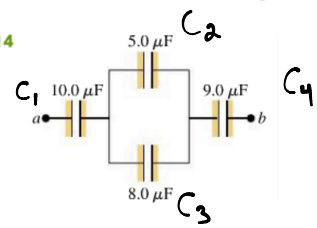
Practice problems on combining capacitors

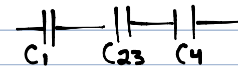
24.14 from ch08 Tutorials

24.14 • Figure E24.14 shows a system of four capacitors, where the potential difference across *ab* is 50.0 V. (a) Find the equivalent capacitance of this system between *a* and *b*. (b) How much charge is stored by this combination of capacitors? (c) How much charge is stored in each of the 10.0-μF and the 9.0-μF capacitors?

Figure **E24.14**



C_2 & C_3 in parallel → $C_{23} = C_2 + C_3 = 13\,\mu F$

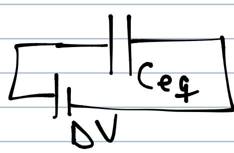


in series → $C_{1234} \neq \frac{C_1 C_{23} C_4}{C_1 + C_{23} + C_4}$

$C_{1234} = \left[\frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4}\right]^{-1} = 3.47\,\mu F$

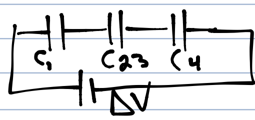
a) $3.47\,\mu F$

b)



$Q = C_{eq}\,\Delta V = 173.5\,\mu C$

c)

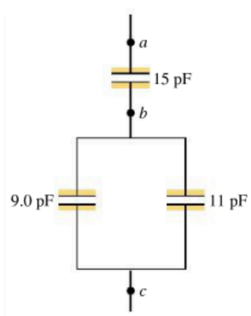


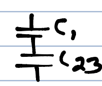
in series, Q_{tot} for C_{eq} is the same as the individual Q's

⇒ $173.5\,\mu C$

24.16 • For the system of capacitors shown in Fig. E24.16, find the equivalent capacitance (a) between *b* and *c*, and (b) between *a* and *c*.

Figure **E24.16**





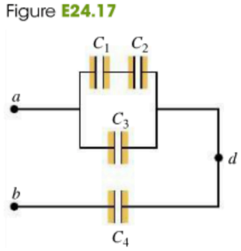
where $C_{23} = 9.0\,pF + 11\,pF = 20\,pF$

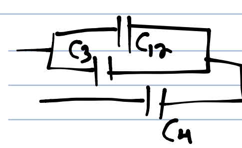
$C_{123} = \left(\frac{1}{C_1} + \frac{1}{C_{23}}\right)^{-1} = 8.6\,pF$

24.17 • In Fig. E24.17, each capacitor has $C = 4.00\,\mu F$ and $V_{ab} = +28.0$ V. Calculate (a) the charge on each capacitor; (b) the potential difference across each capacitor; (c) the potential difference between points *a* and *d*.

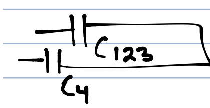
24.18 • In Fig. 24.8a, let $C_1 = 3.00\,\mu F$, $C_2 = 5.00\,\mu F$, and $V_{ab} = +64.0$ V. Calculate

Figure **E24.17**



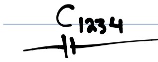


$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = 2\,\mu F$



$C_{123} = C_1 + C_{23} = 6\,\mu F$

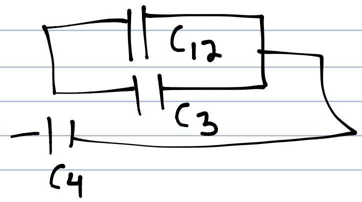
$C_{1234} = \left(\frac{1}{C_{123}} + \frac{1}{C_4}\right)^{-1} = 2.4\,\mu F$



$Q = C_{eq} V = 2.4\mu F \cdot 28.0V = 67.2\mu C$

$Q_4 = 67.2\mu C = Q_{123}$

backing up:



$67.2\mu C = Q_{12} + Q_3 = C_{12} V_{12} + C_3 V_3$

where $V_{12} = V_3$

$V_{tot} = 28V = V_4 + V_{123}$ $(V_{123} = V_{12} = V_3)$

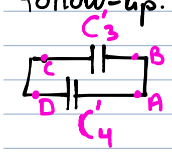
$V_4 = Q_4 / C_4 = 67.2\mu C / 4\mu F = 16.8V \Rightarrow V_{123} = V_3 = V_{12} = 28V - 16.8V = 11.2V$

