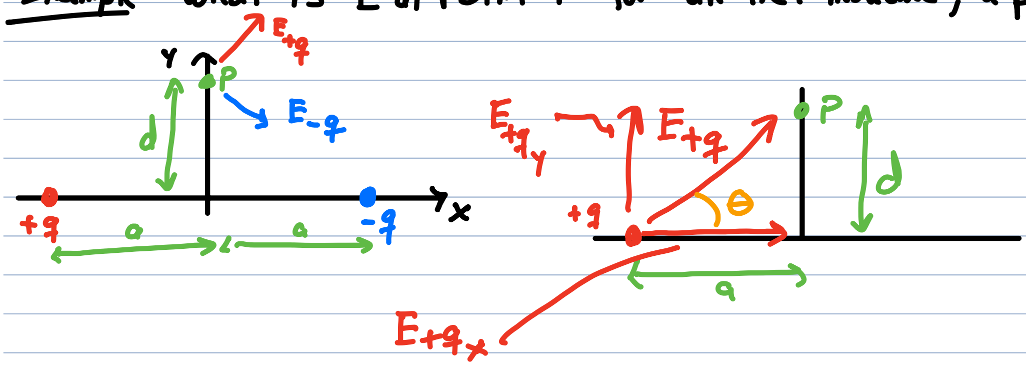
Finishing Chapter 5: Electric charges and fields

Example what is $\vec{E}$ at Point P for an HCl molecule, a permanent "dipole"?
H^+ Cl^-



$$|E_{-q}| = |E_{+q}| = \frac{kq}{r^2} = \frac{kq}{a^2+d^2}$$

x-component $E_{+q_x} = \quad = |E_{+q}| \cos\theta \quad = |E_{+q}| \frac{a}{\sqrt{a^2+d^2}}$

$$E_{-q_x} = |E_{-q}| \cos\theta = |E_{-q}| \frac{a}{\sqrt{a^2+d^2}}$$

$$\rightarrow E_x = E_{+q_x} + E_{-q_x} = \frac{2kq}{a^2+d^2}\frac{a}{\sqrt{a^2+d^2}} = \frac{2kqa}{(a^2+d^2)^{3/2}}$$

$d=0$?
$\rightarrow E_x = 2kq/a^2$

y-component $E_{+q_y} = |E_{+q}| \sin\theta$
$E_{-q_y} = -|E_{-q}| \sin\theta$
$\Big\} \; E_y = E_{+q_y} + E_{-q_y} = 0$

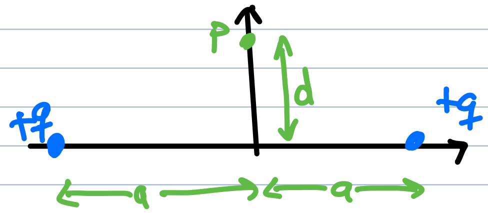
$$\rightarrow E_{net} = \sqrt{E_x^2 + E_y^2} = E_x \xrightarrow{d \gg a} \frac{2kqa}{d^3\left(\frac{a^2}{d^2}+1\right)^{3/2}} \rightarrow \frac{2kqa}{d^3}$$

with numbers: $a \sim 10^{-10}$ m $\quad d \sim 10^{-8}$ m $\quad q = 1.60 \cdot 10^{-19}$ C

$\rightarrow E \sim 3 \cdot 10^5$ N/C

Breakdown of air $3 \cdot 10^6$ N/C
Photocopier drum 10^5 N/C
copper wiring in household circuit 10^{-2} N/C
charged comb 10^3 N/C

challenge: Find E_{net} for 2 positive charges



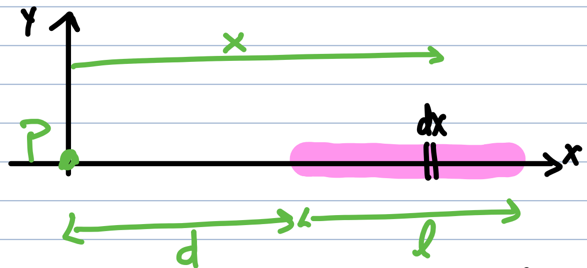
$$E_{net} = E_y = \frac{2kqd}{(a^2+d^2)^{3/2}}$$

