

Identical copper wire loops are placed in different external magnetic fields, as represented by the magnetic field lines shown below.

Draw an arrow on each loop to show the direction of the current induced in that loop. If there is no current induced, state that explicitly.

Loops a, b, and c are stationary. The magnetic fields are not changing in time.

a.

b.

c.

Explain how you determined your answer in each case.

zero

zero

zero

Loops d, e, and f are moving to the right. The magnetic fields are not changing in time.

d.

e.

f.

Explain how you determined your answer in each case.

zero

Loops g, h, and i are stationary. The magnitude of each magnetic field is decreasing in time.

g.

h.

i.

Explain how you determined your answer in each case.

The three wire loops shown at right are made of the same type of wire. Each is placed near the end of identical large solenoids as shown below. The solenoids are connected in series to a battery. Assume that the magnetic field near the end of each solenoid is uniform.

When the switch is closed, the current through the windings of the solenoids begins to increase. The following questions refer to an instant of time at which the current is increasing.

Loop 1: Single loop of radius  $r$

Loop 2: Double loop of radius  $r$  made from a single wire

Loop 3: Single loop of radius  $r/2$



1. Let  $\mathcal{E}_1$  represent the emf of loop 1. Find the total emf of each of the other loops in terms of  $\mathcal{E}_1$ . Explain.

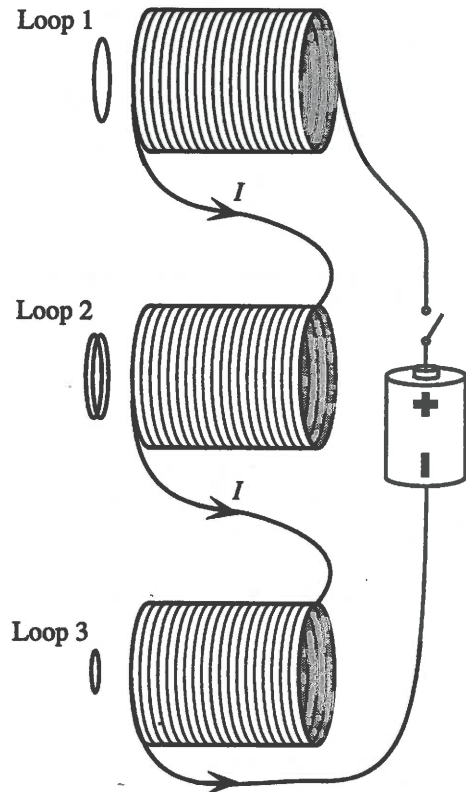
$$\mathcal{E}_2 = 2\mathcal{E}_1 \quad (N_2 = 2N_1)$$

$$\mathcal{E}_3 = \frac{1}{4}\mathcal{E}_1 \quad (A_2 = \frac{1}{4}A_1)$$

2. Let  $R_1$  represent the resistance of loop 1. Find the resistance of each of the other loops in terms of  $R_1$ . Explain.

$$R_2 = 2R_1$$

$$R_3 = \frac{1}{2}R_1$$



3. Find the current through the wire of each of the loops in terms of  $\mathcal{E}_1$  and  $R_1$ .

$$I_2 = \mathcal{E}_2 / R_2 = 2\mathcal{E}_1 / 2R_1 = I_1$$

$$I_3 = \mathcal{E}_3 / R_3 = \frac{\frac{1}{4}\mathcal{E}_1}{\frac{1}{2}R_1} = \frac{1}{2}I_1$$

I. Induced currents

A. A copper wire loop is placed in a uniform magnetic field as shown. Determine whether there would be a current through the wire of the loop in each case below. Explain your answer in terms of magnetic forces exerted on the charges in the wire of the loop.

- The loop is stationary.

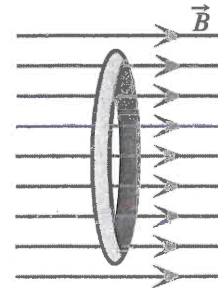
no current

- The loop is moving to the right.

no current

- The loop is moving to the left.

no current



B. Suppose that the loop is now placed in the magnetic field of a solenoid as shown.

1. Determine whether there would be a current through the wire of the loop in each case below. If so, give the direction of the current. Explain in terms of magnetic forces exerted on the charges in the wire of the loop.

