# Equations from the end of each chapter

$$V_{\text{terminal}} = \varepsilon - Ir_{\text{eq}}$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i$$

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R2} + \cdots + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1}$$

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

$$\sum V = 0$$

$$V_{\text{terminal}} = \sum_{i=1}^{N} \varepsilon_i - I\sum_{i=1}^{N} r_i = \sum_{i=1}^{N} \varepsilon_i - Ir_{\text{eq}}$$

$$V_{\text{terminal}} = \varepsilon - I\sum_{i=1}^{N}\left(\frac{1}{r_i}\right)^{-1} = \varepsilon - Ir_{\text{eq}}$$

$$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\tau = RC$$

$$I = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = I_o e^{-\frac{t}{RC}}$$

$$q(t) = Qe^{-\frac{t}{\tau}}$$

$$I(t) = -\frac{Q}{RC}e^{-\frac{t}{\tau}}$$

$$C = \frac{Q}{V}$$

$$I_{\text{ave}} = \frac{\Delta Q}{\Delta t}$$

$$1\ \text{A} = 1\ \text{C/s}$$

$$I = \frac{dQ}{dt}$$

$$\vec{\mathbf{F}} = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$F = qvB\sin\theta$$

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB}$$

$$\vec{\mathbf{F}} = I\,\vec{\mathbf{l}} \times \vec{\mathbf{B}}$$

$$\vec{\boldsymbol{\mu}} = NIA\,\hat{\mathbf{n}}$$

$$\vec{\tau} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$$

$$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$$

$$v_d = \frac{E}{B}$$

$$V = \frac{IBl}{neA}$$

$$V = Blv_d$$

$$\frac{q}{m} = \frac{E}{BB_0R}$$

$$v_{\text{max}} = \frac{qBR}{m}$$

$$\mu_0 = 4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A}$$

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\int_{\text{wire}} \frac{Id\,\vec{\mathbf{l}} \times \hat{\mathbf{r}}}{r^2}$$

$$B = \frac{\mu_0 I}{2\pi R}$$

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$B = \frac{\mu_0 I}{2R}\ \text{(at center of loop)}$$

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{l}} = \mu_0 I$$

$$B = \mu_0 nI$$

$$B = \frac{\mu_o NI}{2\pi r}$$

$$\mu = (1 + \chi)\mu_0$$

$$B = \mu nI$$

$$\Phi_{\text{m}} = \int_S \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\,dA$$

$$\varepsilon = -N\frac{d\Phi_{\text{m}}}{dt}$$

$$\varepsilon = Blv$$

$$\varepsilon = \oint \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{l}} = -\frac{d\Phi_{\text{m}}}{dt}$$

$$\varepsilon = NBA\,\omega\,\sin(\omega t)$$

$$v_d = \frac{I}{nqA}$$

$$I = \iint_{\text{area}} \vec{\mathbf{J}} \cdot d\,\vec{\mathbf{A}}$$

$$\rho = \frac{E}{J}$$

$$V = IR$$

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

$$R \equiv \frac{V}{I}$$

$$R = \rho\frac{L}{A}$$

$$R = R_0(1 + \alpha\Delta T)$$

$$P = IV$$

$$P = I^2R = \frac{V^2}{R}$$

Part I. Questions 1-10. 8 points each. Multiple choice: For **full credit**, circle only the **correct** answer. For **half credit**, circle the **correct** answer **and one incorrect** answer. For ¼ **credit,** circle the **correct** answer **and two incorrect** answers.

**1**. When plugged into a typical 110 V electrical outlet, a particular light bulb has a standard operating power of 100 W. If the same light bulb is put in a different circuit, you find that the bulb operates with a current that is one half of its standard operating current. The power drawn by the bulb in this circuit is approximately

a. 20 W
b. 25 W
c. 33 W
d. 40 W
e. 50 W

**2**. Two pieces of copper wire have the same length, but wire A has a square cross section of width *s* whereas wire B has a circular cross section of diameter *s*. Which of the following statements is true?

a. The resistance of both wires is the same.
b. The resistivity of both wires is the same.
c. Both the resistance and the resistivity of A and B are the same.
d. The resistance of A is greater than that of B.
e. The resistivity of A is greater than that of B.


