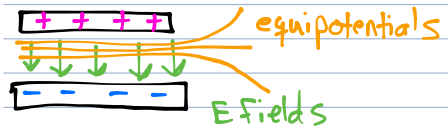
Lines of Equipotential



E is ⊥ to equipotentials

$E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$

Suppose the lower plate is "a" and is at 0 Volts and at $y=0$

" " upper " is "b" and is at 100 Volts and at $y = 1\,m$

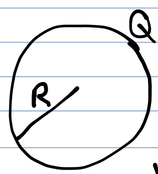
$\Delta V = V_b - V_a = 100\,V$

Q: What is an expression for Voltage as a function of y?

A: $V(y) = 100\,y$ Volts

E field is $E_y = -\frac{\partial V(y)}{\partial y} = -100 \frac{V}{m}\,\hat{j}$

example A solid conducting sphere has charge Q and radius R. What is V(r)?



outside Field is $\vec{E} = \frac{KQ}{r^2}\hat{r}$

choose $V(r\to\infty) = 0$ $\vec{dl} = dr\,\hat{r}$

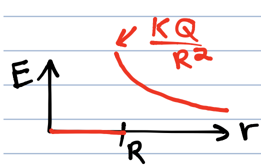
$$V(r) - V(r\to\infty) = V(r) - 0 = \int_r^\infty \vec{E}\cdot\vec{dl} = \int_r^\infty \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}\cdot dr\,\hat{r} = \int_r^\infty \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dr$$

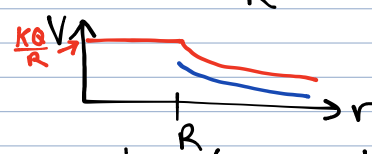
$$= -\frac{Q}{4\pi\epsilon_0 r}\Big|_r^\infty = 0 - -\frac{Q}{4\pi\epsilon_0 r} = \frac{KQ}{r}$$

inside $\vec{E} = 0 \Rightarrow$ no work done in moving a test charge

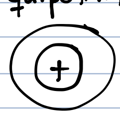
$\Rightarrow$ potential same everywhere

$\Rightarrow V(r<R) = \frac{KQ}{R}$





Equipotentials for a point charge (e.g. proton)



Equipotentials for a charged disk viewed from the "front"
conducting