Equations from the ends of each chapter

Answers 518

$$\vec{F}_{12}(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$\vec{F}(r) = \frac{1}{4\pi\varepsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2}\hat{r}_i$$

$$\vec{F} = Q\vec{E}$$

$$\vec{E}(P) \equiv \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i$$

$$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}\hat{k}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{k}$$

$$\vec{p} \equiv q\vec{d}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$1/(4\pi\varepsilon_0) = 8.99\cdot 10^9$ N m²/C²

$\varepsilon_0 = 8.854\cdot 10^{-12}$ F/m

$q_e = 1.60\cdot 10^{-19}$ C

$$\Phi = \vec{E}\cdot\vec{A} \to EA\cos\theta$$

$$\Phi = \int_S \vec{E}\cdot\hat{n}\,dA = \int_S \vec{E}\cdot d\vec{A}$$

$$\Phi = \oint_S \vec{E}\cdot\hat{n}\,dA = \oint_S \vec{E}\cdot d\vec{A}$$

$$\Phi = \oint_S \vec{E}\cdot\hat{n}\,dA = \frac{q_{enc}}{\varepsilon_0}$$

$$\Phi = \oint_S \vec{E}\cdot\hat{n}\,dA = E\oint_S dA = EA = \frac{q_{enc}}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$v_d = \frac{I}{nqA}$$

$$I = \iint_{area} \vec{J}\cdot d\vec{A}$$

$$\rho = \frac{E}{J}$$

$$V = IR$$

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

$$R \equiv \frac{V}{I}$$

$$R = \rho\frac{L}{A}$$

$$R = R_0(1 + \alpha\Delta T)$$

$$P = IV$$

$$P = I^2 R = \frac{V^2}{R}$$

$$U(r) = k\frac{qQ}{r}$$

$$W_{12\ldots N} = \frac{k}{2}\sum_i^N\sum_j^N \frac{q_i q_j}{r_{ij}} \text{ for } i \neq j$$

$$\Delta V = \frac{\Delta U}{q} \text{ or } \Delta U = q\Delta V$$

$$V = \frac{U}{q} = -\int_R^P \vec{E}\cdot d\vec{l}$$

$$\Delta V_{AB} = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{l}$$

$$V = \frac{kq}{r}$$

$$V_P = k\sum_1^N \frac{q_i}{r_i}$$

$$\vec{p} = q\vec{d}$$

$$V_P = k\frac{\vec{p}\cdot\hat{r}}{r^2}$$

$$V_P = k\int\frac{dq}{r}$$

$$E_x = -\frac{\partial V}{\partial x},\quad E_y = -\frac{\partial V}{\partial y},\quad E_z = -\frac{\partial V}{\partial z}$$

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\vec{E} = -\vec{\nabla}V$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial\varphi} + \hat{z}\frac{\partial}{\partial z}$$

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial\theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}$$

$$C = \frac{Q}{V}$$

$$C = \varepsilon_0\frac{A}{d}$$

$$C = 4\pi\varepsilon_0\frac{R_1 R_2}{R_2 - R_1}$$

$$C = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}$$

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

$$C_P = C_1 + C_2 + C_3 + \cdots$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

$$U_C = \frac{1}{2}V^2 C = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

$$C = \kappa C_0$$

$$U = \frac{1}{\kappa}U_0$$

$$\kappa = \frac{E_0}{E}$$

$$\vec{E}_i = \left(\frac{1}{\kappa} - 1\right)\vec{E}_0$$

Name______________________________

<u>Part I. 10 questions, 8 points each.</u> For full credit, circle only the correct answer. For half credit, circle the correct answer and one incorrect answer. For ¼ credit, circle the correct answer and two incorrect answers.