**3**. Resistors $R_1$ and $R_2 = 2R_1$ are connected in parallel to an emf source that has negligible internal resistance. If resistor $R_2$ is removed from the circuit, the current through $R_1$ would

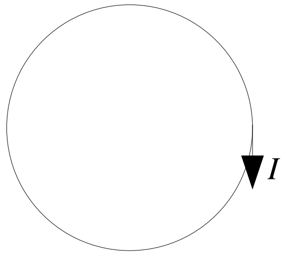
a. increase by a factor of 1.5.
b. increase by a factor of 2.
c. decrease by a factor of 1.5.
d. decrease by a factor of 2.
e. None of the above.

**4**. A resistor of resistance *R*, a capacitor of capacitance *C*, and a battery with emf ε are all connected in series. The capacitor achieves a certain maximum charge after a long time. Another circuit is identical except that it has two resistors of resistance *R* in series. The maximum charge on the capacitor in this second circuit would be _____________ as that in the first circuit.

a. twice as large
b. half as large
c. the same
d. four times as large
e. one fourth as large

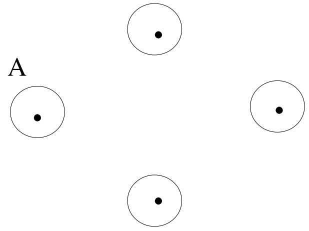
**5**. An elastic, conducting loop of wire carrying current $I$ is placed on a horizontal table. As viewed from above, the current is circulating clockwise. If a uniform magnetic field was introduced pointing straight up out of the table, the loop would

a. slightly expand.
b. slightly contract.
c. remain the same size.
d. reverse its current direction.
e. None of the above.



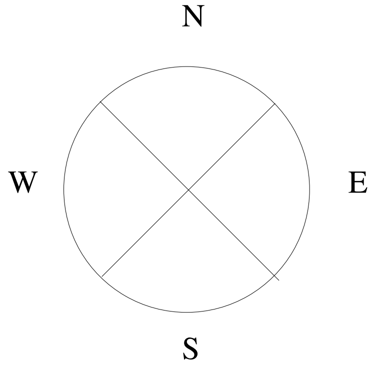

**6**. Four long parallel wires each with current I (coming out of the page) are symmetrically arranged as shown. The net force on wire A is directed

a. to the left
b. to the right
c. up
d. down
e. none of these




**7**. A long wire carries an electric current into the page. What is the direction of the magnetic field due to the current, north of the wire?

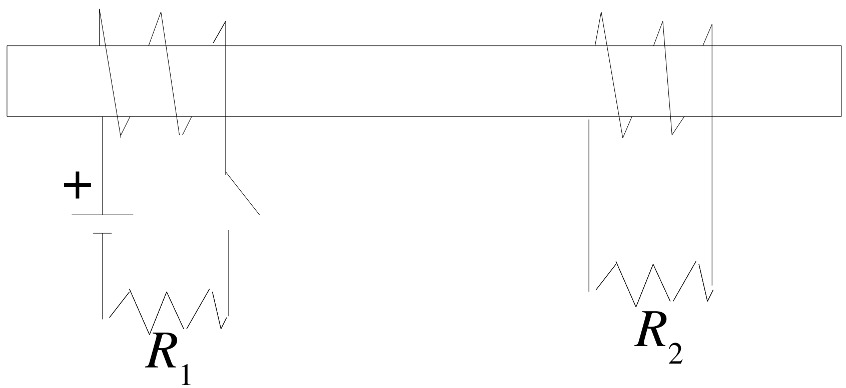
a. north
b. east
c. west
d. south
e. out of the page
f. into the page



**8**.A metallic weight is suspended from a metal spring.  If now a current is passed through the spring,
a. the spring will contract, raising the weight.
b. the spring will elongate, lowering the weight.
c. the weight will not move.
d. whether or not the weight moves up or down depends on what the weight is made of (i.e. whether or not it is magnetizable).
e. None of these is correct.