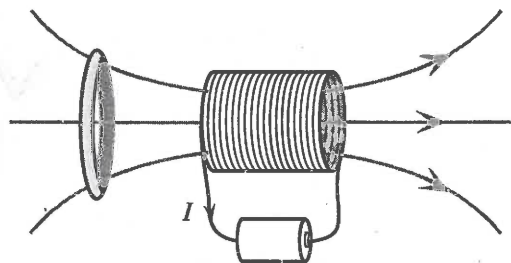
- The loop is stationary.

no current

- The loop is moving toward the solenoid.



- The loop is moving away from the solenoid.



2. For each case above in which there is an induced current, determine:

- the direction of the *magnetic moment* of the loop. (*Hint*: Find the direction of the magnetic field at the center of the loop due to the induced current in the loop. The magnetic moment is a vector that points in this same direction.)



- whether the loop is *attracted toward* or *repelled from* the solenoid.

repelled



attracted

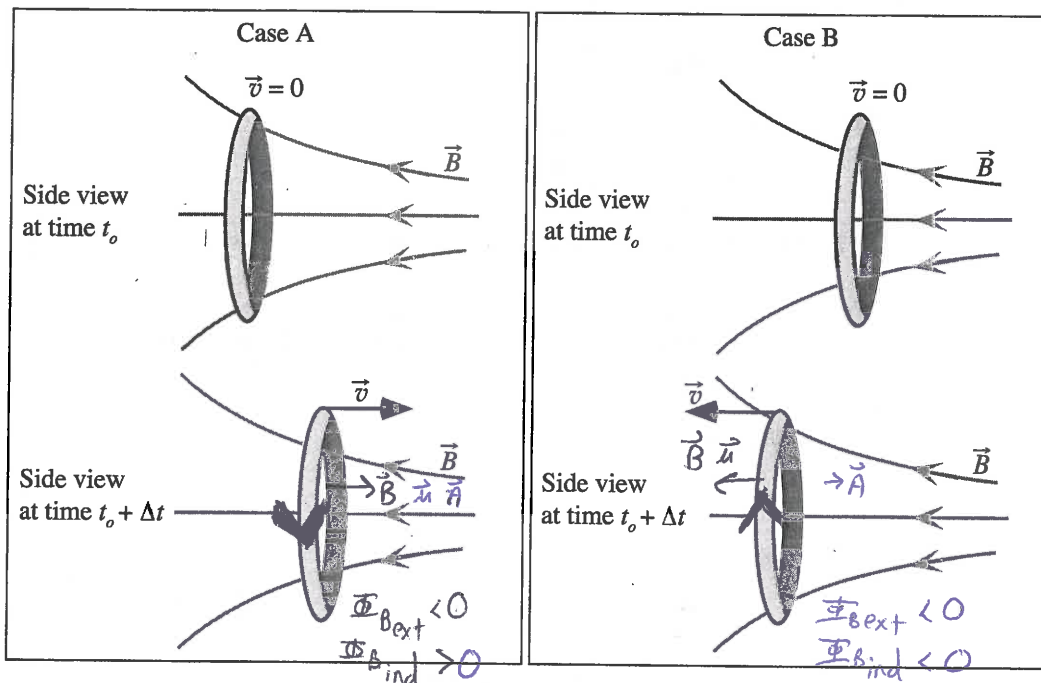


- whether the force exerted on the loop tends to *increase* or to *decrease* the relative motion of the loop and solenoid.

decrease

C. In each of the diagrams below, the position of a loop is shown at two times,  $t_0$  and  $t_0 + \Delta t$ . The loop starts from rest in each case and is displaced to the right in Case A and to the left in Case B. On the diagrams indicate:

- the direction of the induced current through the wire of the loop,
- the magnetic moment of the loop,
- an area vector for each loop,
- the sign of the flux due to the external magnetic field (at both instants), and
- the sign of the induced flux (at both instants).



D. State whether you agree or disagree with each of the students below. If you agree, explain why. If you disagree, cite a specific case for which the student's statement does not give the correct answer. (Hint: Consider cases A and B above.)

Student 1: "The magnetic field due to the loop always opposes the external magnetic field."

False

Student 2: "The flux due to the loop always has the opposite sign as the flux due to the external magnetic field."

