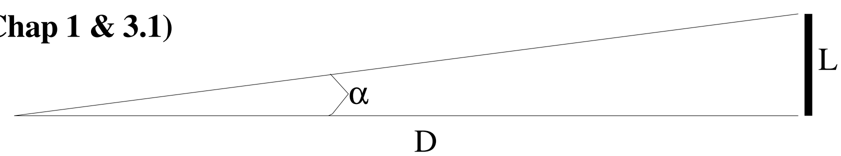
Distances and Units in Astrophysics

Basic Units (commit to memory; astronomers tend to use cgs units)

- solar mass: $2\text{x}10^{33}$ g
- solar luminosity: $4\ \text{x}10^{33}$ erg/s $\qquad 10^{7}$ erg = 1 Joule $\quad 10^{7}$ erg/s = 1 W
- solar radius: $7\text{x}10^{10}$ cm $\qquad$ c: $3\text{x}10^{10}$ cm/s
- earth radius: $6\text{x}10^{8}$ cm $\qquad$ h: $6.62\text{x}10^{-27}$ erg s
- 1 A.U. = $1.5\ \text{x}10^{13}$ cm $\qquad$ k: $1.38\text{x}10^{-16}$ erg /K
- 1 parsec = 3.2 ly = $3\text{x}10^{18}$ cm

Angular size-distance relation (C+O Chap 1 & 3.1)

$$\tan\alpha=\frac{L}{D}=\frac{\sin\alpha}{\cos\alpha}$$



If α is small, then Tan α ≅ α

$$\tan\alpha=\alpha(radians)=\frac{L}{D}\ \rightarrow\ L=D\alpha$$

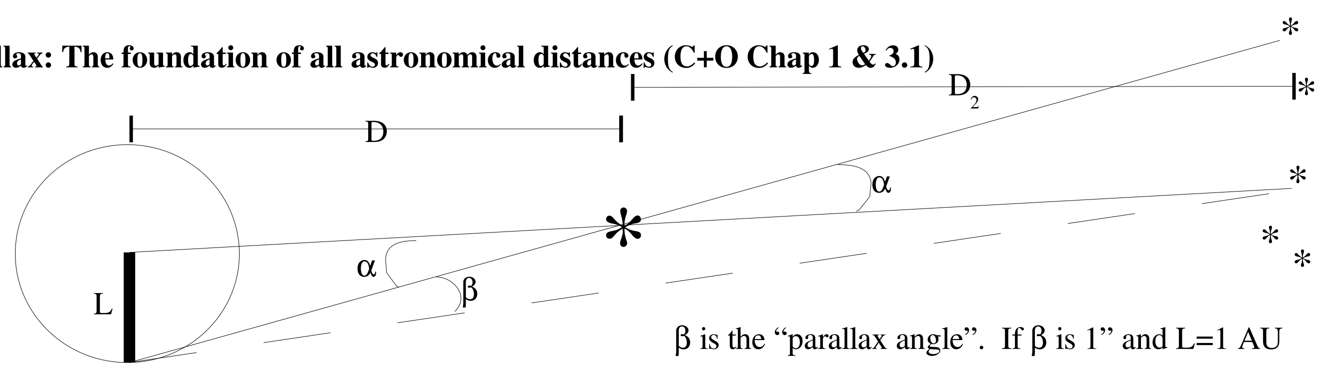
Or, given that there are 206265 arcseconds in a radian

$$\boxed{L=D\frac{\alpha('')}{206562}}$$

Commit to memory some typical distances.

- Nearest star: few pc
- Center of the Milky Way: 8 kpc
- Diameter of a spiral galaxy: 30 kpc
- Magellanic Clouds: 50 kpc
- M31 (Andromeda Galaxy): 0.3 Mpc
- Virgo Cluster: 20 Mpc

Parallax: The foundation of all astronomical distances (C+O Chap 1 & 3.1)



β is the "parallax angle". If β is 1" and L=1 AU

$$If\ D\ll D_2\ \rightarrow\alpha=\beta$$

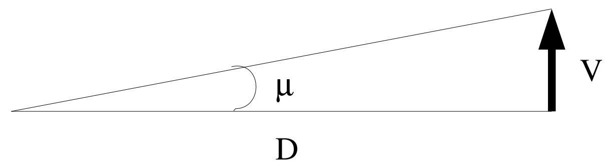
$$D=\frac{1.5\text{x}10^{13}\,cm}{\frac{1''}{206265''/rad}}=3.08\text{x}10^{18}\,cm=1\ parsec\,(1\ parallax\ second)$$

$$\beta=\frac{L}{D}\qquad D=\frac{L}{\beta}$$

Proper Motions(C+O Chap 1)

An object with a true transverse space velocity, V, located at a distance D will appear to move across the sky at a rate, μ, the *proper motion*.

$$\mu(radians\, s^{-1}) = \frac{V(cm\, s^{-1})}{D(cm)}$$



Or in more useful form:

$$\mu(arcsec\, yr^{-1}) = \frac{V(km\, s^{-1})}{D(pc)} 3.33\text{x}10^{-14}\, pc\, km^{-1} \quad x\, \pi\, x 10^{7} s\, yr^{-1} \quad x \quad 206265\, arcsec\, rad^{-1}$$

$$\mu(arcsec\, yr^{-1}) = 0.21 \frac{V(km\, s^{-1})}{D(pc)}$$

Spectroscopic Parallax and Standard Candles, the next rung on the distance ladder. (C&O 3.2)

Once the true luminosity of any type of *standard candle* is established though parallax or another means, the inverse square law of radiation.

Let L be the *luminosity* of a star in erg/s.

Let F be the observed *flux* of the star at some distance, D

$$F(erg\,s^{-1}cm^{-2}) = \frac{L(erg\,s^{-1})}{4\pi D^2(cm^2)}$$

E.g., compute the flux of the sun on earth.

$$F(erg\,s^{-1}cm^{-2}) = \frac{2\text{x}10^{33}(erg\,s^{-1})}{4\pi[1.5\text{x}10^{13}(cm)]^2} = 7.1\text{x}10^5 erg\,s^{-1}cm^{-2}$$

E.g., compute the distance of a G2V star (solar type) if it has a flux of $2\text{x}10^{-7}$ erg/s

$$D(cm) = \sqrt{\frac{L(erg\,s^{-1})}{4\pi F(erg\,s^{-1}cm^{-2})}} = \sqrt{\frac{4\text{x}10^{33}}{4\pi 2\text{x}10^{-7}}} = 4\text{x}10^{19} cm \simeq 10\,pc$$

Certain objects are standard candles with well known luminosities

Cepheid variables: Absolute V magnitude M_V = -3 to -5

RR Lyra variables: Absolute V magnitude M_V = 0 to -2

Brightest blue supergiants: M_V = -10

Type Ia Supernovae: M_V = -20

Recall, absolute magnitude means the magnitude a star would have at a distance of 10 pc. The difference between a star's *apparent magnitude* (m) and *absolute magnitude* (M) is the *distance modulus*.

$$DM = m - M = 5\log(D\frac{(pc)}{10\,pc})$$

E.g., A distant Cepheid variable has an apparent magnitude of m_V = 21. Its distance is

$$D(pc) = 10\,x\,10^{(m-M)/5} = 10*10^{(25/5)} = 10^6\,pc = 1\,Mpc\,(slightly\ outside\ our\ local\ group)$$

Each magnitude represents a factor of 2.5 in relative brightness with smaller numbers being brighter.

$$\Delta m = m_1 - m_2 = -2.5\log(\frac{L_1}{L_2})$$

5 magnitudes is a factor of 100, so that a 5^{th} magnitude star is 100 times fainter than a 0^{th} magnitude star. Another way to say this is that magnitudes are logarithms of the flux. *The difference in magnitudes is the same as a ratio of two fluxes.* A 1 mag difference is a flux ratio of 2.5, etc.