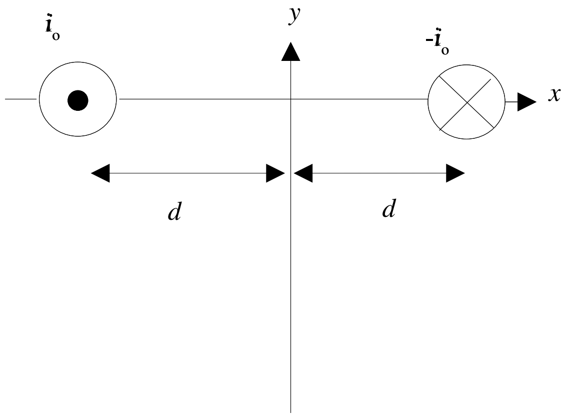
**9**. An airplane is in level flight over Antarctica, where the magnetic field of the earth is mostly directed upward, away from the ground.  As viewed by a passenger facing toward the front of the plane, which wingtip is at a higher electric potential?
a. the left wingtip
b. the right wingtip
c. both wingtips would be at the same potential
d. It depends on which direction the airplane is flying.
e. None of these is correct.

**10**.  Two separate circuits wrap around a metal bar.  The two circuits are electrically isolated from each other since the metal bar is wrapped in a rubber coating.  Just after the switch is closed, how do the currents $I_1$ and $I_2$ flow through the resistors $R_1$ and $R_2$?
a.  $I_1$ flows to the left, $I_2$ flows to the left
b.  $I_1$ flows to the left, $I_2$ flows to the right
c.  $I_1$ flows to the left, $I_2$ is zero
d.  $I_1$ flows to the right, $I_2$ flows to the left
e.  $I_1$ flows to the right, $I_2$ flows to the right
f.  $I_1$ flows to the right, $I_2$ is zero
g.  $I_1$ is zero, $I_2$ is zero



Part II. Short answer/sketch. Answer questions 11-14 as completely as possible. **Show your work to earn partial credit!**

**11**. (25 points)
Two long straight wires carry equal but opposite currents along the $z$-axis (perpendicular to the page) as shown.



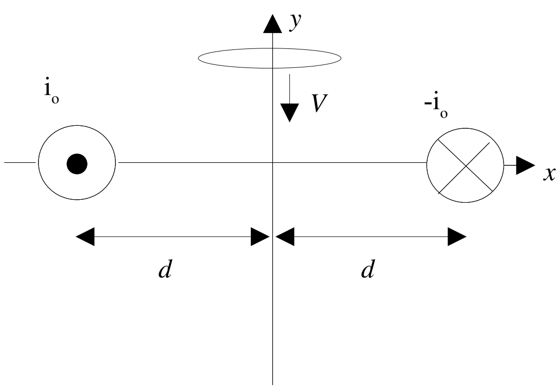

a) At what point on the $y$-axis is the total magnetic field $B$ produced by the wires largest? What is its value $B_{max}$ there, expressed in terms of $i_o$, $d$, and anything else you need? What is the direction of $B$ at this point?

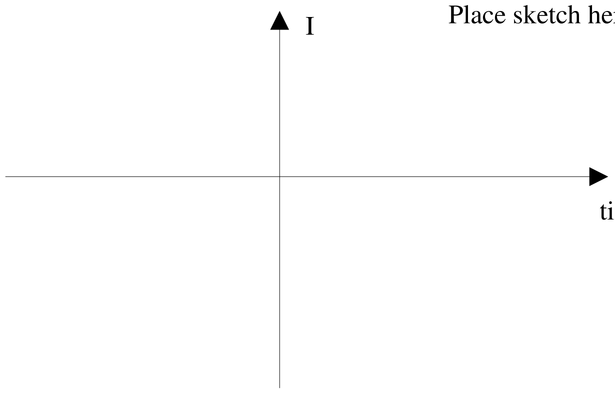
$y$ where $B$ is maximum=____________

direction of $B$ there:________________

$B_{max}$=____________________________

b) A conducting wire loop is lowered at constant speed down the y-axis, toward the two current-carrying wires in part a, as shown. Sketch the current $I$ induced in the loop as a function of time, for $-\infty<t<+\infty$. Assume that the loop crosses $y$=0 at $t$=0, and that a positive current corresponds to counterclockwise current flow as viewed from above the loop. Just sketch the shape; you don't need to label particular values.




c) Does the loop feel a force as it moves toward the wires from above?

d) If you answered "yes" to d), how is this force produced, and what is its direction?
If you answered "no" to d), explain why the loop feels no force.

