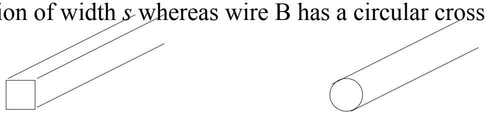
Part I. Questions 1-10. 8 points each. Multiple choice: For **full credit**, circle only the **correct** answer. For **half credit**, circle the **correct** answer **and one incorrect** answer. For ¼ **credit,** circle the **correct** answer **and two incorrect** answers.

1. When plugged into a typical 110 V electrical outlet, a particular light bulb has a standard operating power of 100 W. If the same light bulb is put in a different circuit, you find that the bulb operates with a current that is one half of its standard operating current. The power drawn by the bulb in this circuit is approximately

a. 20 W
b. 25 W
c. 33 W
d. 40 W
e. 50 W

2. Two pieces of copper wire have the same length, but wire A has a square cross section of width *s* whereas wire B has a circular cross section of diameter *s*. Which of the following statements is true?

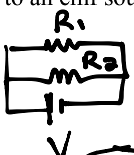
a. The resistance of both wires is the same.
b. The resistivity of both wires is the same.
c. Both the resistance and the resistivity of A and B are the same.
d. The resistance of A is greater than that of B.
e. The resistivity of A is greater than that of B.

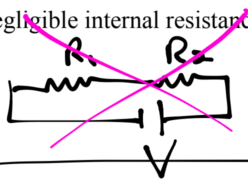


A B

3. Resistors R_1 and $R_2 = 2R_1$ are connected in parallel to an emf source that has negligible internal resistance. If resistor R_2 is removed from the circuit, the current through R_1 would

a. increase by a factor of 1.5.
b. increase by a factor of 2.
c. decrease by a factor of 1.5.
d. decrease by a factor of 2.
e. None of the above; ~~we need to know the emf value~~.





(But I through battery changes!)

⇒ potential across R_1 doesn't change ⇒ $I_1 = \frac{V}{R_1}$ for both situations

4. A resistor of resistance *R*, a capacitor of capacitance *C*, and a battery with emf ε are all connected in series. The capacitor achieves a certain maximum charge after a long time. Another circuit is identical except that it has two resistors of resistance *R* in series. The maximum charge on the capacitor in this second circuit would be ______________ as that in the first circuit.

a. twice as large
b. half as large
c. the same
d. four times as large
e. one fourth as large

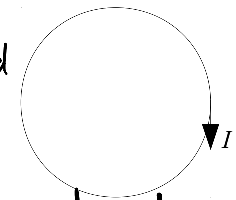
$\tau_1 = RC$ $\tau_2 = 2RC$ so 2nd circuit takes longer to charge (but final charge is still $Q = CV$)

$Q(t) = CV(1-e^{-t/2RC})$

5. An elastic, conducting loop of wire carrying current *I* is placed on a horizontal table. As viewed from above, the current is circulating clockwise. If a uniform magnetic field was introduced pointing straight up out of the table, the loop would

a. slightly expand.
b. slightly contract.
c. remain the same size.
d. reverse its current direction.
e. None of the above.

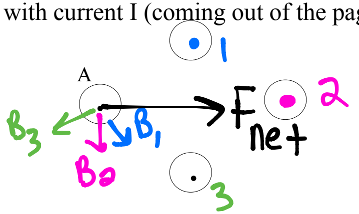
$\vec{F} = I\vec{L} \times \vec{B} \Rightarrow$ F always points inward



Lenz's Law/Faraday's Law $\Rightarrow$ shrinks in order to maintain a small flux

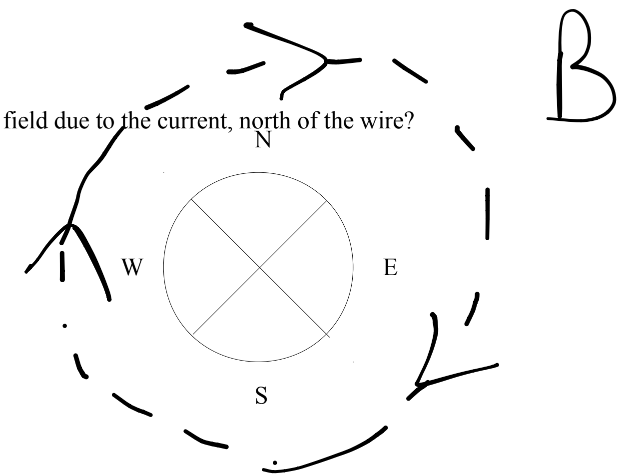
6. Four long parallel wires each with current I (coming out of the page) are symmetrically arranged as shown. The force on wire A is directed

a. to the left
b. to the right
c. up
d. down
e. none of these



7. A long wire carries an electric current into the page. What is the direction of the magnetic field due to the current, north of the wire?