Converting between magnitudes and real flux units**.**

The (antiquated) magnitude scale astronomers (still) use is based on a comparison to the star Vega, an A0V star, defined to be 0^{th} magnitude at all wavelengths.

($1 eV = 1.6 \times 10^{-12}$ erg)

($1 Jy = 10^{-26} W/m^2/Hz = 10^{-23}$ erg s^{-1} cm^{-2} Hz^{-1})

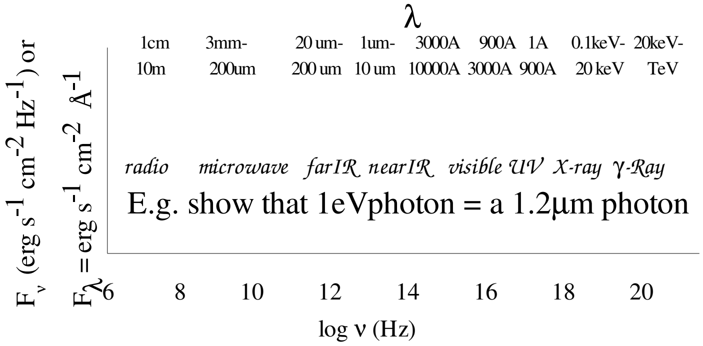
Flux or Flux density: F_ν = erg s^{-1} cm^{-2} Hz^{-1}

or F_λ = erg s^{-1} cm^{-2} Å^{-1}

To convert between F_ν units and F_λ units use

F_λ (erg s^{-1} cm^{-2} Å^{-1}) = 3×10^{-13} x F_ν (Jy)/ λ^2 (µm)

E.g., find the energy in eV of a 912 Å photon.
Does this number seem familiar? Why?

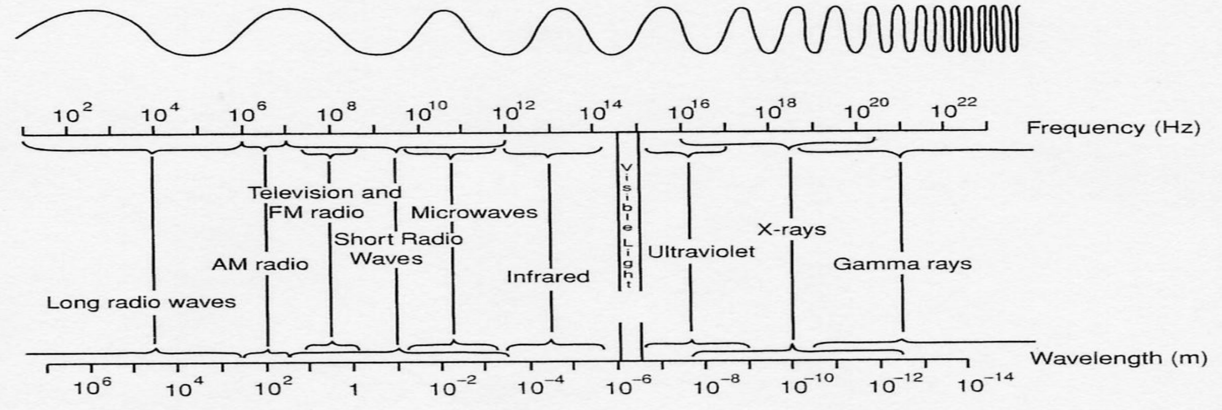


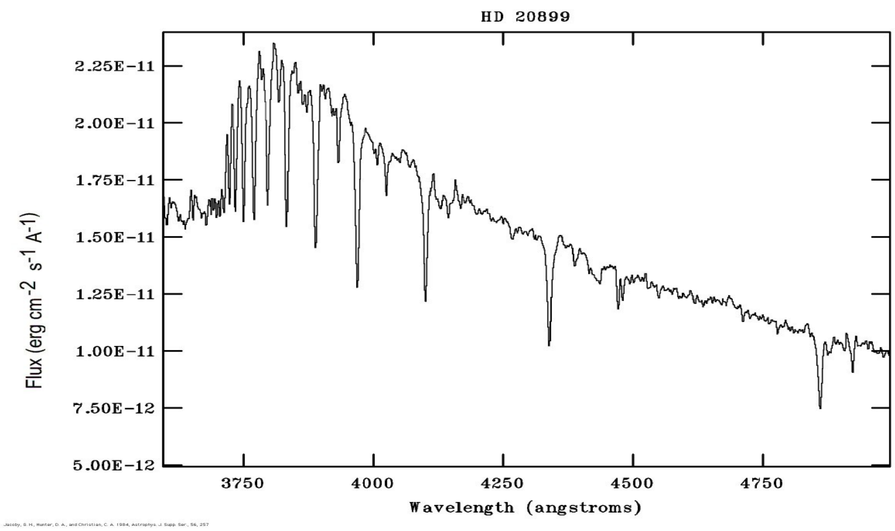
To convert between F_ν or F_λ you need to realize that 1 Angstrom worth of spectrum contain a different number of Hz, depending on what part of the EM spectrum you are in.

$$c = \nu\lambda \qquad \nu = \frac{c}{\lambda} \qquad \rightarrow \qquad \frac{\delta\nu}{\delta\lambda} = \frac{-c}{\lambda^2}$$

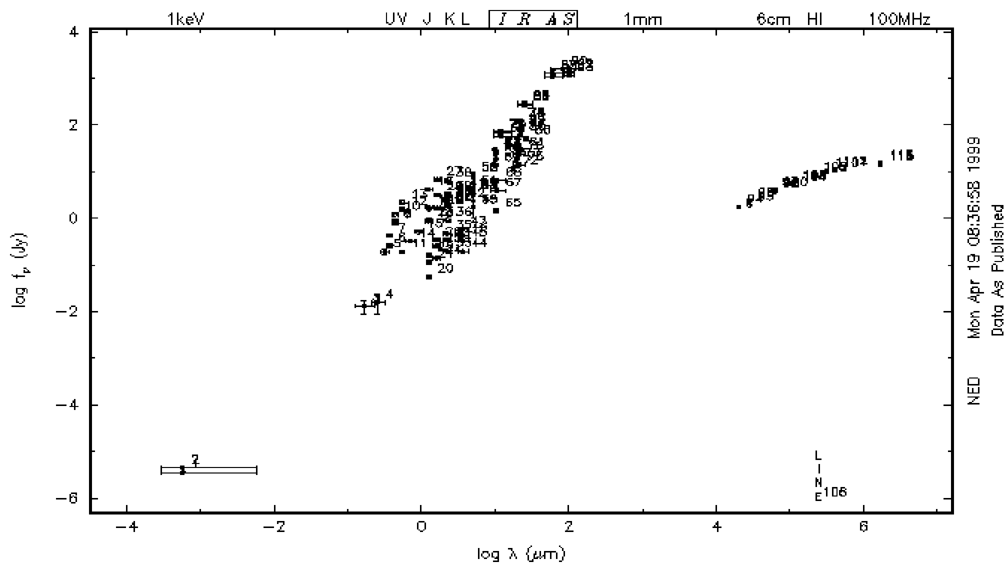
E.g., A source has a flux of 3×10^{-14} erg s^{-1} cm^{-2} Å^{-1} at 5000 Å. How many erg s^{-1} cm^{-2} Hz^{-1} is this?
Therefore there are $\delta\lambda/\delta\nu = -\lambda^2 / c$ = -(5000 Å)^2 / (3x18 Å/s) = -8.33×10^{-12} Å/Hz
(the negative sign just means that as wavelength increases, frequency decreases!) which results in
3×10^{-14} erg s^{-1} cm^{-2} Å^{-1} x 8.33×10^{-12} Å/Hz = 2.5×10^{-25} erg s^{-1} cm^{-2} Hz^{-1} .