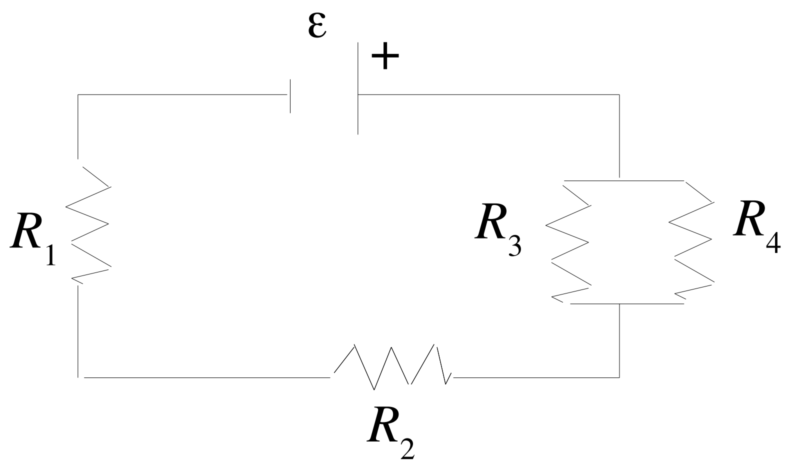
**12**. (25 points)
The circuit below contains four resistors and a battery. The current through each resistor is $I_1$, $I_2$, $I_3$, and $I_4$.
a. Circle the relations that are always correct: $I_1=I_2$; $I_2=I_3$; $I_3=I_4$; $I_1=I_4$; $I_1=I_2+I_3$

b. Combine $R_3$ and $R_4$ into one equivalent resistor. Call this $R_{34}$.

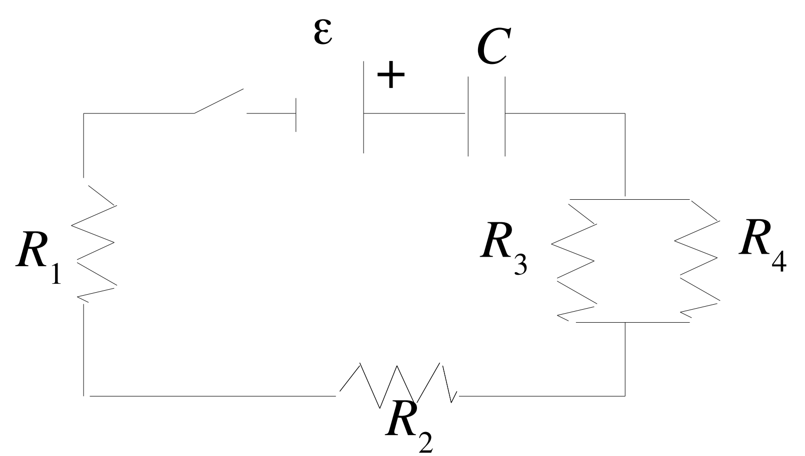
c. Combine all the resistors into one equivalent resistor. Call this $R_{1234}$.

d. Compute $I_4$ in terms of $R_1$, $R_2$, $R_3$, $R_4$, $\varepsilon$ and no other parameters.

e. You want to maximize the lifetime of the battery, and you are free to remove one resistor only. If all four resistors are equal, which one would you remove? Justify your answer. $P=IV=I^2R=V^2/R$