1. When plugged into a typical 120 V electrical outlet, a particular light bulb has a standard operating power of 100 W. If the same light bulb is put in a different circuit, you find that the bulb operates with a current that is one half of its standard operating current. The power drawn by the bulb in this circuit is approximately

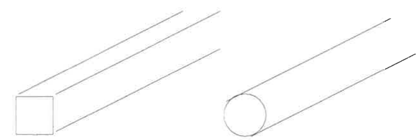
$P = 100\,W = I^2R$

$P' = 25W = I'^2R = (\frac{1}{2}I)^2R = \frac{1}{4}P$

a. 20 W
+8 (b.) 25 W
c. 33 W
d. 40 W
e. 50 W

2. Two pieces of copper wire have the same length, but wire A has a square cross section of width *s* whereas wire B has a circular cross section of diameter *s*. Which of the following statements is true?

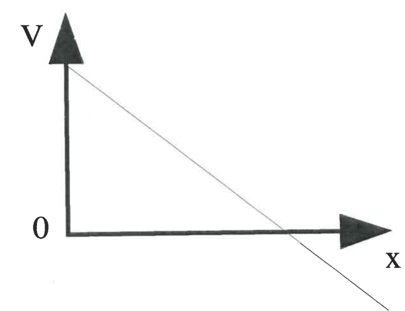
a. The resistance of both wires is the same.
+8 (b.) The resistivity of both wires is the same.
+4 c. Both the resistance and the resistivity of A and B are the same.
d. The resistance of A is greater than that of B.
e. The resistivity of A is greater than that of B.

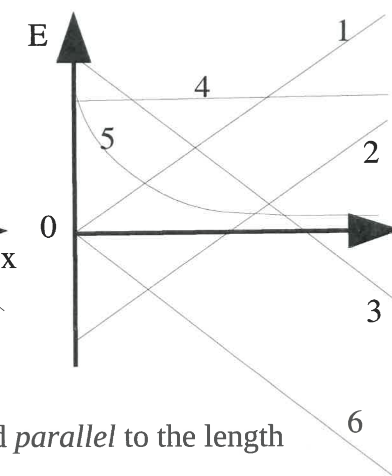


$R = \frac{\rho L}{A}$

3. Which electric field graph best corresponds to the potential in the first graph?

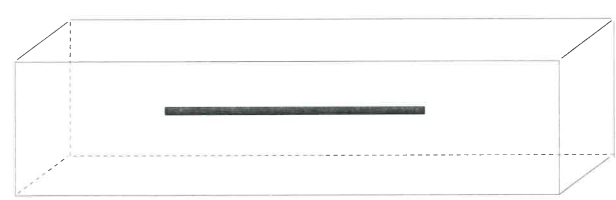
a. 1
b. 2.
c. 3
+8 (d.) 4
e. 5
f. 6



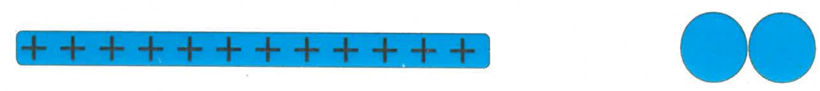


4. A long box of length *l*, width *w*, and height *h* contains a line of positive charge density +λ. The line of charge is oriented *parallel* to the length of the box, is centered within the box, and is length *l*/2. The net electric flux through the box is

a. zero
8 (b.) positive
2 c. negative
d. not enough information given
e. positive through one end, and negative through the other

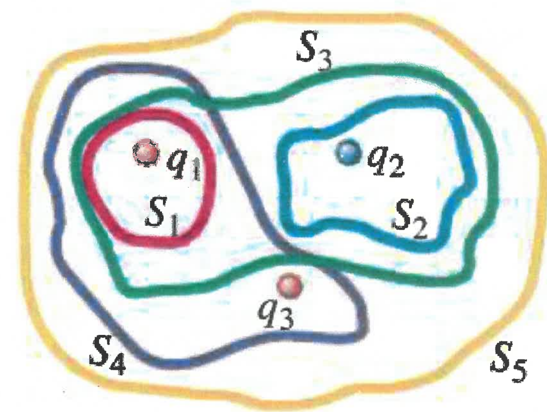


5. If you bring a positively charged insulator near two uncharged metallic spheres that are in contact and then separate the spheres, The sphere on the right will have



a. no net charge.
b. a positive charge.
c. a negative charge.
d. either a positive or a negative charge.
e. None of these is correct.