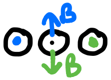
a. north
b. east
c. west
d. south
e. out of the page
f. into the page



8.A metallic weight is suspended from a metal spring. If now a current is passed through the spring,

a. the spring will contract, raising the weight.
b. the spring will elongate, lowering the weight.
c. the weight will not move.
d. whether or not the weight moves up or down depends on what the weight is made of (i.e. whether or not it is magnetizable).
e. None of these is correct.

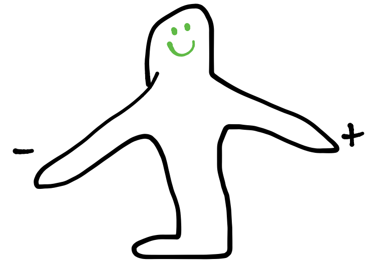
But remember: only the outer 2 loops will actually contract



9. An airplane is in level flight over Antarctica, where the magnetic field of the earth is mostly directed upward, away from the ground. As viewed by a passenger facing toward the front of the plane, which wingtip is at a higher potential?

a. the left wingtip
b. the right wingtip
c. both wingtips would be at the same potential
d. It depends on which direction the airplane is flying.
e. None of these is correct.

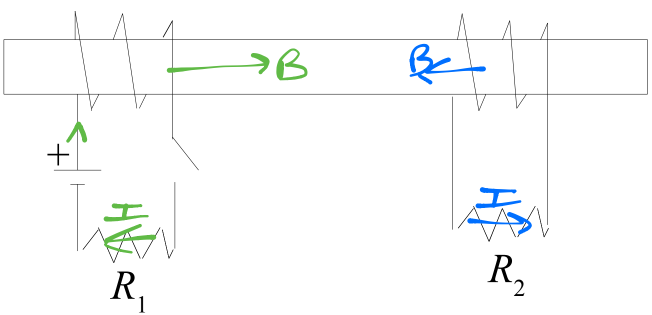
q v x B



10. Two separate circuits wrap around a metal bar. Just after the switch is closed, how do the currents I_1 and I_2 flow through the resistors R_1 and R_2?

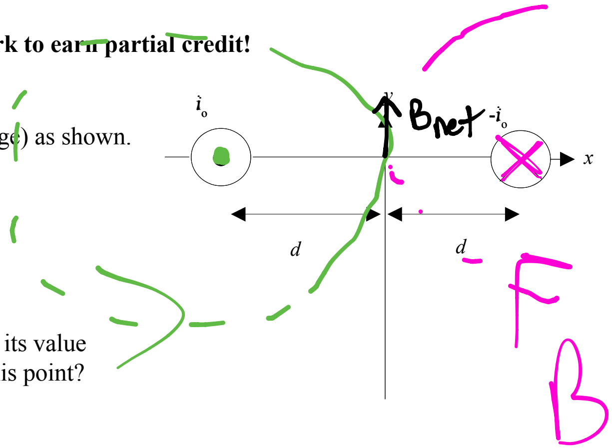
a. I_1 flows to the left, I_2 flows to the left
b. I_1 flows to the left, I_2 flows to the right
c. I_1 flows to the left, I_2 is zero
d. I_1 flows to the right, I_2 flows to the left
e. I_1 flows to the right, I_2 flows to the right
f. I_1 flows to the right, I_2 is zero
g. I_1 is zero, I_2 is zero

There's no cross product business here . . .



Part II. Short answer/sketch. Answer questions 11-14 as completely as possible. **Show your work to earn partial credit!**

11. (25 points)
Two long straight wires carry equal but opposite currents along the z-axis (perpendicular to the page) as shown.



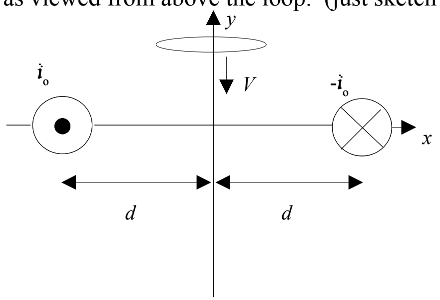
a) At what point on the y-axis is the total magnetic field B produced by the wires largest? What is its value B_{max} there, expressed in terms of i_o, d, and anything else you need? What is the direction of B at this point?