Region of spectrum	Units
Gamma rays	MeV, GeV
X-ray (hard and soft)	keV
Ultraviolet, optical	Å, nm
Infrared (near-IR, IR, far-IR)	microns (µm), mm, cm^{-1}
Millimeter, microwave	mm
Radio	cm, m, MHz, GHz

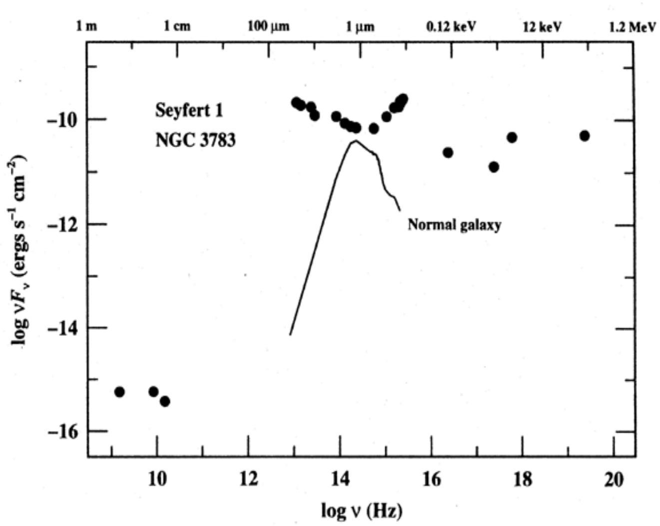




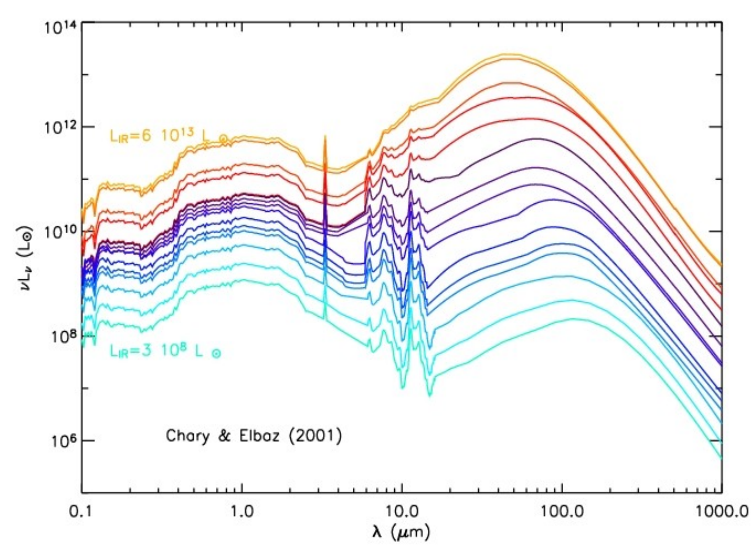
Example of a star's spectral energy distribution (SED) using λ and f_λ units.



Example of a spectral energy distribution using mixed λ and f_ν units.



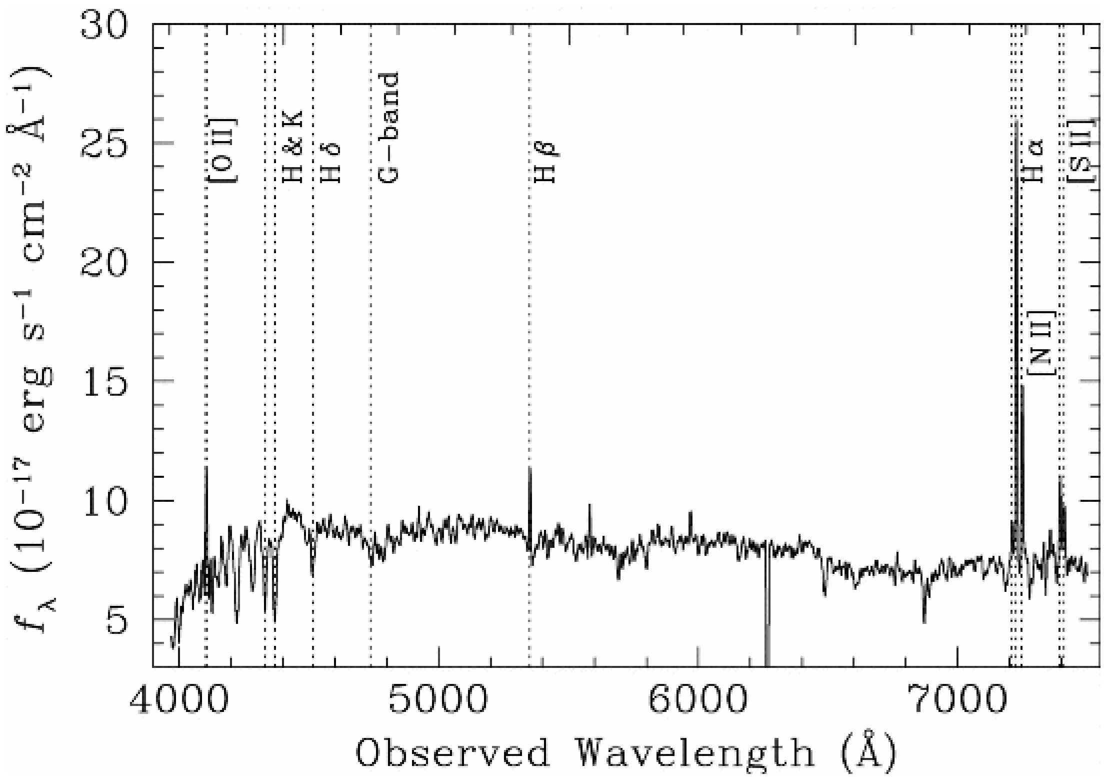
Example of a galaxy Spectral Energy Distribution (SED) plot using ν and νF_ν units.



Example of a stellar spectral energy distribution using mixed λ and νL_ν units.

Below is a typical spectrum of a spiral galaxy in the optical portion of the electromagnetic spectrum.

Note in particular the bright emission line from Hydrogen α (the Balmer α transition) which comes from ionized Hydrogen in nearly all types of galaxies. This line is bright and can make the galaxy visible even from very far away. It can also be used to estimate the distance of the galaxy using the Hubble Flow and the Tully-Fisher methods of distance determination, described below.