False

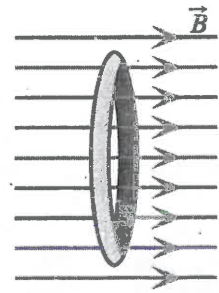
Student 3: "The flux due to the loop always opposes the change in the flux due to the external magnetic field."

True

⇒ Before continuing, check your answers to parts C and D with a tutorial instructor.

**II. Lenz' law**

A. The diagram at right shows a stationary copper wire loop in a uniform magnetic field. The magnitude of the field is *decreasing* with time.



1. Would you predict that there would be a current through the loop:

- if you were to use the idea that there is a magnetic force exerted on a charge moving in a magnetic field? Explain your reasoning:

No, since  $q\vec{v} \times \vec{B} = 0$  since  $\vec{v} = 0$

- if you were to use the reasoning of the student in part D of section I with whom you agreed? Explain.

Yes

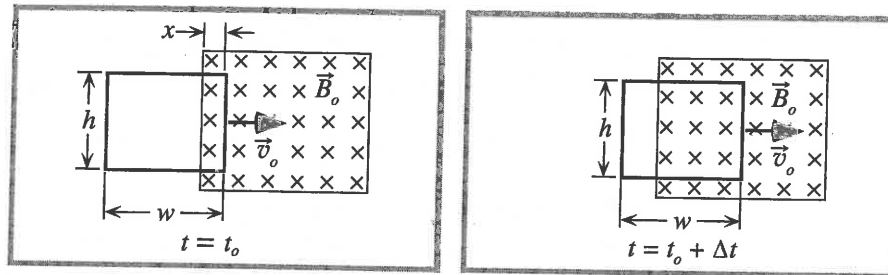
2. It is *observed* that there is an induced current through the wire loop in this case. Use the appropriate reasoning above to find the direction of the current through the wire of the loop.



To understand the interaction between the wire loops and solenoids in section I, we can use the idea that a force is exerted on a charged particle moving in a magnetic field. In each of those cases there was an induced current when there was relative motion between the solenoid and the wire loop. In other situations such as the one above, however, there is an induced current in the wire loop even though there is no relative motion between the wire loop and the solenoid. There is a general rule called *Lenz' law* that we can use in *all* cases to predict the direction of the induced current.

B. Discuss the statement of Lenz' law in your textbook with your partners. Make sure you understand how it is related to the statement by the student with whom you agreed in part D of section I.

C. A wire loop moves from a region with no magnetic field into a region with a uniform magnetic field pointing into the page.

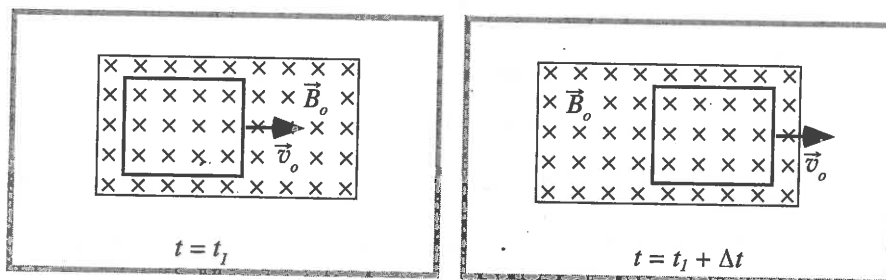


The loop is shown at two instants in time,  $t = t_0$  and  $t = t_0 + \Delta t$ .

Let's call  $\Phi > 0$  ~~into the page~~  
for  $\vec{B}$  fields out of the page

1. Is the magnetic flux through the loop due to the external field *positive, negative, or zero*:
  - a. at  $t = t_0$ ? *negative*
  - b. at  $t = t_0 + \Delta t$ ? *negative*
2. Is the *change* in flux due to the external field in the interval  $\Delta t$  *positive, negative, or zero*? *negative*
3. Use Lenz' law to determine whether the flux due to the induced current in the loop is *positive, negative, or zero*. *positive*
4. What is the direction of the current in the loop during this time interval? *ccw*

D. At two later instants,  $t = t_1$  and  $t = t_1 + \Delta t$ , the loop is located as shown.



1. Use Lenz' law to determine whether the flux due to the current induced in the loop is *positive, negative, or zero*. Explain. *zero*
2. Describe the current in the loop during this time interval. *none*
3. Consider the following student dialogue:

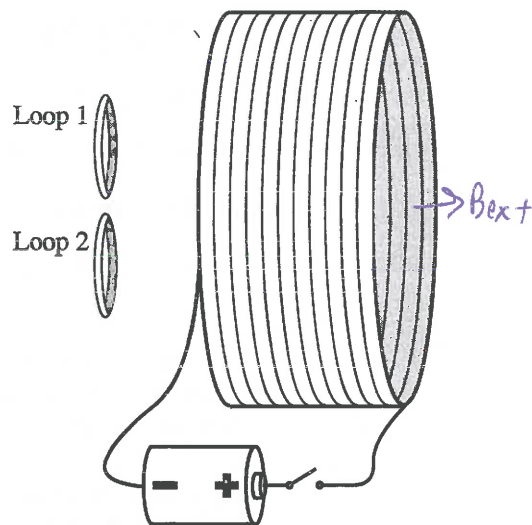
Student 1: "The sign of the flux is the same as it was in part C. So the current here will also be counter-clockwise."

Student 2: "I agree. If I think about the force on a positive charge on the leading edge of the loop, it points towards the top of the page. That's consistent with a counter-clockwise current."

Do you agree with either student? Explain. *False*

**I. Faraday's law**

Two loops of the same radius are held near a solenoid. Both loops are the same distance from the end of the solenoid and are the same distance from the axis of the solenoid.



A. The resistance of loop 2 is greater than that of loop 1. (The loops are made from different materials.)

1. Is there a current induced through the wire of either of the loops:

- before the switch is closed? Explain.

No

- just after the switch is closed? Explain.

Yes

- a long time after the switch is closed? Explain.

No

2. For the period of time that there is a current induced through the wire of the loops, find the direction of the current.

⌚

3. The ratio of the induced currents for the two loops is found by experiment to be equal to the inverse of the ratio of the resistances of the loops.

What does this observation imply about the ratio of the induced *emf* in loop 1 to the induced *emf* in loop 2?

$$I = \mathcal{E}/R \Rightarrow \mathcal{E}_1 = \mathcal{E}_2$$

B. Suppose that loop 2 were replaced by a wooden loop.

- Would there still be an *emf* in the loop?

Yes

- Would there still be a current induced in the wood loop?

No infinite R

C. Suppose that loop 2 were removed completely. Consider the circular path that the wire of loop 2 used to occupy.

- Would there still be an *emf* along the path? Explain.

Yes; see section 29.5 p. 971

- Would there still be a current along the path? Explain.

No

The results of the previous exercises are consistent with the idea that a change in the magnetic flux through the surface of a loop results in an *emf* in that loop. If there is a conducting path around the loop (e.g., a wire), there will be a current. The *emf* is independent of the material of which the loop is made; the current is not. It is found by experiment that the induced *emf* is proportional to the rate of change of the magnetic flux through the loop. This relationship is called *Faraday's law*. The direction of any induced current is given by Lenz' law.

- D. Three loops, all made of the same type of wire, are placed near the ends of identical solenoids as shown. The solenoids are connected in series. Assume that the magnetic field near the end of each of the solenoids is uniform.

Loop 2 consists of two turns of a single wire that is twice as long as the wire used to make loop 1. Loop 3 is made of a single wire that is half as long as the wire used to make loop 1.

Just after the switch has been closed, the current through the battery begins to increase. The following questions concern the period of time during which the current is increasing.

- Let  $\mathcal{E}$  represent the induced *emf* of loop 1. Find the induced *emf* in each of the other loops in terms of  $\mathcal{E}$ . Explain your reasoning.

$$\mathcal{E}_2 = 2\mathcal{E}_1$$

$$\mathcal{E}_3 = \frac{1}{4}\mathcal{E}_1$$

- Let  $R$  represent the resistance of loop 1. Find the resistance of each of the other loops in terms of  $R$ . Explain.

$$R_2 = 2R_1$$

$$R_3 = \frac{1}{2}R_1$$

- Find the current induced through the wire of each of the loops in terms of  $\mathcal{E}$  and  $R$ .

$$I_2 = I_1$$

$$I_3 = \frac{1}{2}I_1$$

