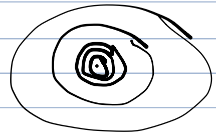
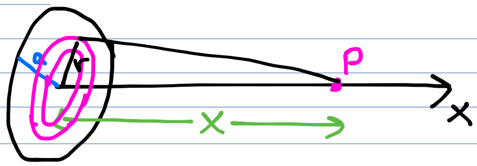
lines of equipotential → think of a topo map

a single positive charge would have contours like a mountain or a topo



example Potential of a uniformly charged disk of radius a and surface charge density sigma σ at the point P



$dV = \frac{k\,dq}{dist}$ dq is from an annulus of radius r and thickness dr

$dist = \sqrt{x^2+r^2}$ $dq = \sigma\, dA = \sigma\, 2\pi r\, dr$

$$V = \int dV = \int_0^a \frac{k\sigma 2\pi r\, dr}{\sqrt{r^2+x^2}} = \pi k\sigma \int_0^a 2r\,dr\,(r^2+x^2)^{-1/2}$$

$$= \pi k\sigma \frac{(r^2+x^2)^{1/2}}{1/2}\Big|_0^a = 2\pi k\sigma\left[(a^2+x^2)^{1/2} - (0+x^2)^{1/2}\right]$$

$$= 2\pi k\sigma\left[\sqrt{a^2+x^2} - x\right]$$

Does this make sense?

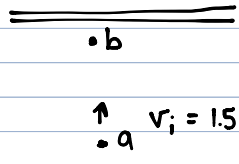
Take a limiting case where $\frac{x}{a} \gg 1$

$$V = 2\pi k\sigma x\left[\left(\left(\frac{a}{x}\right)^2+1\right)^{1/2} - 1\right] \xrightarrow[x \gg a]{} 2\pi k\sigma x\left[\frac{1}{2}\left(\frac{a}{x}\right)^2 + 1 - 1\right]$$

$$\left[\begin{array}{l} \text{Taylor expansion: } f(y)_{@\,0} \simeq f(0) + f'(0)\,(y-0) \\ \quad f(y) = (1+y)^n \qquad f'(y) = n(1+y)^{n-1} \\ \quad f(y)_{@\,0} \approx 1 + ny \end{array}\right]$$

$$V = \frac{k\pi a^2\sigma}{x} = \frac{kQ}{x} \checkmark$$

example Long charged rod with lambda = 5.00 10^-12 C/m. A positive charge is shot toward it at 1.5 km/s from a distance of 18.0 cm. How close does it get?



$q = 1.60\cdot 10^{-19}\,C$

$m = 1.67\cdot 10^{-27}\,Kg$

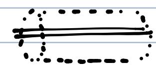
$v_i = 1.50\ Km/s$

$\Delta k + \Delta U = 0$ $\Delta k = 0 - \frac{1}{2} m v_i^2 = \frac{-1 m v_a^2}{2}$

$\Delta U = U_b - U_a = q\Delta V = q(V_b - V_a)$

$\Rightarrow \Delta K = -q(V_b - V_a) = -\frac{1}{2} m v_a^2$ & $V_b - V_a = \int_a^b \vec{E}\cdot d\vec{l}$

aside: $E(r)$ for a long line of charge



Gaussian cylinder $\oint \vec{E}\cdot d\vec{A} = Q_{encl}/\epsilon_0$

$\Rightarrow Q_{encl} = \lambda l$ and $\oint \vec{E}\cdot d\vec{A} = E\,2\pi r l = \frac{\lambda l}{\epsilon_0}$

$E(r) = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{r}$

$V_b - V_a = \int_b^a \vec{E}\cdot d\vec{l} = \int_b^a E_r\,dr = \frac{\lambda}{2\pi\epsilon_0}\int_b^a \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0}\ln r_a/r_b$

$\Rightarrow \frac{\lambda}{2\pi\epsilon_0}\ln r_a/r_b = V_b - V_a = 0.0117V \rightarrow r_b = r_a e^{-2\pi\epsilon_0(V_b - V_a)/\lambda}$

$= 0.158\text{ m}$