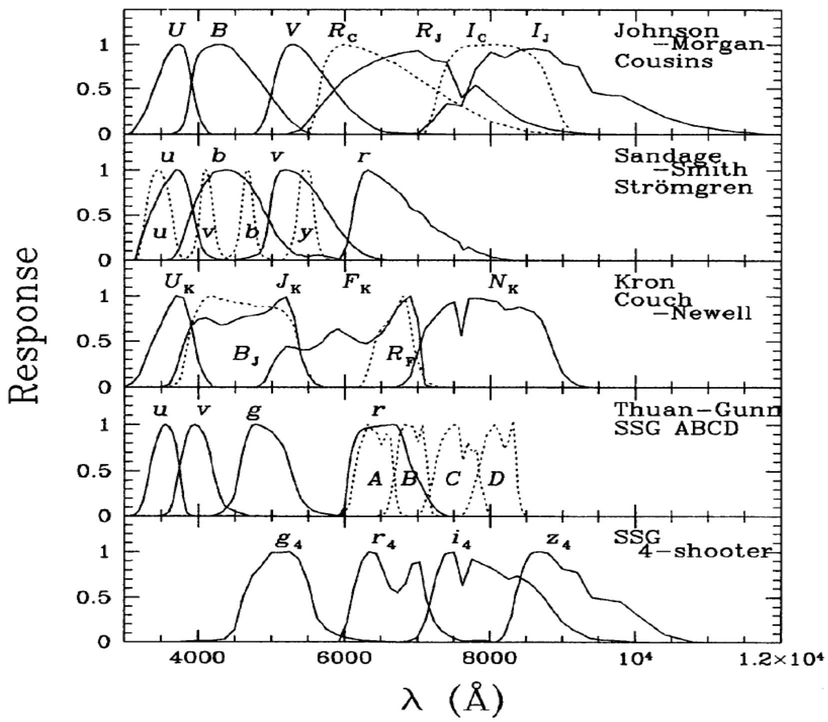


To convert from a magnitude to flux units such as erg s^{-1} cm^{-2} $Angstrom^{-1}$ or erg s^{-1} cm^{-2} Hz^{-1} you need a table of zero points for a given wavelength or band of observation. (see the Hubble Space Telescope NICMOS calibration manual).

To convert between magnitudes and flux units use

$F_\nu = F_0 \times 10^{-m/2.5}$ or $F_\nu = F_0 \times 2.5^{-m}$

Band	λ (μm)	F_0 (Jy)
U	0.36	1880
B	0.44	4440
V	0.56	3540
R	0.70	2870
I	0.90	2250
J	1.25	1670
H	1.65	980
K	2.2	620
L	3.4	280
IRAC1	[3.6]	280.9
IRAC2	[4.5]	179.7
IRAC3	[5.8]	115.0
IRAC4	[8.0]	64.1
MIPS24	[24]	



where F_0 is the *zero point flux* for that bandpass. The Table below shows common astronomical bandpasses. E.g., the sun has an apparent V magnitude of -26. Find its flux in F_ν units and F_λ units at V band.

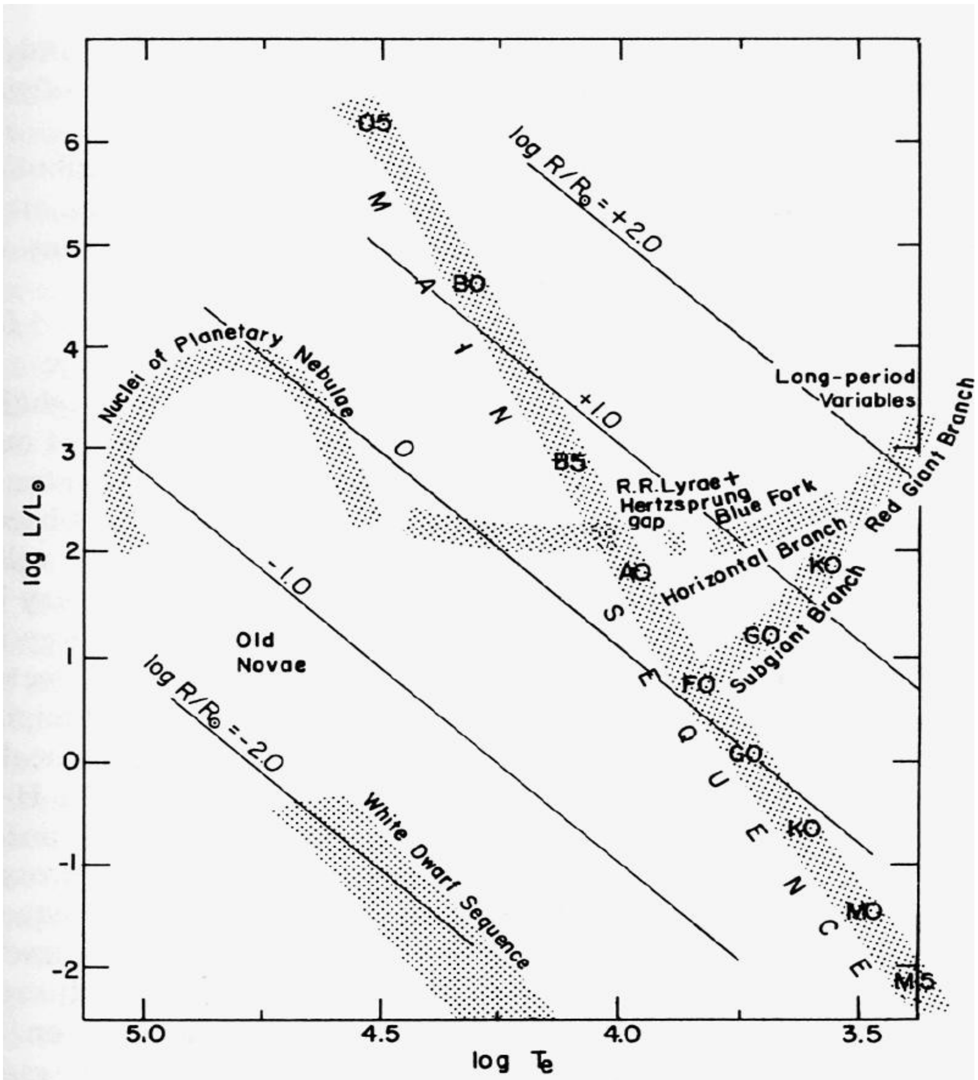
$F_\nu = F_0 \times 10^{-m/2.5} = 3540 \times 10^{-(-26/2.5)} = 8.9 \times 10^{13}$ Jy

$= 8.9 \times 10^{13}$ Jy x 10^{-23} erg s^{-1} cm^{-2} Hz^{-1} Jy^{-1} = 8.8×10^{-10} erg s^{-1} cm^{-2} Hz^{-1}