$C_3 = Q_3 / V_3 =$

$Q_3 = C_3 V_3 = 4\mu F (11.2V) = 44.8\mu C$

<u>Concept Q ch08/s3.html</u> (b) parallel → same ΔV!

follow-up: what are their new ΔV's and Q's? Q_3' and Q_4'



$\underline{V_B - V_C}$ $C_{eq} = C_3' + C_4' = C_3 + C_4$ since $C_3 = C_3'$ $C_4 = C_4'$

$Q_{total} = Q_3' + Q_4' = Q_3 + Q_4$ cons. of charge

and since $\frac{Q_{total}}{C_{eq}} = \Delta V$ & $\Delta V = V_B - V_C = V_A - V_D \rightarrow V_B - V_C = \frac{Q_3 + Q_4}{C_3 + C_4}$

$Q_3' = C_3'(V_B - V_C) = C_3 \frac{Q_3 + Q_4}{C_3 + C_4}$ $Q_4' = C_4'(V_A - V_D) = C_4 \frac{(Q_3 + Q_4)}{C_3 + C_4}$

Numbers: Suppose $C_3 = 2\mu F$ $C_4 = 4\mu F$

$Q_3 = 24\mu C$ $Q_4 = 12\mu C$

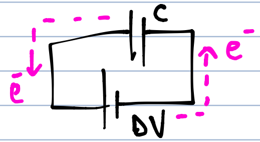
$\Delta V_3 = 12V$ $\Delta V_4 = 3V$

$\rightarrow Q_3' = 2\mu F \frac{36\mu C}{6\mu F}$ $Q_4' = 4\mu F \frac{36\mu C}{6\mu F}$

$= 12\mu C$ $= 24\mu C$

$Q_3' + Q_4' = 36\mu C = Q_3 + Q_4$ ✓

<u>Energy storage in capacitors</u>



At some time t, charge on plates is q

To transfer charge dq from one plate to another,

requires $dW = \Delta V dq = \frac{q}{C} dq$

Total work to transfer Q is $W = \int dW = \int \frac{q}{C} dq = \frac{1}{C} \int dq\, q$

$= \frac{1}{2}\frac{Q^2}{C}$

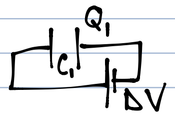
How long can we illuminate a light bulb that generates 2.4 Watts when connected to a 450,000 μF capacitor and 4.5 V?

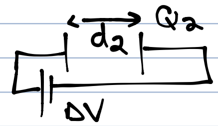
energy stored in capacitor is $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\,450000\mu F \cdot (4.5V)^2 = 4.6\,J$

$\Delta t = \frac{\text{energy stored}}{\text{energy loss rate}} = \frac{4.6J}{2.4W} = 1.9s$

Take two capacitors, one with separation d of the plates, and one with separation 2d

$C \propto \frac{1}{d}$





$d_2 = 2d_1$

$C_1 = 2C_2$

If $\Delta V_1 = \Delta V_2$, then

$Q_1 = 2Q_2 \rightarrow U_1 = \frac{1}{2}Q_1^2/C_1 = \frac{1}{2}\frac{4Q_2^2}{2C_2} = 2\left(\frac{1}{2}\frac{Q_2^2}{C_2}\right) = 2U_2$ ✓

Portable heart defibrillator

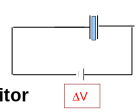
$C = 70\mu F \quad \Delta V = 5000\,V \quad U = \frac{1}{2}CV^2 = 875\,J$

about 200J of this is sent in a ~2.0ms pulse

$\rightarrow$ Power $= \frac{\text{energy}}{\text{time}} = 100kW$ (much greater than the battery could provide)

$U = \frac{1}{2}CV^2 = \frac{1}{2}Q^2/C = \frac{1}{2}QV$

Suppose a dielectric placed between two plates of a capacitor increases the overall capacitance by a factor of two.



The amount of stored energy in the capacitor

a) doubles
b) halves
c) quadruples
d) is four times smaller