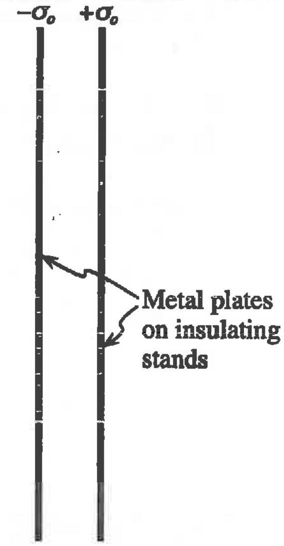
Pretest: Capacitance

Name *Answers*

Two thin metal plates on insulating stands are placed side by side as shown. Assume the plates are large and close enough together that fringing effects can be ignored and that all the charge resides on the *inner surfaces* of the plates.



1. The charge density on one plate is $+\sigma_o$; on the other, $-\sigma_o$. How does each of the following quantities change (if at all) when the two plates are moved closer together? Explain.

- the charge density on each plate

  remains unchanged

- the electric field between the plates

  $E = \sigma/\epsilon_0$ (see example 22.8)

  remains unchanged

- the potential difference between the plates

  $V_a - V_b = \int \vec{E} \cdot d\vec{l}$

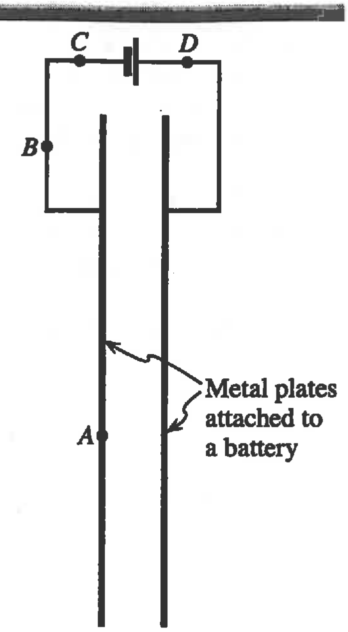
  → ΔV decreases

  → less distance over which E could hypothetically accelerate a charged particle

- the capacitance of the pair of plates

  $C = \frac{k \epsilon_0 A}{d}$ → increases

2. The two metal plates are discharged and then connected to a battery as shown.



How does each of the following quantities change (if at all) when the two plates are moved closer together? Explain.

- the potential difference between the plates

ΔV = constant — given the presence of the battery

- the electric field between the plates

$E = \frac{\Delta V}{\Delta x}$ → increases

- the charge density on each plate

$\sigma = E\sigma_0$ → increases

[also: $Q = CV$ → Q increases]

- the capacitance of the pair of plates

$C = \frac{k\varepsilon_0 A}{d}$ — C increases

3. Rank the absolute values of the electric potential differences ΔV_{AB}, ΔV_{BC}, ΔV_{CD}, and ΔV_{DA} between points *A*, *B*, *C*, and *D* in the diagram in question 2. Explain.

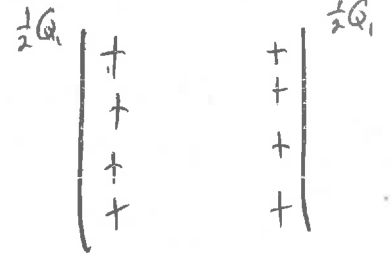
$|\Delta V_{CD}| = |\Delta V_{DA}| > |\Delta V_{AB}| = |\Delta V_{BC}| = 0$

I. The electric field near conducting plates

A. A small portion near the center of a large thin conducting *plate* is shown magnified at right. The portion shown has a net charge Q_1 and each side has an area A_1.

Side view of thin charged plate

Write an expression for the charge density on each side of the conducting plate.



$\sigma_{left} = \sigma_{right} = \dfrac{\frac{1}{2}Q_1}{A_1}$

B. Use the principle of superposition to determine the electric field inside the conductor (if you have not done so already).

$E_{inside} = 0$

Is your answer consistent with your knowledge of the electric field inside a conductor? Explain.

Yes

C. Use the principle of superposition to determine the electric ~~field on each side of~~ the plate.

$E_{outside} = \frac{1}{2}\frac{\sigma_{left}}{\epsilon_0} + \frac{1}{2}\frac{\sigma_{right}}{\epsilon_0} = \frac{1}{2}\frac{Q_1/A_1}{\epsilon_0}$

Does the charge on the *right* surface contribute to the electric field to the *left* of the plate (even though metal separates the two regions)? Explain.

