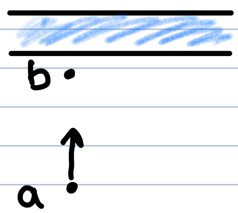
Example

A long charged rod has lambda = 5.00 10^-12 C/m. A positive charge is shot toward it at 1.5 km/s from a distance of 18.0 cm. How close does it get?

$q = 1.60 \cdot 10^{-19}\,C \qquad m = 1.67 \cdot 10^{-27}\,kg$



$V_a = 1.50\ km/s$

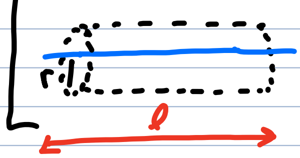
$\Delta K + \Delta U = 0 \rightarrow \Delta K = \frac{1}{2} m \cancel{V_b^2}^{\,0} - \frac{1}{2} m V_a^2 = -\frac{1}{2} m V_a^2$

$\Delta U = U_b - U_a = q\,\Delta V = q\,(V_b - V_a)$

$\Rightarrow \Delta K = -q(V_b - V_a) = -\frac{1}{2} m V_a^2$

also, $V_b - V_a = \int_b^a \vec{E}\,d\vec{l}$

[$E(r)$ for a long line of charge:



Gaussian cylinder $\quad \int \vec{E} \cdot d\vec{A} = Q_{encl}/\epsilon_0 \qquad Q_{encl} = \lambda l$

$\int \vec{E}\,d\vec{A} = E \int dA = E\, 2\pi r l \Rightarrow E(r) = \frac{\lambda}{2\pi\epsilon_0 r}$]

$V_b - V_a = \int_b^a \vec{E}\,d\vec{l} = \int_b^a E(r)\,dr = \frac{\lambda}{2\pi\epsilon_0} \int_b^a \frac{dr}{r} = \frac{\lambda}{2\pi\epsilon_0} \ln r_a/r_b$

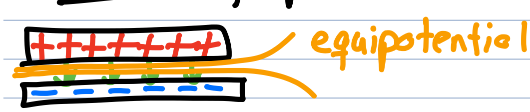
$\Rightarrow \frac{\lambda}{2\pi\epsilon_0} \ln r_a/r_b = V_b - V_a = 0.0117\,V$

$\Rightarrow r_b = r_a\, e^{-2\pi\epsilon_0 (V_b - V_a)/\lambda} = 0.158\,m$

note on signs:

$W_{1\to 2} = \Delta K = K_2 - K_1 = -\Delta U = -(U_2 - U_1) = -(V_2 - V_1)\,q_0 = (V_1 - V_2)\,q_0$

lines of equipotential



E is $\perp$ to equipotential

$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y}$

Suppose lower plate is "a" and at 0 Volts.
upper plate "b" is at 100 Volts. $\Delta y = 1m$

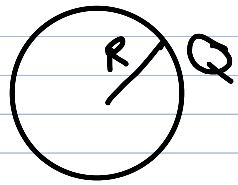
$$\Delta V = V_b - V_a = 100V$$

Q: what is an equation for $V(y)$?

A: $V(y) = 100\, y(m)$ Volts

$$E_x = -\frac{\partial V}{dx} = 0 \qquad E_y = -\frac{\partial V}{\partial y} = -100 \frac{V}{m}\,\hat{j}$$

<u>example</u> solid conducting sphere, charge Q, radius R



<u>E(r>R)</u> $\vec{E} = kQ/r^2$ and chose $V(r=\infty)=0$

$$V(r) - V(r\to\infty) = V(r) - 0 = -\int_{\infty}^{r} \vec{E}\cdot\vec{dl}$$

$$= -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,\hat{r}\cdot dr\,\hat{r} = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0} Q \frac{dr}{r^2}$$

$$= -\frac{Q}{4\pi\epsilon_0}\frac{1}{r}\Big|_r^{\infty} = 0 - -\frac{Q}{4\pi\epsilon_0 r} = \frac{kQ}{r}$$

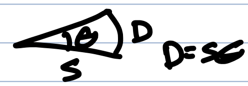
<u>E(r<R)</u> $= 0$ $\Rightarrow$ work done in moving a charge anywhere within $r<R$ is zero.

Potential same everywhere $\left(E_r = -\frac{\partial V}{\partial r}\right)$

and equals $\frac{kQ}{R}$ $\left(\frac{kQ}{r}\Big|_{r=R}\right)$

<u>example</u> ch 5 #90

Find E and V at P



arc length is $2\theta_0 R$

$$\lambda = \frac{Q}{2\theta_0 R} \qquad dq = \lambda R\, d\beta$$

$$dE = \frac{k\,dq}{dist^2} = \frac{k\lambda R\,d\beta}{R^2} = \frac{k\lambda\,d\beta}{R}$$

$$E_x = \int dE_x = 0 \qquad E_y = \int dE_y = -2\int_0^{\theta_0} dE\cos\beta\,\hat{j} = \frac{-2k\lambda}{R}\int_0^{\theta_0} d\beta\cos\beta\,\hat{j}$$

$$= -\frac{2k\lambda}{R}\sin\theta_0(\hat{j}) = -\frac{kQ}{R^2\theta_0}\sin\theta_0(\hat{j})$$

<u>Now find V</u> $V = \int dV = \int \frac{k\,dq}{R} = \frac{k}{R}\int dq = \frac{k}{R}Q$

OR $V = \int dV = \int \frac{k\,dq}{R} = 2\int_0^{\theta_0} \frac{k\lambda R\,d\beta}{R} = 2k\lambda\int_0^{\theta_0} d\beta = 2k\lambda\theta_0 = \frac{kQ}{R}$

white board challenge:

1.) Find V at center of a ring of charge (radius r, charge +Q)

2) Find V at center – same as #1, but also a 2nd concentric ring of charge -2Q and radius 2r

1) $V = \int dV = \int \frac{k\,dq}{r} = \frac{k}{r}\int dq = \frac{kQ}{r}$

2) $V = \int dV = \int_0^{Q} \frac{k\,dq}{r} + \int_0^{-2Q} \frac{k\,dq}{2r} = \frac{kQ}{r} + \frac{-2kQ}{2r} = 0$

<u>example ch5 #95</u>

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