ASTRONOMY

University of Wyoming
Department of Physics & Astronomy
Written Preliminary Examination - Part II.
For ASTRONOMY students.
Saturday, January 28, 1995
9:00 am - 2:00 pm

Instructions for Part II

1. Do six (6) problems as instructed on the following pages.

2. Answer each problem on a separate set of papers.

3. Enter the identification number assigned to you on the upper right hand corner of each answer page. Do not write your name on these pages.

4. Some integrals, numerical constants, and mathematical expressions which might be useful are provided on the data pages at the end of the examination.

5. Each problem will contribute equally to your grade on this part of the examination.

6. Partial credit will be given for orderly progress towards a correct solution of a problem; however no credit will be given for an illogical, disorganized presentation or for irrelevant material.

Part II : Section A

Do each of the four (4) problems in Section A, numbered 1 through 4 inclusive.

1. Using contour integration, evaluate the definite Riemann integral

$$\int_0^\infty \frac{\sin x}{x} \, dx.$$

Carry out all parts of the calculation explicitly and rigorously.

2. (a) The one dimensional motion of a classical particle is governed by the Lagrangian

$$L = \frac{1}{2} m\dot{x}^2 + m\,A\,x\,t$$

where x is the position of the particle, t is the time, and $\dot{x} = \frac{dx}{dt}$ is the velocity. Introduce L into Lagrange's equations of motion and solve the resulting differential equation for the initial conditions $x = 0$ and $\dot{x} = v_0$ at $t = 0$.

(b) Derive the canonical momentum for this system and obtain the Hamiltonian. Show that applying Hamilton's equations of motion will lead to the same differential equation found in (a).

(c) Introduce Hamilton's principal function into the Hamiltonian found in (b) and hence obtain the Hamilton - Jacobi equation. Solve this equation subject to the same initial conditions as used in (a).

Hint: You might use the solution for p from (a) to guess an ansatz for Hamilton's principal function.

3. (a) Derive an expansion in spherical coordinates for the Green function for the Laplace - Poisson operator in the space outside a sphere of radius R, which obeys boundary conditions

$$\hat{n} \cdot \nabla G(\vec{r}, \vec{r}\,') = 0, \; r = R, \text{ and}$$
$$\lim_{\vec{r} \to \infty} G(\vec{r}, \vec{r}\,') = 0$$

by reducing the problem to a one dimensional Green function equation in the r coordinate.

(b) If the electrostatic potential at infinity is zero, if the space outside R contains no charge, and if

$$E_r = E_0 \, \frac{1}{2} \, (3 \cos^2 \theta - 1), \text{ at } r = R,$$

then use the expansion of the Green function found in (a) to calculate the electrostatic potential outside R.

4. (a) A quantum mechanical system with a Hamiltonian H is in a state ψ which obeys the time dependent Schroedinger equation. Derive an equation for the total time derivative of the expectation value of an operator A on this system when it is in the state ψ.

(b) If the system consists of a single particle in a potential, then the Hamiltonian is given by

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}).$$

For this single particle system, evaluate the equation found in (a) for the operator

$$A = \vec{p} \cdot \vec{r}$$

in terms of the expectation value for kinetic energy and an expectation value involving the potential as well as other quantitives.

(c) If the potential in (b) is the Coulomb potential which is central and if the system is in an eigenstate of the Hamiltonian, use the equation found in (b) to derive a relation between the expectation values of the kinetic energy and the potential energy. This relation is called the virial theorem.

Part II : Section B (Astronomy)

Do any two (2) of the three (3) problems in Section B, numbers 5 through 7 inclusive.

5. Suppose that the principal contribution to the noise of a detector is from the flux of photons from a source whose fluctuation may be assumed to be governed by Poission statistics. The NEP for the detector is defined as

$$NEP = \frac{P}{(S/N)\sqrt{\Delta f}}$$

where P = incident power in energy/time, S/N = signal - to - noise ratio, and Δf = measurement bandwidth. Derive an expression for NEP in terms of the flux of photons ϕ in photons / (area·time) and the area of the detector A in which the dependence of Δf has been eliminated. You may ignore considerations of spectral bandwidth and quantum efficiency of the detector.