Yes

D. Consider instead a portion near the center of a large *sheet* of charge. Like the plate in part A, the portion of the sheet has a net charge Q_1 and area A_1.

How does the charge density σ' on this sheet compare to the charge density on each side of the plate above? Explain.

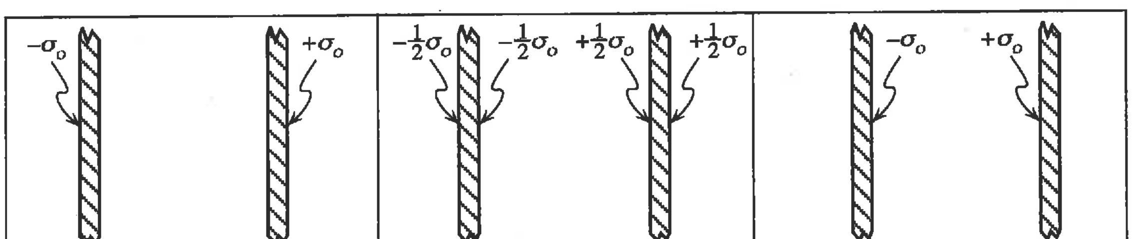
it would be twice as large

How does the electric field on one side of the *sheet of charge* compare to the electric field on the same side of the *charged plate?* Explain.

$E_{\text{due to sheet}} = E_{\text{due to plate}}$

E. A second plate with the same magnitude charge as the first, but opposite sign, is now held near the first. The plates are large enough and close enough together that fringing effects near the edges can be ignored.

The diagrams below show various distributions of charge on the two plates. Decide which arrangement is physically correct. Explain.



(a) (Excess charge is on outside surfaces)

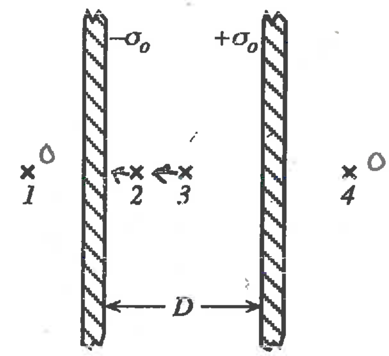
(b) (Excess charge is evenly distributed on both surfaces)

(c) (Excess charge is on inside surfaces)

opposite charges attract

II. Parallel plates and capacitance

Two very large thin conducting plates are a distance D apart. The surface area of the face of each plate is A_o. A side view of a small portion near the center of the plates is shown.



A. The inner surface of one plate has a uniform charge density of $+\sigma_o$; the other, $-\sigma_o$. The charge density on the outer surface of each plate is zero.

1. At each labeled point, draw vectors to represent the electric field at that point due to each charged plate.

2. Write expressions for the following quantities in terms of the given variables:

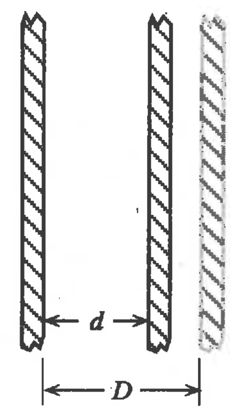
- the electric field at points *1*, *2*, *3*, and *4*

$\vec{E}_1 = \vec{E}_4 = 0$ $\vec{E}_2 = \vec{E}_3 = \frac{\sigma_o}{\epsilon_o} \; -\hat{i}$

- the potential difference between the plates

$\Delta V = E \Delta x = \frac{\sigma_o}{\epsilon_o} D$

3. The right plate is moved to the left as shown. Both plates are kept insulated. Describe how each of the following quantities will change (if at all). Explain.