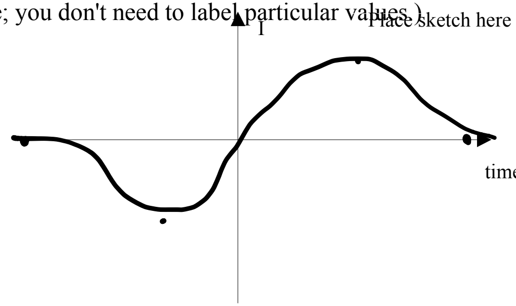
y where B is maximum= origin

direction of B there: $+\hat{j}$

B_{max}= $2\frac{\mu_0 I}{2\pi d}$

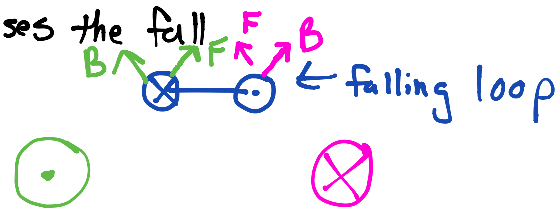
b) A conducting wire loop with its axis along the y-axis is lowered at constant speed down the y-axis, as shown. Sketch the current I induced in the loop as a function of time, for $-\infty<t<+\infty$. Assume that the loop crosses y=0 at t=0, and that a positive current corresponds to counterclockwise current flow as viewed from above the loop. (just sketch the shape; you don't need to label particular values.)





c) Does the loop feel a force as it moves toward the wires from above?

yes, opposes the fall



d) If you answered "yes" to d), how is this force produced, and what is its direction?
If you answered "no" to d), explain why the loop feels no force.

I've ignored the other two forces here since they are smaller

12. (25 points)

The circuit below contains four resistors and a battery. The current through each resistor is I_1, I_2, I_3, and I_4.

a. Circle the relations that are always correct: $I_1=I_2$; $I_2=I_3$; $I_3=I_4$; $I_1=I_4$; $I_1=I_2+I_3$

b. Combine R_3 and R_4 into one equivalent resistor. Call this R_{34}.

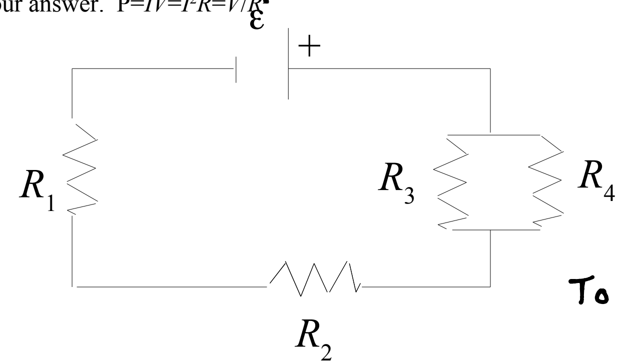
$$R_{34} = \frac{R_3 R_4}{R_3 + R_4}$$

c. Combine all the resistors into one equivalent resistor. Call this R_{1234}.

$$R_{1234} = R_1 + R_2 + R_{34}$$

d. Compute I_4 in terms of R_1, R_2, R_3, R_4, and ε.

e. You want to maximize the lifetime of the battery, and you are free to remove one resistor only. If all four resistors are equal, which one would you remove? Justify your answer. P=$IV=I^2R=V^2/R$



a) $I_1 = I_3 + I_4$

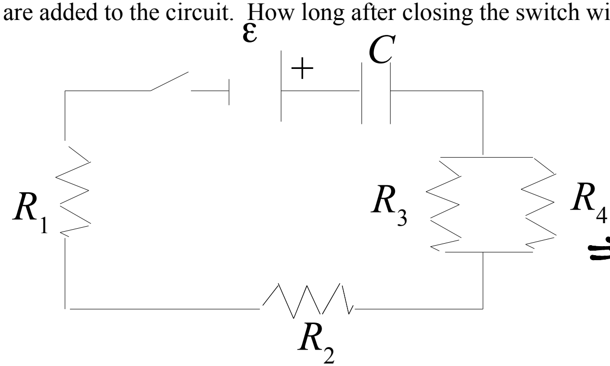
b) $I_1 = \varepsilon / R_{1234}$

c) $I_3 R_3 = I_4 R_4$

$$I_4 = \frac{\varepsilon}{R_{1234}} \left(\frac{1}{1 + R_4/R_3} \right)$$

To minimize power, maximize R_{1234} ($P = V^2/R$) ⇒ remove either R_3 or R_4

f. A switch and a capacitor of capacitance *C* are added to the circuit. How long after closing the switch will it take to charge up the capacitor to ≈ 63.2% of its final value?