6. Three point charges have charge q1 = -1 nC, q2 = +2 nC, and q3 = -3 nC. Which of the 5 surfaces has the largest *magnitude* (absolute value) of net electric flux passing through it?

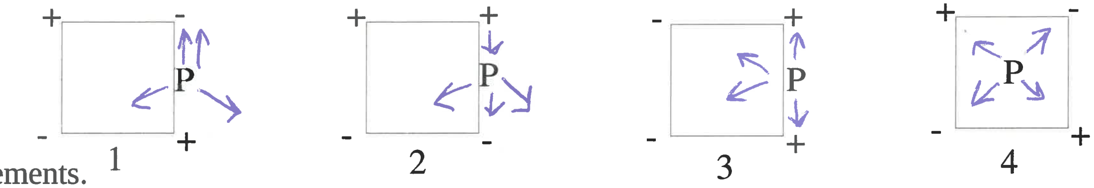


a. S_1
b. S_2
c. S_3
d. S_4
e. S_5
f. Not enough information

7. The charge on each capacitor in a set of capacitors in parallel is
a. directly proportional to its capacitance.
b. inversely proportional to its capacitance.
c. independent of its capacitance.
d. the same.
e. None of these is correct.

8. The capacitance of a capacitor
a. is defined as the amount of work required to move a charge from one plate to the other.
b. decreases if a dieletric is placed between its plates.
c. is independent of the distance between the plates.
d. has units of energy/charge.
e. is independent of the charge on the capacitor.

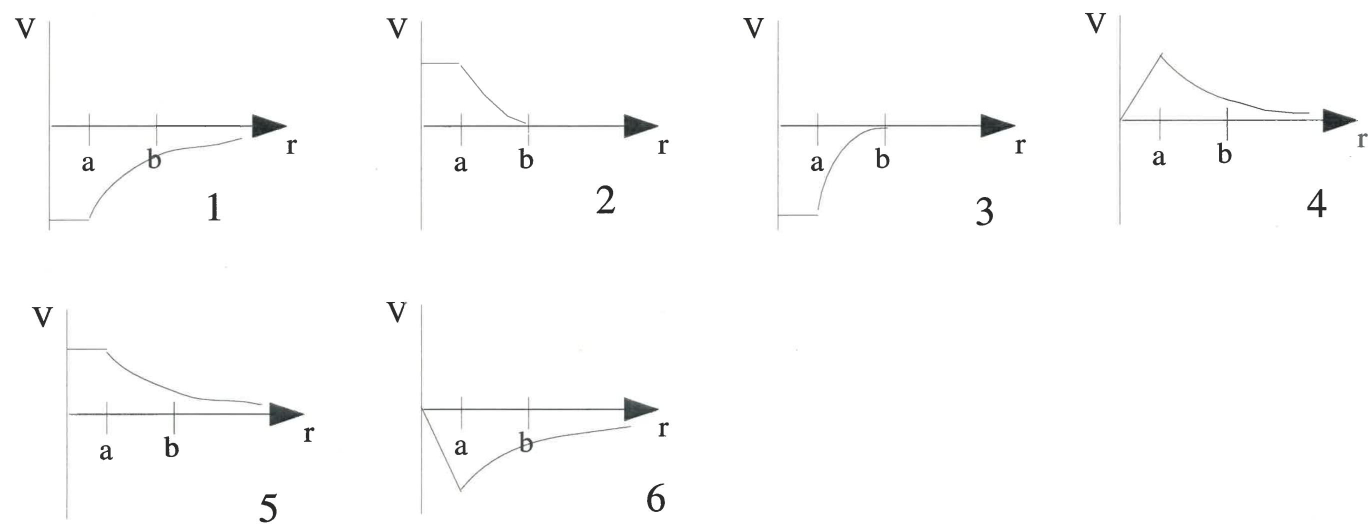
9. Four charges of equal magnitude are arranged at the corners of a square. In which arrangement is the magnitude of the electric field at point P a maximum?