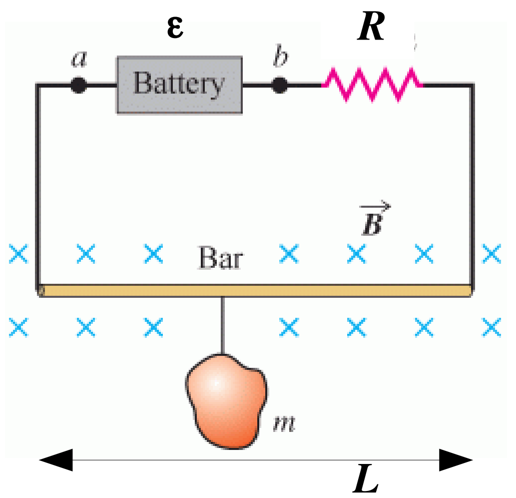



f. A switch and a capacitor of capacitance $C$ are added to the circuit. How long after closing the switch will it take to charge up the capacitor to ≈63.2% of its final value?



**13**. (25 points)
A magnetic balance is used to weigh objects. The mass $m$ to be measured is hung from the center of the bar of length $L$ suspended in a uniform magnetic field $B$ directed into the page. The battery voltage $\varepsilon$ can be adjusted to vary the current in the circuit. The horizontal bar is made of extremely lightweight conducting material, and is connected to the battery by thin elastic conducting wire.

a) To measure the mass, an upward force must balance the downward force due to gravity. What is this upward force? You can either use words or an equation for your answer.

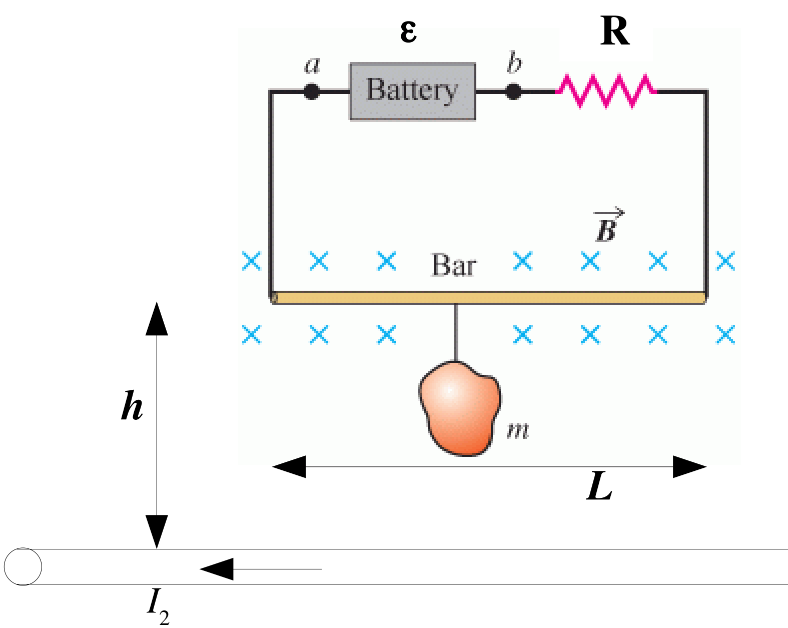



b) Which point, $a$ or $b$, should be the positive terminal of the battery?

c) If the maximum terminal voltage of the battery is $\varepsilon_{max}$, what is the greatest mass $m_{max}$ that this instrument can measure?

d) Now suppose a long wire with current $I_2$ is placed at a fixed distance $h$ directly below the magnetic balance. How does this additional current affect the net magnetic field at the location of the bar? (i.e., stronger, weaker, same, same but different direction, ...)

e) Derive an expression for the new maximum mass that can be measured in this situation.



**14**. (25 points)
A long metal bar is pulled to the right at a steady speed perpendicular to a uniform magnetic field. The bar rides on parallel metal rails connected through a resistor, so the apparatus makes a complete circuit. Ignore the resistance of the rails.

a) Calculate the magnitude of the emf induced in the circuit (in terms of the variables given in the figure below).

b) What is the direction of the current induced in the circuit?

c) Compute the current through the resistor (in terms of the variables in the figure below).

d) Repeat part c) with numbers:
- bar resistivity $=1.72 \cdot 10^{-8}\ \Omega$ m
- bar radius = 0.1 mm
- bar length = 3.50 m
- bar speed= 5.0 m/s
- magnetic field = 0.750 T
- resistor R=1.00 $\Omega$