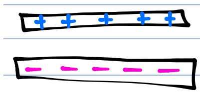
Chapter 08 – Capacitance

From the "Charges and Fields" PhET simulation we saw that $\Delta V \propto Q$ on plate

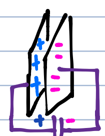


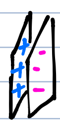
proportional increases in $Q \longleftrightarrow$ proportional increases in ΔV

$\frac{Q}{\Delta V}$ = constant $\equiv$ "C" capacitance
↳ "capacity" to hold charge

Some basics:

Take a pair of parallel conducting slabs. Hook up a battery





Remove the battery → charges are held apart ⇒ potential energy is stored

Q: How can we retrieve this U?

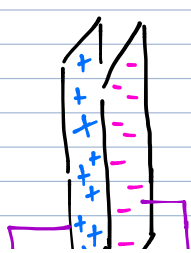
A: Put a charge between the plates – it will move

A: hook up a light bulb → negative charges move toward the positive charges via the conducting wire → bulb glows

Q: How can we increase the stored charge?

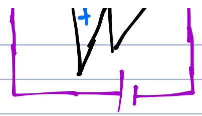
A: Increase plate area

only so many charges can hop onto the plates before "Coulombic saturation"



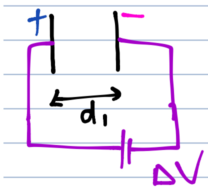
more area, more charge

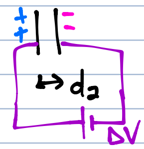
$\longrightarrow C \propto A$



Same battery, same ΔV

A: move plates closer together

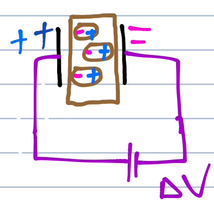




I haven't changed the battery, yet there is more attraction between charges

$$C \propto \frac{1}{d}$$

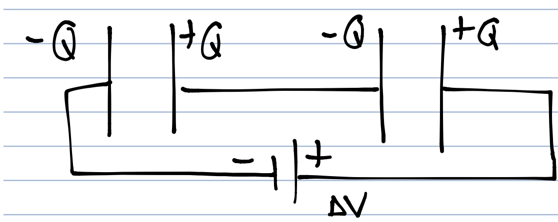
A: Place an insulator between plates



the atoms within the insulator do not move by definition, but they can rotate

$$C \propto K$$

2 identical capacitors in series



Concept question ch08/s1.html

answer = a

Concept Question follow-up

How does the magnitude of the charge change?

Can answer in two ways

i) conceptually, inserting dielectric increases capacitance $\Rightarrow Q = C\,\Delta V$ means more charge

ii) quantitative

case a

potential difference on each plate is $\Delta V/2$

$$Q = C_1 \Delta V_1 = C_2 \Delta V_2 = C_1 \frac{\Delta V}{2} = C_2 \frac{\Delta V}{2}$$

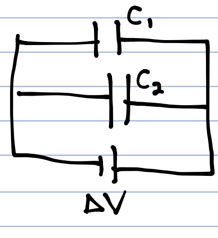
also, $\Delta V = \Delta V_1 + \Delta V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} \longrightarrow \frac{\Delta V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \equiv \frac{1}{C_{eq}} \Rightarrow C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$

if $C_1 = C_2$, then $C_{eq} = \frac{1}{2} C$

$\rightarrow Q = C_{eq} \Delta V = \frac{C}{2} \Delta V$ for $C_1 = C_2 = C$

case b: $C_{eq} = \frac{KC^2}{KC + C} = \frac{KC}{K+1} = \frac{C}{\frac{1}{K}+1} > \frac{C}{2}$ i.e. $C_{case\,b} > C_{case\,a}$

Capacitors in parallel



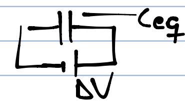
Now the capacitors have the same voltage drop

We know $\Delta V = \Delta V_1 = \Delta V_2$

$$\begin{array}{ccc} \Delta V & \Delta V_1 & \Delta V_2 \\ \| & \| & \| \\ Q/C_{eq} & Q/C_1 & Q/C_2 \end{array}$$

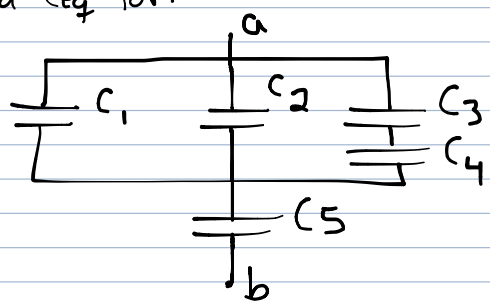
Also, $Q_{total} = Q_1 + Q_2 = C_1 \Delta V_1 + C_2 \Delta V_2 = C_1 \Delta V + C_2 \Delta V = (C_1 + C_2) \Delta V$

$\rightarrow C_1 + C_2 = Q/\Delta V \Rightarrow C_{eq} = C_1 + C_2$



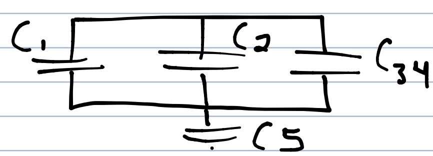
Group work

Find C_{eq} for:

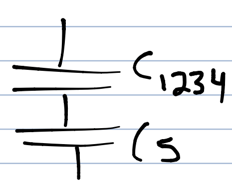


$C_1 = 3\mu F$ $C_2 = 11\mu F$ $C_3 = 12\mu F$ $C_4 = 6\mu F$

$C_5 = 9\mu F$

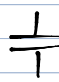


$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = 4\mu F$



$C_{1234} = C_1 + C_2 + C_{34}$

$= 18\mu F$



$C_{12345} = \frac{C_{1234}\, C_5}{C_{1234} + C_5} = 6\mu F$