Finish the last circuit

a) $I_2 = \mathcal{E} - I(R_1 + R_4)/R_2$

b) $I_3 = I_2 R_2/R_3 = \dfrac{\mathcal{E} - I(R_1+R_4)}{R_2}\dfrac{R_2}{R_3} = \dfrac{\mathcal{E} - I(R_1+R_4)}{R_3}$

c) $I = \dfrac{\mathcal{E} - I(R_1+R_4)}{R_2} + \dfrac{\mathcal{E} - I(R_1+R_4)}{R_3} \Rightarrow I = \dfrac{\mathcal{E}(R_2+R_3)}{R_2R_3 + (R_1+R_4)(R_2+R_3)}$

$\mathcal{E} = 12\,V \quad R_1 = R_2 = 20\,\Omega \quad R_3 = 30\,\Omega \quad R_4 = 80\,\Omega$

$I = 0.107\,A$

It's much easier to first combine R_2 & R_3: $R_{23} = \dfrac{R_2R_3}{R_2+R_3}$

$R_{eq} = R_1 + R_{23} + R_4 \Rightarrow I = \dfrac{V}{R_{eq}}$

Suppose $R_4 \gg R_1, R_2, R_3$

$$I = \frac{\mathcal{E}}{R_4}\frac{(R_2+R_3)}{\frac{R_2R_3}{R_4} + \left(\frac{R_1}{R_4}+1\right)(R_2+R_3)} \longrightarrow \frac{\mathcal{E}}{R_4}\frac{(R_2+R_3)}{0 + (0+1)(R_2+R_3)} = \mathcal{E}/R_4$$

Suppose $R_1 = R_2 = R_3 = R_4 \equiv R$ and you want to maximize battery life.

If you could remove one R, which would it be?

$P = \mathcal{E}^2/R_{eq}$ Minimizing $P \Rightarrow$ maximizing R_{eq}

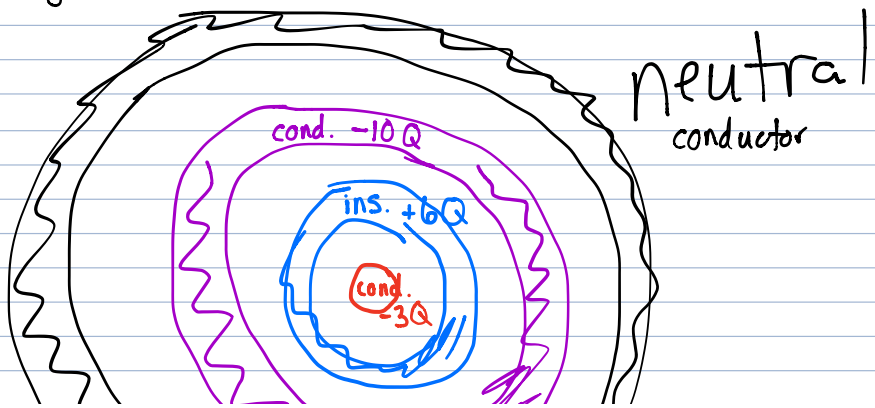
$R_{eq} = R_1 + \dfrac{R_2R_3}{R_2+R_3} + R_4$

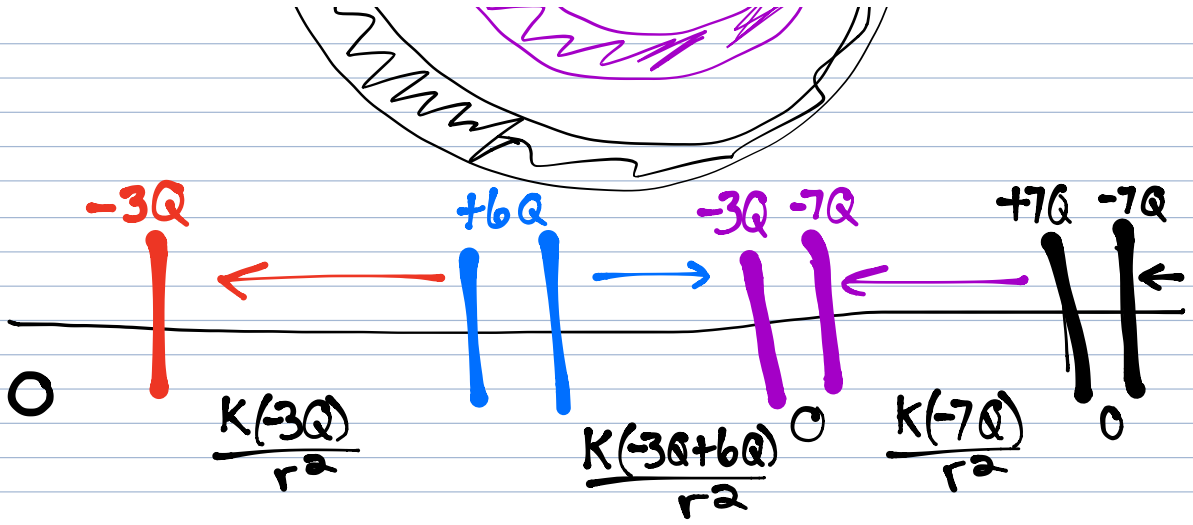
removing R_1 or $R_4 \Rightarrow R_{eq} = \frac{3}{2}R$

removing R_2 or $R_3 \Rightarrow R_{eq} = 3R$

Practice Problem

Find $\vec{E}$ and the charge distributions




-3Q
+6Q
-3Q
-7Q
+7Q
-7Q
O
O
O
K(-3Q)/r²
K(-3Q+6Q)/r²
K(-7Q)/r²