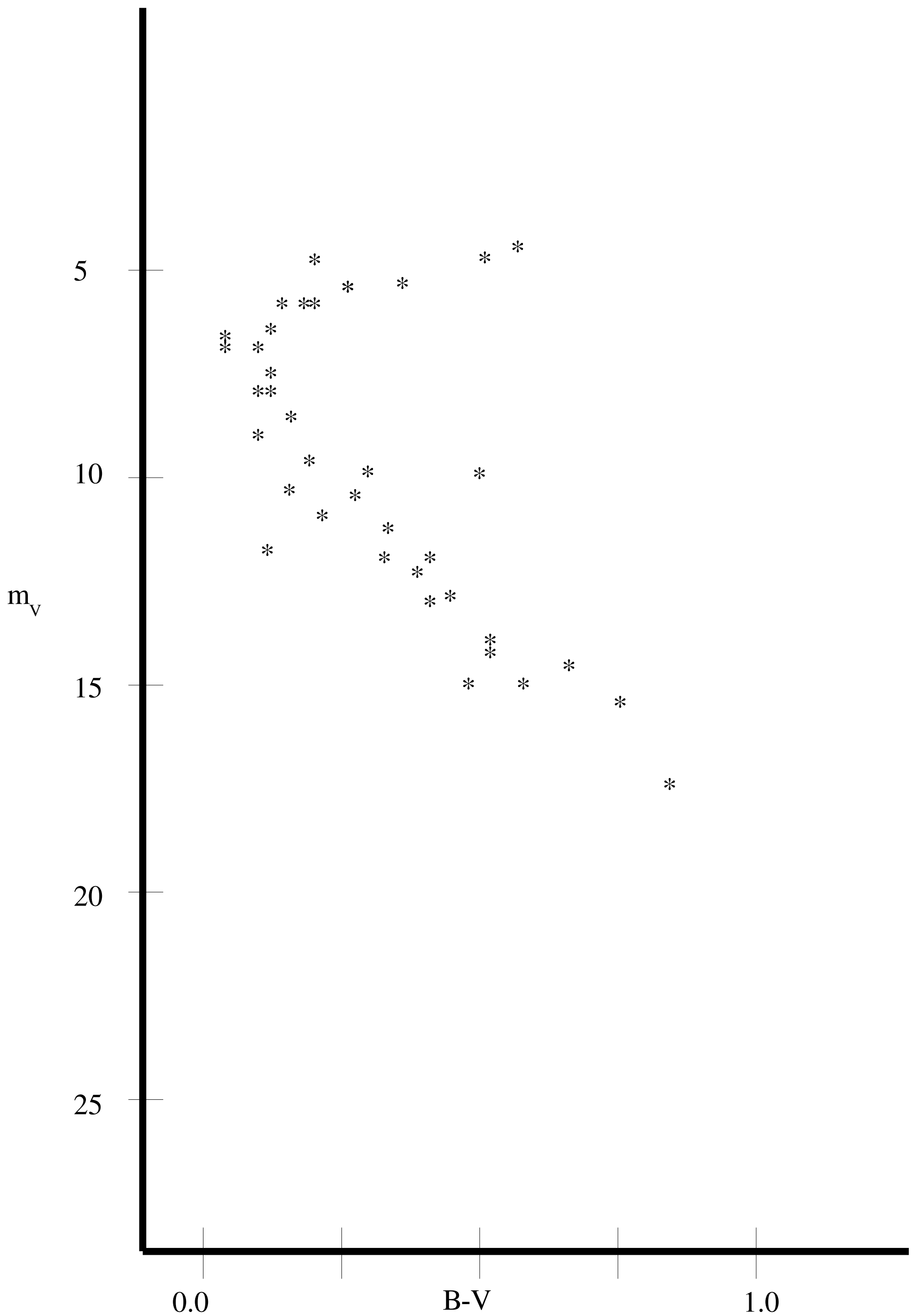
F_λ (erg s^{-1} cm^{-2} $Å^{-1}$)= $3 \times 10^{-13} \times F_\nu$ (Jy)/ λ^2 (μm) = $3 \times 10^{-13} \times 8.9 \times 10^{13}$ (Jy)/ $(0.56)^2$ (μm) =85 erg s^{-1} cm^{-2} $Å^{-1}$

Main Sequence Fitting

One representation of the HR Diagram (Harwitt, Figure 1.5; C&O Chap 8.2, Chap 13; Ryden & Peterson Chap 14, 17). Note the lines of constant radius. What can you say about the progression of radius along the main sequence? Temperature?



Once the structure of the *main sequence* in an *HR diagram* is known, an observed cluster at arbitrary distance can be compared to the "ideal" main sequence by shifting the observed cluster along the vertical axis (the magnitude axis) until the morphology of the clusters match. The difference between the observed magnitudes, m_V, of the observed cluster and the absolute magnitudes, M_V, of the ideal main sequence gives the *Distance Modulus*. This is illustrated below.