- the charge density on each plate

unchanged

- the electric field both outside and between the plates

unchanged

- the potential difference between the plates

decreases $\Delta V = E \Delta x$

4. Write expressions for the following quantities in terms of σ_o and d (the new distance between the plates).

   - the magnitude of the electric field between the plates

     $E = \frac{\sigma_o}{\epsilon_o}$

   - the potential difference between the plates

     $\Delta V = \frac{\sigma_o}{\epsilon_o} d$

5. Find $\frac{Q}{\Delta V}$ (the ratio of the net charge on one plate to the potential difference between the plates).

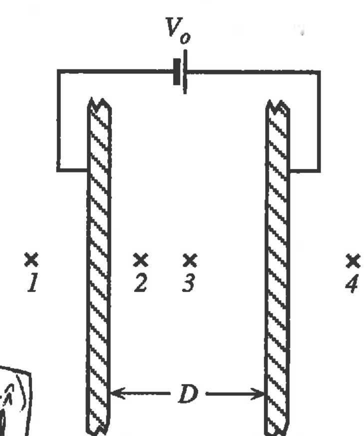
   $\frac{Q}{\Delta V} = \frac{\sigma_o A_o}{\frac{\sigma_o}{\epsilon_o} d} = \frac{A_o \epsilon_o}{d} \equiv C$

   How, if at all, would this ratio change if the charge densities on the plates were $+2\sigma_o$ and $-2\sigma_o$?

   unchanged

➪ Check your results for part A with a tutorial instructor before you continue.

B. Suppose the plates are discharged, then held a distance D apart and connected to a battery. (Ignore the fringing fields near the plate edges.)



1. Write expressions for the following quantities in terms of the given variables. Explain your reasoning in each case.

   - the potential difference ΔV between the plates

     $\Delta V = V_o$

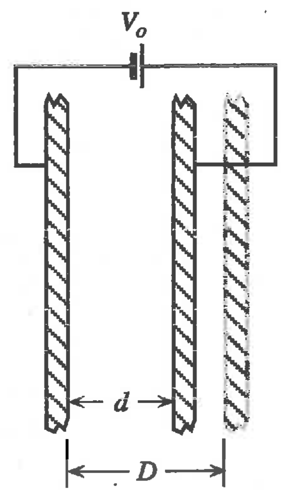
   - the electric field at points *1*, *2*, *3*, and *4*

     $E_1 = E_4 = 0 \qquad \vec{E}_2 = \vec{E}_3 = \frac{V_o}{D}(-\hat{x})$

   - the charge density on each plate

     $\sigma = E\epsilon_o = \frac{V_o}{D}\epsilon_o$

2. The right plate is moved to the left. Describe how each of the following quantities changes (if at all). Explain.



   - the potential difference ΔV between the plates

     $\Delta V = V_o$

   - the electric field both outside and between the plates

     $E = \frac{V_o}{d}$ increases

   - the charge density on each plate

     $\sigma = E\epsilon_o$ increases

3. Write expressions for the following quantities in terms of V_o and d (the new distance between the plates).

   - the magnitude of the electric field between the plates

     $\frac{V_o}{d}$

   - the charge density on each plate

     $E\epsilon_o = \frac{V_o}{d}\epsilon_o$

4. Find $\frac{Q}{\Delta V}$ (the ratio of the net charge on one plate to the potential difference between the plates).

   $C = \frac{Q}{\Delta V} = \frac{\sigma A}{V_o} = \frac{E\epsilon_o A}{V_o} = \frac{\frac{V_o}{d}\epsilon_o A}{V_o} = \frac{\epsilon_o A}{d}$

   How, if at all, would this ratio change if the voltage of the battery was $2V_o$?

   unchanged

⇨ Check your results for part B with a tutorial instructor before you continue.

C. Compare the ratio $\frac{Q}{\Delta V}$ that you calculated for two insulated plates (part A) to the same ratio for two plates connected to a battery (part B).

1. Does the ratio $\frac{Q}{\Delta V}$ depend on whether or not the plates are connected to a battery?

No   capacitance is an intrinsic property

2. Does the ratio $\frac{Q}{\Delta V}$ depend on the distance between the plates?

yes

The potential difference ΔV between two isolated conductors depends on their net charges and their physical arrangement. If the conductors have charge $+Q$ and $-Q$, the ratio $\frac{Q}{\Delta V}$ is called the *capacitance* (C) of the particular arrangement of conductors.

D. For the following cases, state whether each of the quantities q, σ, E, ΔV, and C changes or remains fixed:

1. two insulated conducting plates are moved farther apart

q unchanged
σ unchanged
E unchanged
ΔV increases
C drops

2. two conducting plates connected to a battery are moved farther apart

q drops
σ drops
E drops
ΔV unchanged
C drops