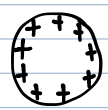
Sticky tape activity:

ripping tape off table induces same sense of charge

stacking the tape strips on top of each other, and then ripping $\rightarrow$ opposite charge

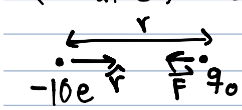
Electric Field



$q_0 \bullet \longrightarrow \vec{E} = \vec{F}/q_0$

Field: how a charge distribution affects a positive test charge

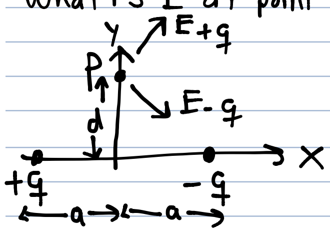
Q: What is the electric field near a point charge of $-10e$?

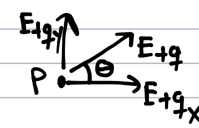


$$\vec{E} = \vec{F}/q_0 = \frac{1}{4\pi\epsilon_0}\frac{|-10e^-|(-\hat{r})}{r^2}$$

example: Consider an HCl molecule (permanent dipole of H^+ and Cl^-)

What is $\vec{E}$ at point P?





$$|E_{-q}| = |E_{+q}| = \frac{kq}{r^2} = \frac{kq}{a^2+d^2}$$

x-components: $E_{+q_x} = E_{+q}\cos\theta = E_{+q}\frac{a}{\sqrt{a^2+d^2}}$,

$$E_{-q_x} = E_{-q}\cos\theta = E_{-q}\frac{a}{\sqrt{a^2+d^2}},$$

$$\rightarrow E_x = E_{+q_x} + E_{-q_x} = \frac{2kq}{a^2+d^2}\frac{a}{\sqrt{a^2+d^2}}, = \frac{2kqa}{(a^2+d^2)^{3/2}}$$

sanity check: if $d=0$, $E_x = \frac{2kq}{a^2}$

y-components: $E_{+q_y} = E_{+q}\sin\theta$

$E_{-q_y} = -E_{-q}\sin\theta$

and they point in opposite directions

$$E_y = E_{+q_y} + E_{-q_y} = 0$$

$$E_{total} = \sqrt{E_x^2+E_y^2}, = E_x = \frac{2kqa}{d^3\left(\left(\frac{a}{d}\right)^2+1\right)^{3/2}} \xrightarrow{d \gg a} \frac{2kqa}{d^3}$$

Numbers: $a = 10^{-10}$ m $\quad d = 10^{-8}$ m $\quad E = 3\cdot10^5$ N/C

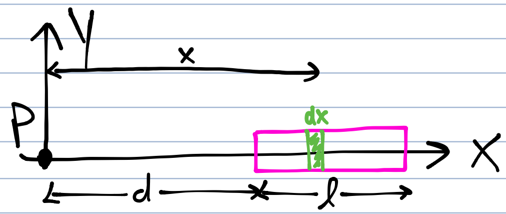
compare this to: $3\cdot10^6$ N/C breakdown of air

10^{-2} N/C $\vec{E}$ in copper wiring in your house

	symbol	SI unit	differential charge	picture
charge	q	C	dq	
linear charge density	λ	C/m	$dq = \lambda\, dx$	
Surface charge density	σ	C/m^2	$dq = \sigma\, dA$	
volume charge density	ρ	C/m^3	$dq = \rho\, dV$	

example $\vec{E}$ due to a positively-charged rod

total charge Q
length ℓ



$$dq = \lambda\, dx = \frac{Q}{\ell}\, dx$$

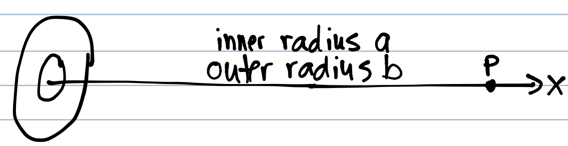
$$dE \text{ at Point } P = \frac{K\,dq}{x^2} = \frac{K\lambda\, dx}{x^2}$$

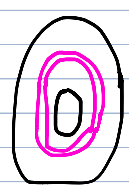
$$E = \int dE = \int_{x=d}^{x=d+\ell} \frac{K\lambda\, dx}{x^2} = K\lambda \int_{d}^{d+\ell} \frac{dx}{x^2} = K\lambda\left[\frac{-1}{x}\right]_{d}^{d+\ell}$$

$$= K\lambda\left[\frac{-1}{d+\ell} - -\frac{1}{d}\right] = K\lambda \frac{\ell}{d(d+\ell)}$$

$$\Rightarrow \vec{E} = K\lambda\frac{\ell}{d(d+\ell)}(-\hat{i}) = K\lambda \frac{\ell}{d^2}\frac{1}{\left(1+\frac{\ell}{d}\right)} \xrightarrow[d \gg \ell]{} = K\lambda\frac{\ell}{d^2} = \frac{KQ}{d^2}$$

Q: What is $\vec{E}$ at P? A disk with surface ch. density σ, with a hole in center





draw an infinitely thin annulus of thickness dr