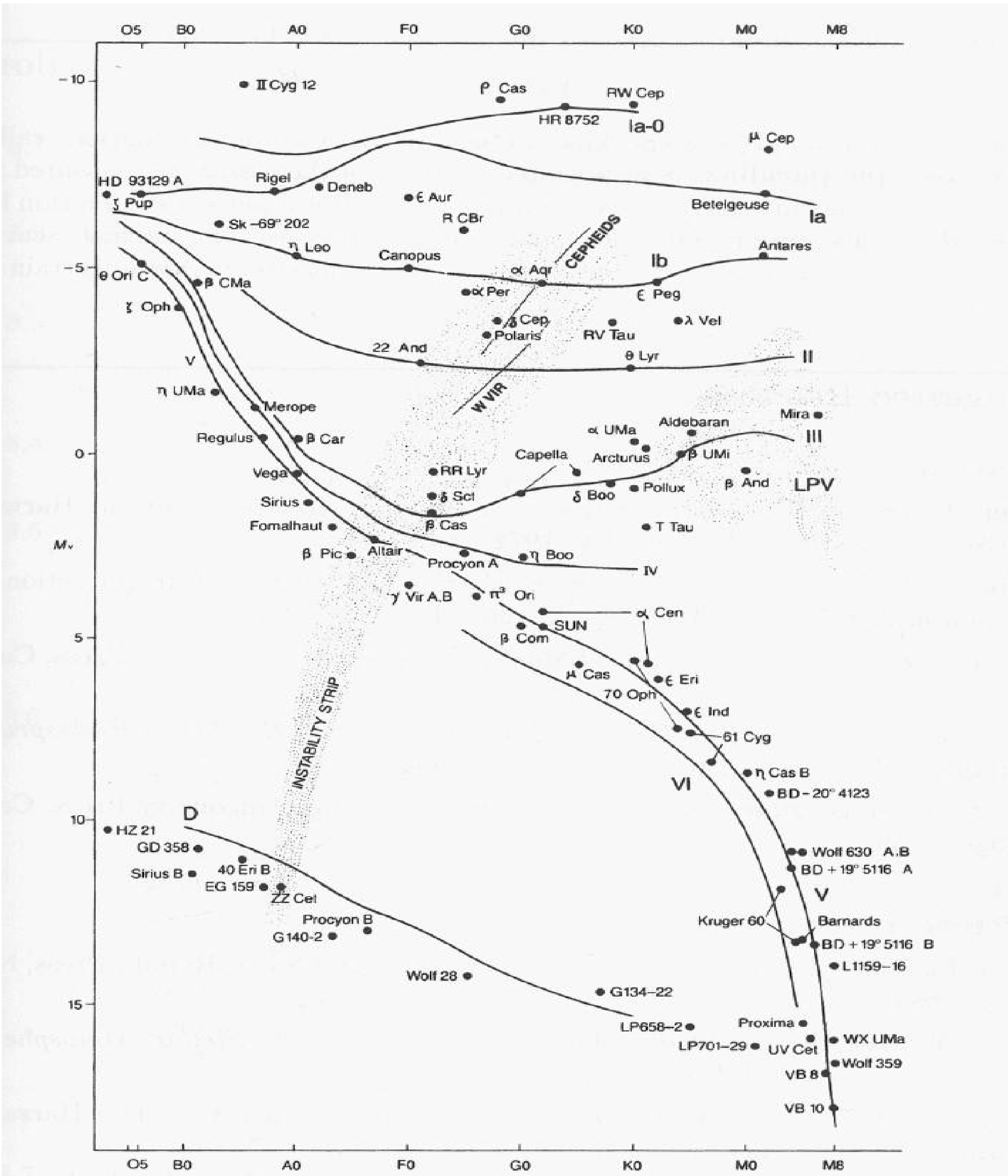


5
10
15
20
25
m_V
0.0
B-V
1.0

Carroll&Ostlie
Fig. 8.15

Or
$\log(L/L_\odot)$

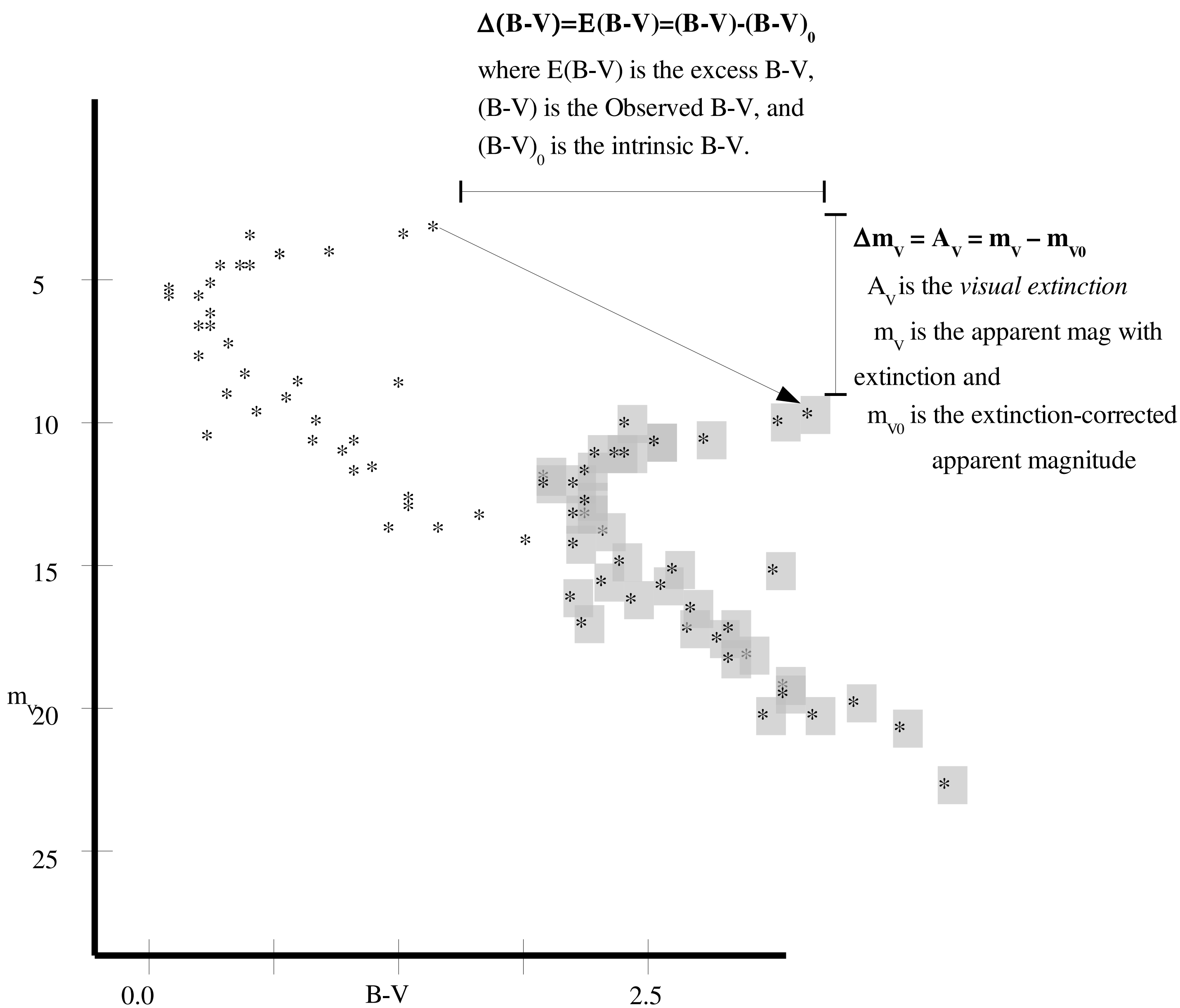
6
5
4
3
2
1
0
-1
-2
-3



Also							
40,000		10,000		6600	4000	3000	Temp(K)
50	10	4	2	1	0.5	0.3	Mass (Msun)
-0.5		0.0		1.0	2.0	3.0	(B-V) color index

Reddening (C+O 12.1; Ryden & Peterson 16.1)

One problem with photometry of clusters of any object is the effect of *interstellar dust* which obscures objects. Dust both 1) extinguishes the light, that is, makes the apparent magnitude fainter than it would otherwise be, and it 2) reddens the objects, affecting the blue (B) magnitudes more than the red (R) magnitudes. This effect is illustrated below.



The slope of the extinction vector is R, and it relates the amount of extinction to the amount of reddening.

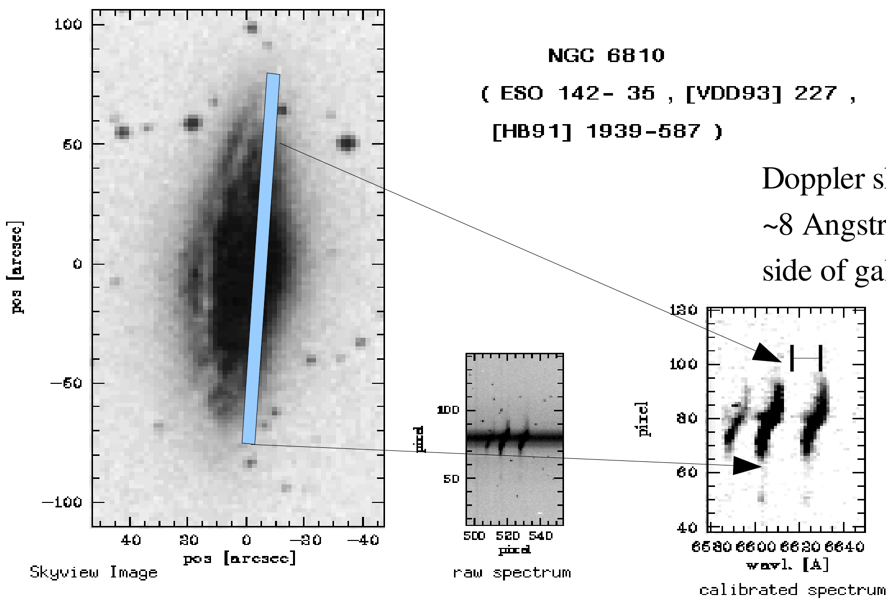
A_V = R x E(B-V)

For most places in the Milky Way, the value of R is about 3.1. The absolute magnitude is found from

$M_V = m_V - A_V - D.M.$

The Tully-Fisher Relation (C+O Chap 25; Ryden & Peterson Chap 20)

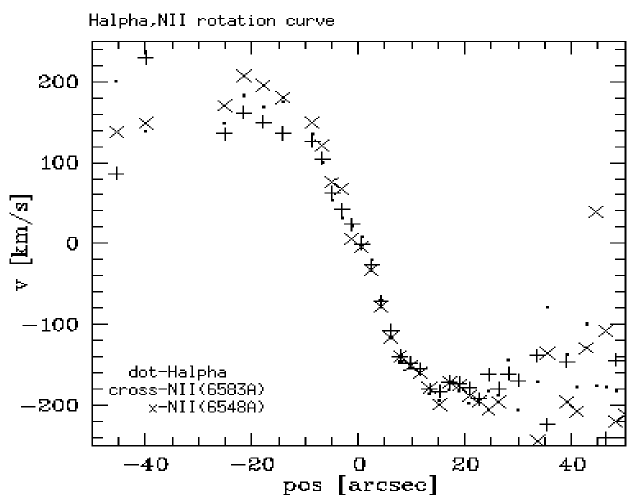
The *Tully-Fisher relation* is a correlation between the rotational velocity of a spiral galaxy and its luminosity (i.e., absolute magnitude). Once the T-F relation is calibrated in nearby galaxies (using distances obtained from Cepheid variables, for instance) then the relation can be used to infer the luminosities of more distant galaxies since it is relatively easy to measure rotational velocities from spectra of the ionized gas.