Electric field of a continuous charge distribution

$$\vec{E} = k\int \frac{dq}{r^2}\hat{r} = \int d\vec{E}$$

	symbol	MKS unit	differential charge	Picture
charge	q	C	dq	•
linear ch. density	λ	C/m	$dq = \lambda\, dx$	—
surface ch. density	σ	C/m^2	$dq = \sigma\, dA$	
volume ch. density	ρ	C/m^3	$dq = \rho\, dV$	

Example $\vec{E}$ due to a positively-charged rod $\left(\frac{\text{charge}}{\text{length}}\ \lambda\right)$



$$dE = \frac{k\,dq}{x^2} = \frac{k\lambda\, dx}{x^2}$$

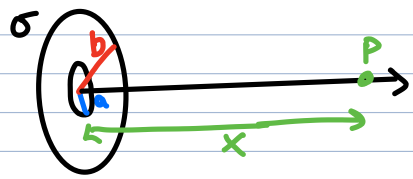
(gets equation in terms of one unknown)

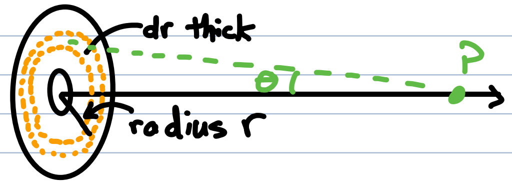
$$E = \int d\vec{E} = \int_{x=d}^{x=d+\ell} \frac{k\lambda\, dx}{x^2} = k\lambda \int_d^{d+\ell} \frac{dx}{x^2} = k\lambda\left[\frac{-1}{x}\right]_d^{d+\ell}$$

$$= k\lambda\left[-\frac{1}{d+\ell} - -\frac{1}{d}\right] = \frac{k\lambda\ell}{d(d+\ell)}$$

$$\rightarrow \vec{E} = \frac{k\lambda\ell}{d(d+\ell)}(-\hat{\imath}) = \frac{k\lambda\ell}{d^2}\frac{1}{\left(1+\frac{\ell}{d}\right)} \xrightarrow{d \gg \ell} \frac{k\lambda\ell(-\hat{\imath})}{d^2} = \frac{kQ}{d^2}(-\hat{\imath})$$

Example $\vec{E}$ due to a uniformly and positively-charged annulus





$dq = \sigma dA$

[by symmetry, $E_y = 0$]

$$dE = k\frac{dq}{\text{distance}^2} = k\frac{2\pi r\,dr\,\sigma}{x^2+r^2}$$

$$dE_x = dE\cos\theta = \frac{k\,2\pi r\,dr\,\sigma}{r^2+x^2}\frac{x}{\sqrt{r^2+x^2}} = \frac{k\,2\pi r\,dr\,\sigma\,x}{(r^2+x^2)^{3/2}}$$

$$E = \int dE = \int dE_x = k\pi\sigma x\int_a^b \frac{2r\,dr}{(x^2+r^2)^{3/2}} = k\pi\sigma x\frac{(x^2+r^2)^{-1/2}}{-1/2}\Bigg|_a^b$$

$$= 2k\pi\sigma x\left[-\frac{1}{\sqrt{x^2+b^2}} - -\frac{1}{\sqrt{x^2+a^2}}\right]$$

if $a=0$ $\vec{E} = 2k\pi\sigma x\left[\frac{1}{x} - \frac{1}{x\sqrt{1+\frac{b^2}{x^2}}}\right] \xrightarrow[x\gg b]{} 2\pi k\sigma\frac{x}{x}\left[1-\left(1-\frac{1}{2}\frac{b^2}{x^2}\right)\right]$

$\rightarrow 2\pi k\sigma\frac{1}{2}\frac{b^2}{x^2}$

$\rightarrow \frac{kQ}{x^2}$

$(1+x)^n$ if $x \ll 1 \rightarrow 1+nx$