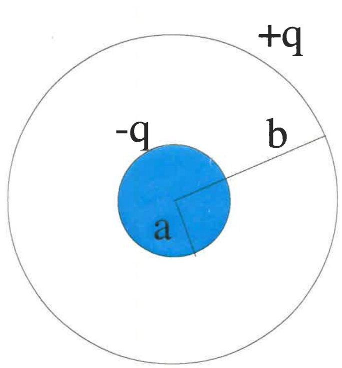


a. 1
b. 2
c. 3
d. 4
e. The maximum electric field occurs in more than one of these arrangements.

10. A spherical shell of radius b contains at its center a metal sphere with radius a. The sphere has charge $-q$ whereas the shell has charge $+q$. Which graph best represents the potential as a function of radius? Assume the potential is zero at infinity.

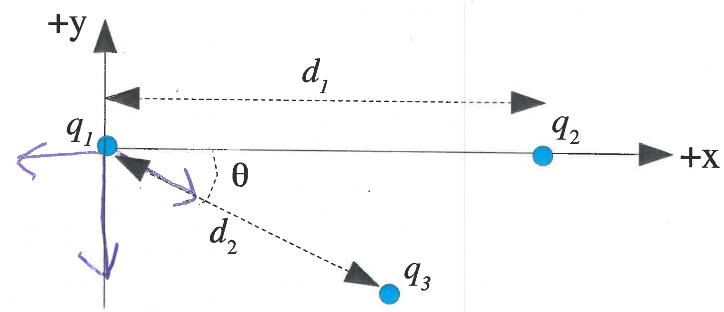
a. 1
b. 2
c. 3
d. 4
e. 5
f. 6





Part II. Short answer/sketch. Answer all four questions as completely as possible.
If you can't answer quantitatively (with numbers), then try to answer qualitatively. **Show your work to earn partial credit!**

11. (20 points)
Charge q_1 is positive and charges q_2 and q_3 are of unknown charge. All three charges are fixed in place. The net electric force on q_1 is in the negative *y*-direction.



a. What is the sign of the charge q_2?

2

positive

b. What is the sign of the charge q_3?

2

negative

c. What is the force on q_1 due to q_2?

4

$\frac{Kq_1q_2}{d_1^2}(-\hat{i})$

d. What is the force on q_1 due to q_3?

8

$\frac{Kq_1q_3}{d_2^2}\left[\cos\theta(+\hat{i}) + \sin\theta(-\hat{j})\right]$

e. Suppose that the *magnitude* of q_3 is $2q_1$. What is q_2?

4

$|F_{12,x}| = |F_{23,x}| \Rightarrow \frac{Kq_1q_2}{d_1^2} = \frac{Kq_1 2q_1\cos\theta}{d_2^2} \Rightarrow \boxed{q_2 = 2q_1\cos\theta\frac{d_1^2}{d_2^2}}$

12. (25 points)
A hollow cylinder of radius *r* and height *h* has a total charge *q* uniformly distributed over its surface. The axis of the cylinder coincides with the *z*-axis, and the cylinder is centered at the origin. We will calculate the electric potential at the origin by summing up the individual potentials due to infinitesimally thin rings.

a. Consider a thin ring of thickness dz at a distance *z* above the origin, as shown. What is the distance from the origin to the ring?

3

$\sqrt{z^2+r^2}$

b. How much charge is contained in the ring?