Doppler shift of Hα line is ~8 Angstroms from one side of galaxy to other

Non-relativistic Doppler shift

$$\frac{\Delta\lambda}{\lambda}=\frac{\lambda_2-\lambda_1}{\lambda_1}=\frac{\Delta v}{c}$$



ngc 6810

Coord 2000.0 =
19 43 34 -58 39.3

morph. type = Sb

heliocentric velocity
(Halpha,NII)
2033 km/s

$$\Delta v=\frac{\lambda_2-\lambda_1}{\lambda_1}c=\frac{6610-6602}{6602}3x10^5=363\,km/s$$

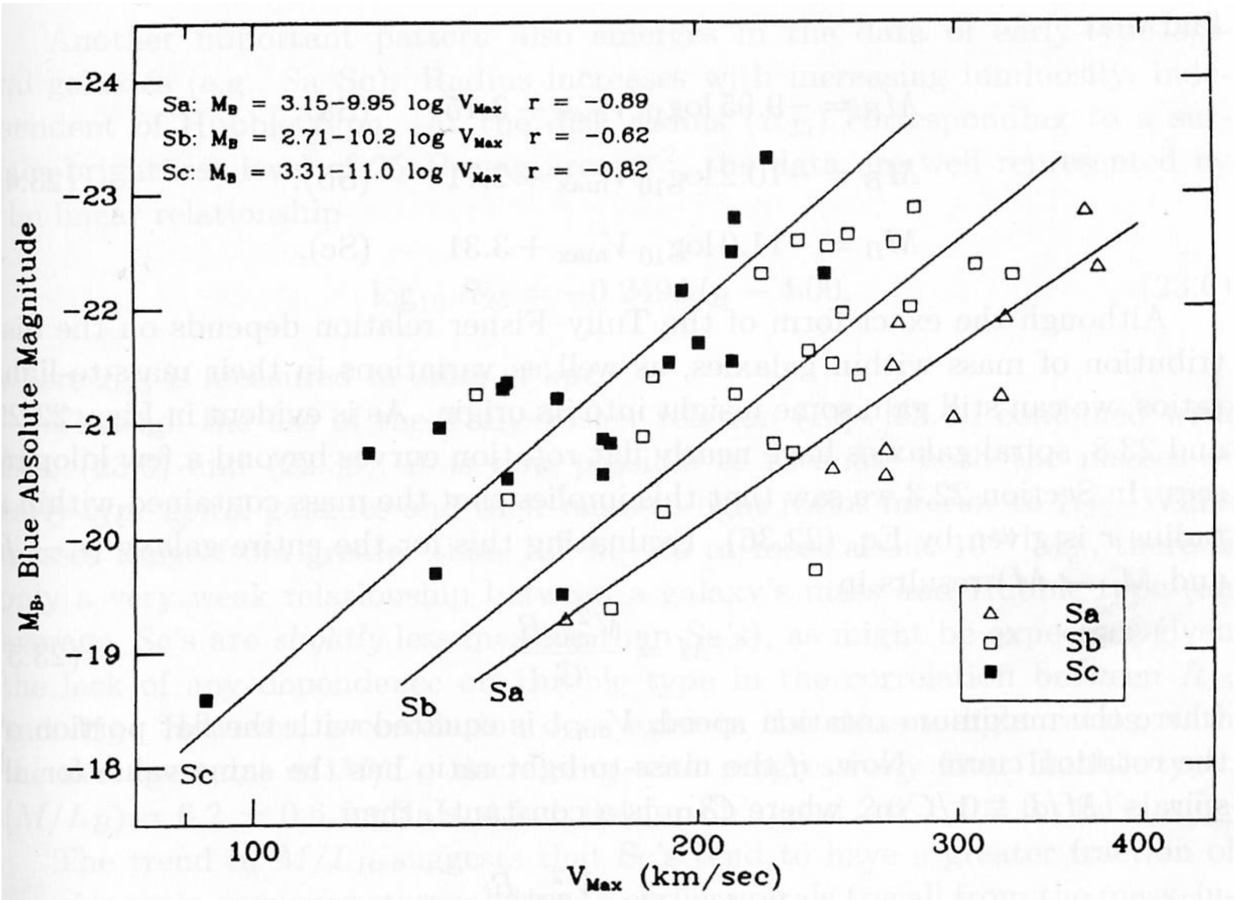
Doppler shift expressed as redshift, z

$$\frac{\Delta\lambda}{\lambda}=z=v/c$$

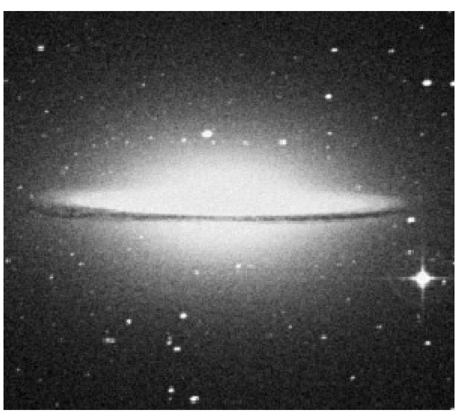
The full rotational amplitude of this galaxy is about 360-400 km/s. The rotational velocity is half this, about 180 km/s. Really what we observe is the projected velocity, $v_{proj} = v_{rot} \sin i$ where i is the inclination of the disk. By estimating i from images, we can recover v_{rot} from v_{proj}

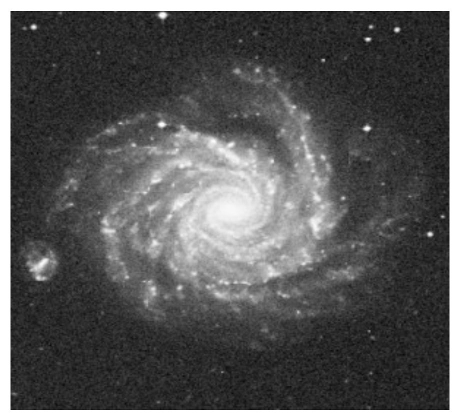
Using the calibrated T-F diagram below, we can infer that the luminosity of this galaxy is about M_B=-20 since it is most similar to an Sb type spiral. If, for example, we observe this galaxy to have an apparent B magnitude of m_B=15, we infer a distance modulus of DM=15 - -20 = 35 mag. The estimated distance is

$$D(pc) = 10\,x\,10^{(m-M)/5} = 10\,x\,10^{(35/5)} = 10^6\,pc = 100\,Mpc$$



Galaxy Types (See C+O Chap 25.1 for types of galaxies!)