6. Among the most important direct observations of stellar processes are the measurements of solar neutrino flux with two very different types of detectors. In the longest running experiment, neutrinos have been observed with chlorine based detectors which find only $\frac{1}{4}$ the flux originally expected. More recently neutrinos have been observed with gallium based detectors which find a flux closer to that expected but still somewhat low in the preliminary analysis. Discuss the implications of these experiments for both astronomy and particle physics. In particular, what are possible explanations for the chlorine based results, if the gallium based results are ignored? What are the further implications if the gallium based results are confirmed?

7. An isothermal, homogenous, spherical cloud of radius R is composed of N_H atoms of atomic hydrogen. If the temperature of the cloud T and the distance to the cloud D are known, if the cloud is optically thin, and if the total flux in a specific emission line from atomic hydrogen is observed, derive an expression for N_H in terms of these quantities, fundamental physical constants, and physical constants specific to hydrogen and the energy levels involved. You may also assume that no absorption intervenes between the cloud and the earth, that the gas is in LTE, and that ionization may be neglected.

   Hint: You may consider that, in an optically thin cloud, every transition is seen directly by the observer, and may avoid using a solution to the radiative transfer equation.

Mathematical Relations

$$\int_0^\infty e^{-ax^2} x^{2n} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}} \frac{\Gamma(n+\frac{1}{2})}{a^n \Gamma(\frac{1}{2})}, \; (a > 0; \; n \text{ an integer} \geq 0)$$

$$\int_0^\infty e^{-ax} x^n dx = n!/a^{n+1}, \; (a > 0; \; n \text{ an integer } \geq 0)$$

$$n! \approx \sqrt{2\pi} \quad n^{n+\frac{1}{2}} e^{-n}, \; for \quad n >>>> 1$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

$$\int e^{ax} \sin(bx) \; dx = (a^2+b^2)^{-1} \; e^{ax}(a \; \sin(bx) - b \; \cos(bx))$$

$$\int x \; \sin^2(ax) \; dx = \frac{x^2}{4} - \frac{x \; \sin(2ax)}{4a} - \frac{cos(2ax)}{8a^2}$$

$$\int \sin^3 x \; dx = -\frac{1}{3}\cos(x)(\sin^2 x + 2)$$

$$\int \sin^4 x \; dx = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32}$$

$$\int_0^\infty e^{-ax^2} \cos(bx) \quad dx = \frac{1}{2}(\pi/a)^{\frac{1}{2}} e^{-b^2/4a}$$

Cylindrical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}(rV_r) + \frac{1}{r}\frac{\partial V_\theta}{\partial \theta} + \frac{\partial V_z}{\partial z}$$

$$\nabla \times \vec{V} = \hat{r}(\frac{1}{r}\frac{\partial V_z}{\partial \theta} - \frac{\partial V_\theta}{\partial z}) + \hat{\theta}(\frac{\partial V_r}{\partial z} - \frac{\partial V_z}{\partial r}) + \hat{z}\frac{1}{r}(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial \theta})$$

$$\nabla \Phi = \hat{r} \; \frac{\partial \Phi}{\partial r} + \hat{\theta} \; \frac{1}{r} \; \frac{\partial \Phi}{\partial \theta} + \; \hat{z} \; \frac{\partial \Phi}{\partial z}$$

$$\nabla^2 \Phi = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \Phi}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2}$$

Spherical Coordinates

$$\nabla \cdot \vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta V_\theta) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial\phi}$$

$$\nabla \times \vec{V} = \hat{r}\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\sin\theta V_\phi) - \frac{\partial V_\theta}{\partial\phi}\right) + \hat{\theta}\left(\frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rV_\phi)\right) + \hat{\phi}\frac{1}{r}\left(\frac{\partial}{\partial r}(rV_\theta) - \frac{\partial V_r}{\partial\theta}\right)$$

$$\nabla\Phi = \hat{r}\,\frac{\partial\Phi}{\partial r} + \hat{\theta}\,\frac{1}{r}\,\frac{\partial\Phi}{\partial\theta} + \hat{\phi}\,\frac{1}{r\sin\theta}\,\frac{\partial\Phi}{\partial\phi}$$

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}\right)$$

The Dirac matrices are given by

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

where σ is one of the Pauli spin matrices and I is the identity matrix.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$