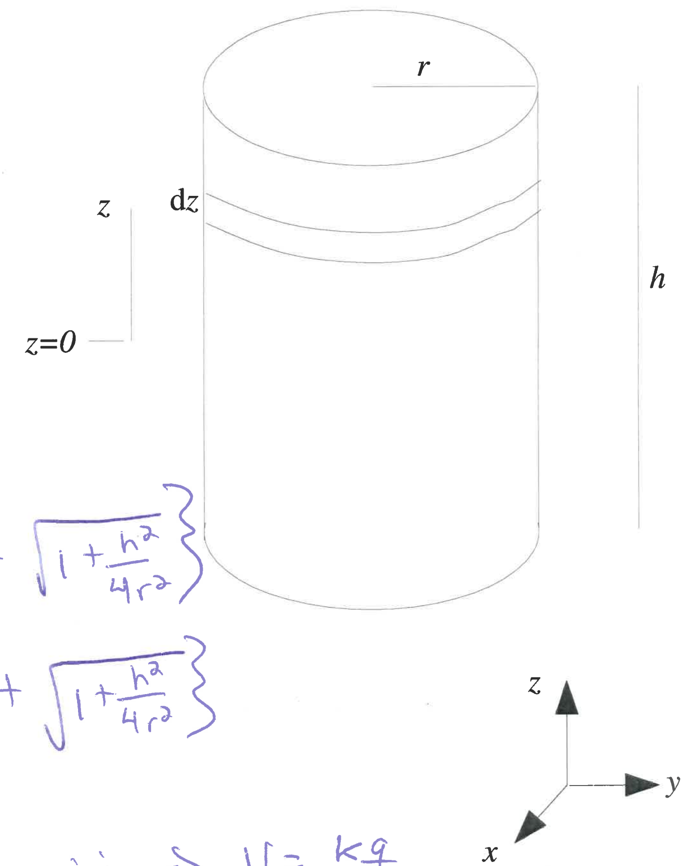
3

$dq = \sigma dA = \frac{q}{2\pi r h} dz \, 2\pi r = \frac{q}{h} dz$

c. What is the potential d*V* at the origin due to this ring?

5

$dV = \frac{k\,dq}{\text{distance}} = \frac{k\, q/h \, dz}{\sqrt{z^2+r^2}}$



d. To obtain the total potential *V*, you need to integrate d*V*. What are the lower and upper limits of this integral?

2

$z = -\frac{h}{2}$ to $+\frac{h}{2}$ or 2 times the integral from 0 to $h/2$

e. Evaluate *V*. You may find one of the following useful:

6

$\int da/(a^2+b^2)^{1/2} = \ln\{a/b+(1+a^2/b^2)^{1/2}\}$ $\int da/(a^2+b^2) = 1/b \tan^{-1}(a/b)$ $\int da/a = \ln(a)$

$V = \int dV = 2\int_0^{h/2} \frac{kq\,dz}{h\sqrt{z^2+r^2}} = 2kq \int_0^{h/2} \frac{dz}{\sqrt{z^2+r^2}} = \frac{2kq}{h} \ln\left\{\frac{h}{2r} + \sqrt{1+\frac{h^2}{4r^2}}\right\}$

$= \frac{q}{2\pi\epsilon_0 h} \ln\left\{\frac{h}{2r} + \sqrt{1+\frac{h^2}{4r^2}}\right\}$

f. What is the potential in the limit that $h \ll r$? This can be solved mathematically or conceptually.

6

conceptually, collapsing to a ring centered on the origin $\Rightarrow V = \frac{kq}{r}$

mathematically, $V \xrightarrow{h \ll r} \frac{2kq}{h}\frac{h}{2r} = \frac{kq}{r}$

13. (25 points)
A ball of insulating material and radius r_a has charge +Q distributed throughout its volume. It is centered within two spherical conducting shells, the inner one with radii r_b and r_c and charge -Q, and the other with radii r_d and r_e and charge +Q.