Sa

Large bulge to disk ratio

Tight arms

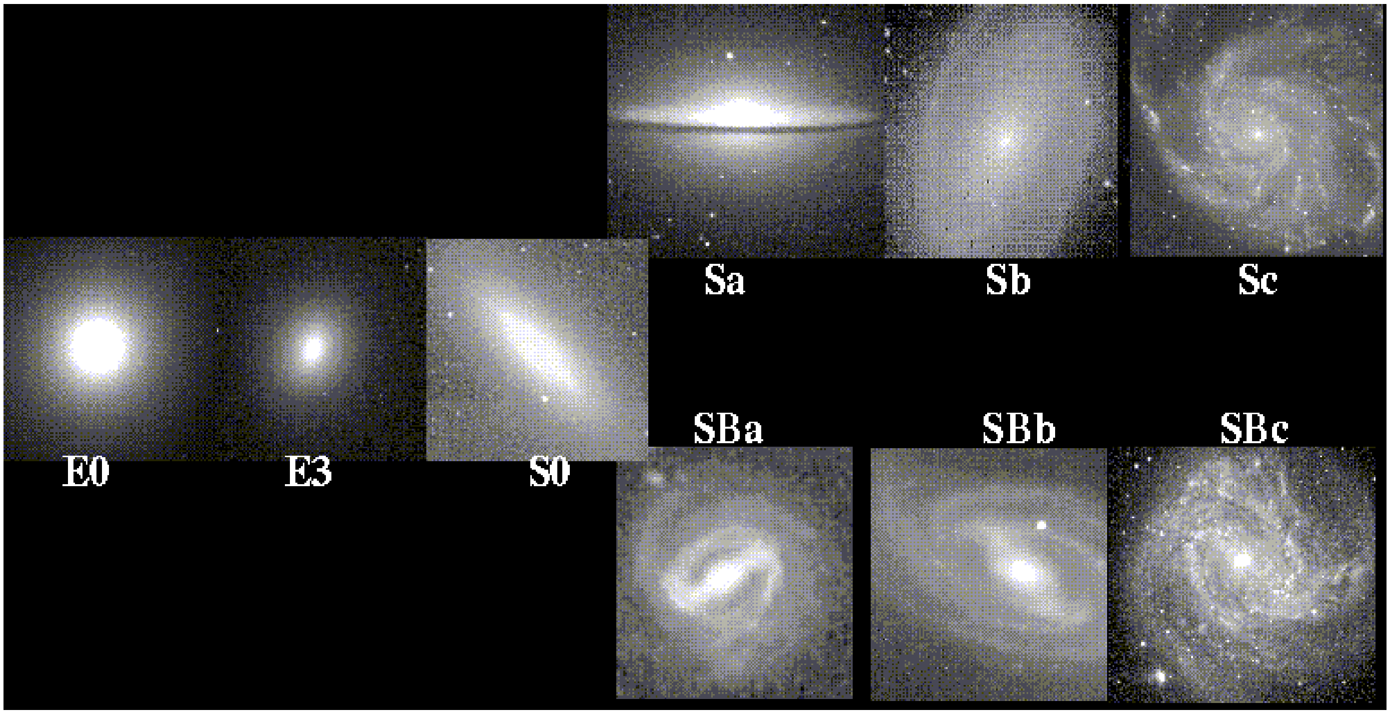
Sb

Sc

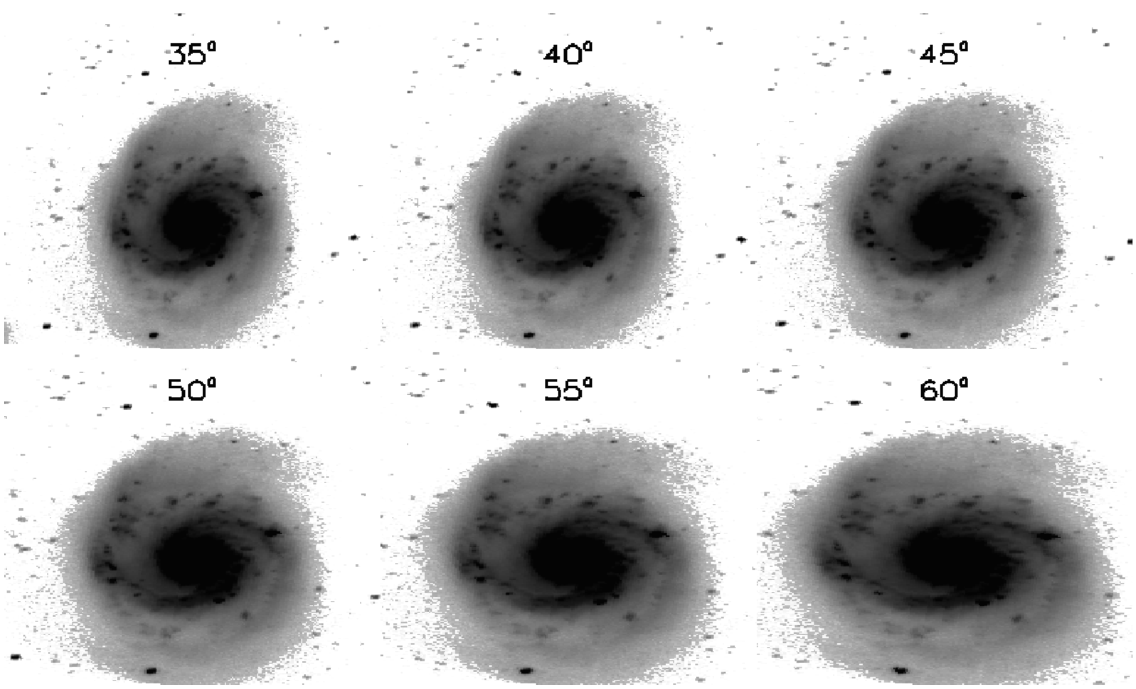
Small bulge to disk ratio

Diffuse arms

Below is the galaxy "tuning fork" diagram attributed to Edwin Hubble for classifying galaxies.



Note that any of these galaxies could be viewed from any inclination angle, so below is a view of a spiral galaxy from different inclination angles (0° is face-on; 90° is edge-on). Generally, it is possible to estimate the inclination angle from the shape of the galaxy.



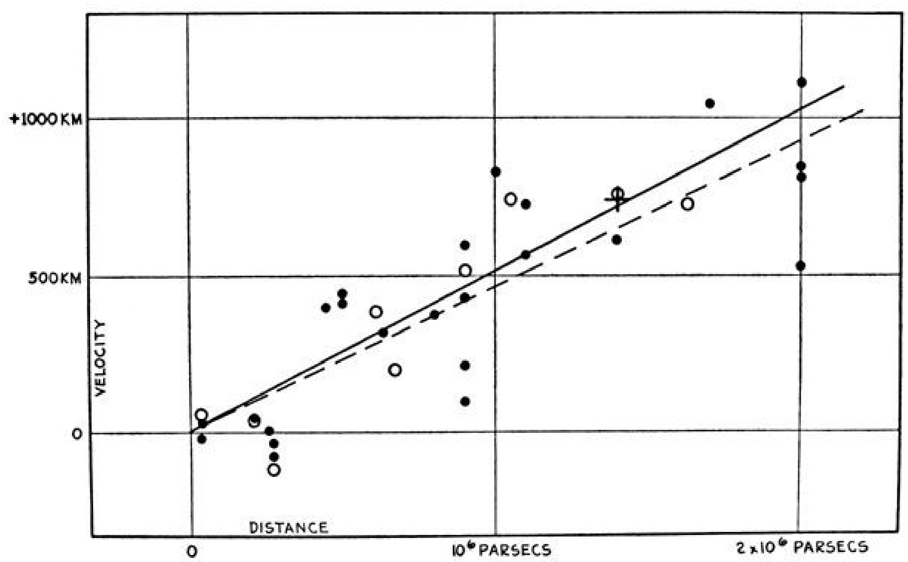
The Redshift-Distance Relation (Hubble's Law; C+O chap 27.2; Ryden & Peterson 20.5)

Edwin Hubble is credited with discovering the relation between recessional velocity and distance. More distant galaxies are moving away more quickly, due to the overall expansion of the universe. Once this relation is calibrated locally using other tools such as Cepheids and the T-F relation, the Hubble Relation or redshift-distance relation can be used to find distances to galaxies, because it is relatively easy to measure a redshift for a galaxy. Below is Hubble's original 1914 plot. The slope of the best fit line is approximately

$$\frac{\Delta Y}{\Delta X} = \frac{1000\, km\, s^{-1}}{2\, Mpc} = 500\, km\, s^{-1}\, Mpc^{-1}$$

This slope is known as *Hubble's constant*, H_0. The distance, D, and velocity, V, are related by

$$V = H_0 D$$



Most modern determinations of the Hubble Constant give $H_0 \sim 70$ km s^{-1} Mpc^{-1}.