$$q(t) = Q_f\left(1 - e^{-t/R_{eq}C}\right)$$

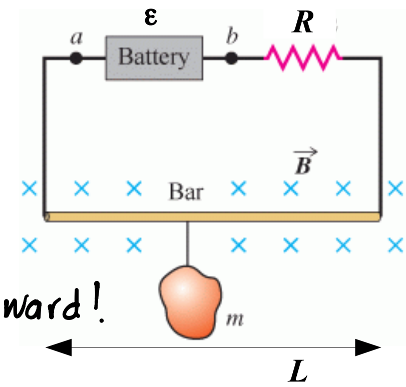
$$\rightarrow t = -R_{eq}C \ln\left(1 - \frac{q(t)}{Q_f}\right)$$

$$\Rightarrow t_{0.632} = -R_{eq}C \ln(1 - 0.632)$$

$$= R_{eq}C = R_{1234}C$$

13. (25 points)

A magnetic balance is used to weigh objects. The mass ***m*** to be measured is hung from the center of the bar of length ***L*** suspended in a uniform magnetic field ***B*** directed into the page. The battery voltage **ε** can be adjusted to vary the current in the circuit. The horizontal bar is made of extremely lightweight conducting material, and is connected to the battery by thin elastic conducting wire.



a) To measure the mass, an upward force must balance the downward force due to gravity. What is this upward force? You can either use words or an equation for your answer.

magnetic force

b) Which point, *a* or *b*, should be the positive terminal of the battery?

need current in the bar flowing to the right, to get $\vec{I}L \times \vec{B}$ upward!

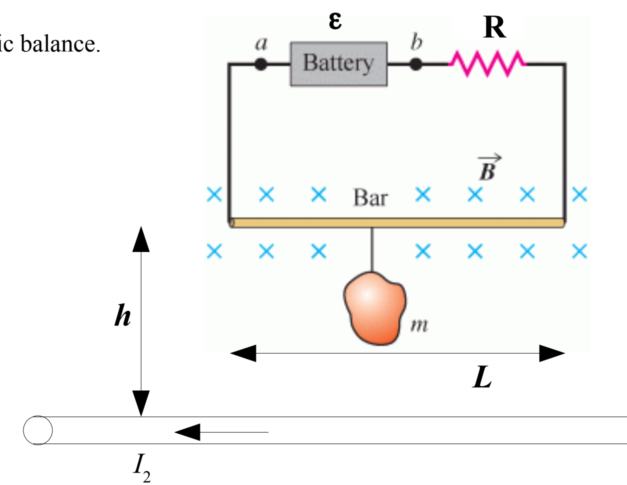
c) If the maximum terminal voltage of the battery is ε_{max}, what is the greatest mass m_{max} that this instrument can measure?

$$I l B \sin 90^\circ = mg$$

$$M_{max} = \frac{I_{max} l B}{g} = \frac{\frac{\varepsilon_{max}}{R} l B}{g}$$

d) Now suppose a long wire with current I_2 is placed at a fixed distance ***h*** directly below the magnetic balance. How does this additional current affect the net magnetic field at the location of the bar? (i.e., stronger, weaker, same, same but different direction, ...)

e) Derive an expression for the new maximum mass that can be measured in this situation.



$$M_{max} = \frac{\frac{\varepsilon_{max}}{R} l \left(B + \frac{\mu_0 I}{2\pi h}\right)}{g}$$

14. (25 points)

A long metal bar is pulled to the right at a steady speed perpendicular to a uniform magnetic field. The bar rides on parallel metal rails connected through a resistor, so the apparatus makes a complete circuit. Ignore the resistance of the bar and the rails.

a) Calculate the magnitude of the emf induced in the circuit (in terms of the variables given in the figure below).

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\oint \vec{B}\cdot d\vec{A} = -\frac{d}{dt}\oint B\,dA = -B\frac{d}{dt}\int dA = -B\frac{d}{dt}A(t) = -B\frac{d}{dt}Lx = -BLv \Rightarrow BLv$$

b) What is the direction of the current induced in the circuit?

CCW

c) Compute the current through the resistor (in terms of the variables in the figure below).

$$I = \frac{\mathcal{E}}{R} = \frac{BvL}{R}$$

d) Repeat part c) with numbers:

bar resistivity $= 1.72\cdot10^{-8}\ \Omega$ m
bar radius = 0.1 mm
bar length = 3.50 m
bar speed= 5.0 m/s
magnetic field = 0.750 T

$I = 6.85$ A

$$R_{bar} = \frac{\rho L}{A_{bar}}$$