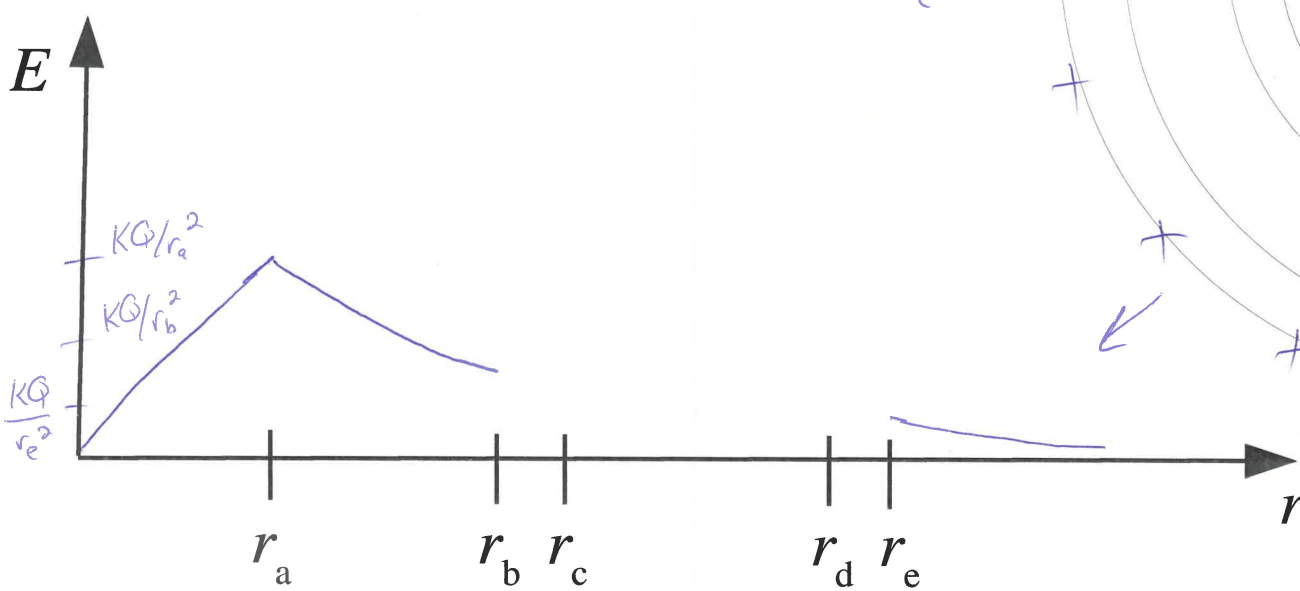
a. Show how the charges are distributed on the two shells. Use '+' for positive charges, '-' for negative charges, and no symbol for neutral regions.

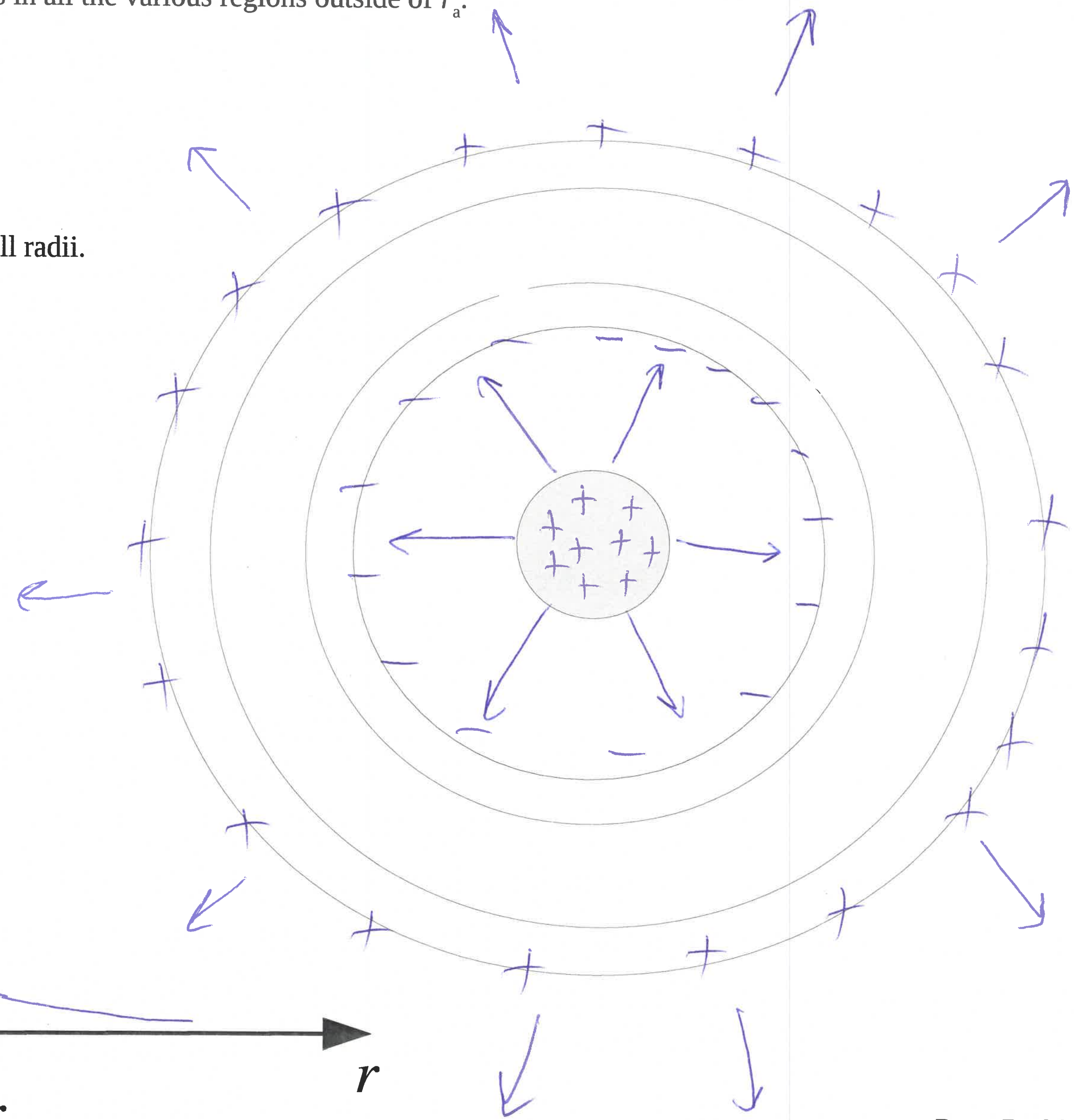
b. Use arrows to indicate the direction of the (non-zero) electric fields in all the various regions outside of r_a.

c. Provide an expression for the electric field beyond r_e:

$E(r>r_e) =$ $\frac{KQ}{r^2}\hat{r}$

d. Use the below graph to plot the amplitude of the electric field for all radii.





14. (25 points)
Three capacitors are connected as shown.

a. What can you say about the potential across C_1 compared to the potential across C_2? nothing

b. What can you say about the charge on C_1 compared to the charge on C_2? they are equal

c. Indicate on the diagram below where the positive and negative charges reside on the capacitors.

d. Write down an expression for combining the three capacitors into one equivalent capacitor C_{123}.

$C_{123} = C_{12} + C_3 = \frac{C_1C_2 + C_1C_3 + C_2C_3}{C_1 + C_2}$ $\quad C_{12} = \frac{C_1C_2}{C_1+C_2}$

e. What is the total energy stored in the system in terms of C_{123}?

$U_{total} = \frac{1}{2} C_{123} \mathcal{E}^2$

f. Which would store more energy: inserting a dielectric into C_1 or C_3? For this part assume all capacitors are originally identical. Justify your answer.

into #1: $\frac{c(k+k+1)}{k+1}$ $\quad \frac{\text{into \#1}}{\text{into \#2}} = \frac{(2k+1)/(k+1)}{(2k+1)/2} = \frac{2}{k+1} < 1 \Rightarrow$ insert into #3

into #3: $\frac{c(1+k+k)}{2}$

g. Find expressions for the charges on each capacitor.

$Q_1 = C_{12}\mathcal{E}$

$Q_2 = C_{12}\mathcal{E}$

$Q_3 = C_3\mathcal{E}$