Radiatively-Excited Regions

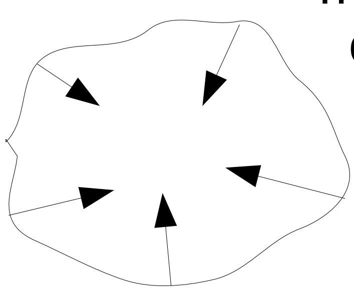
A high temperature star will ionize the surrounding gas, creating what is known as an HII region. In the special case where this gaseous envelope has been ejected by an evolved star, the resulting nebula is called a planetary nebula. Technically, SNe also create HII regions through photo- and collisional-ionization.

	HII regions	Planetary Nebulae
T_* (K)	30,000-50,000	80,000-600,000
n_H (cm^{-3})	$10\text{-}10^5$	$10^3\text{-}10^7$
T_{e-} (K)	5,000-10,000	10,000-95,000

Why are the gaseous envelopes surrounding PNe hotter?

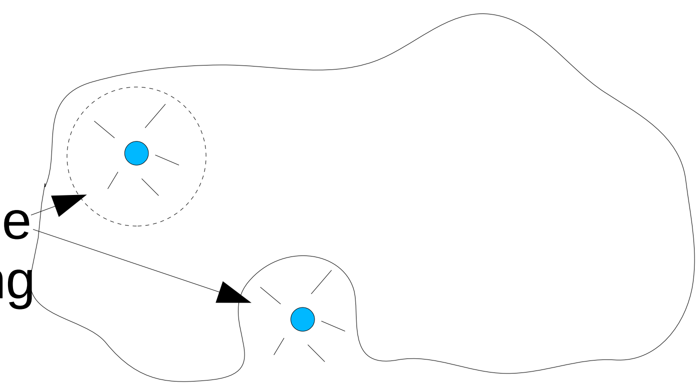
HII regions are the remnant material left over from the process of star formation that is ionized by the newly-formed star.

HII region:



molecular cloud collapses

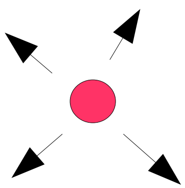
newly-formed stars ionize the surrounding material



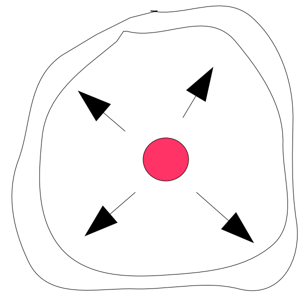
10^4 K maintained by excess kinetic energy of photo-ionization. Cooling by O^+, O^{++}, N^+

Radiatively-Excited Regions

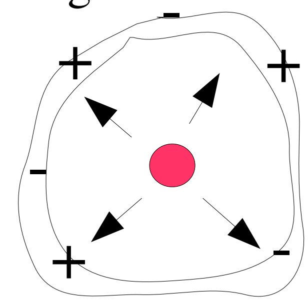
Planetary nebula:



Red giant forms as star evolves and expands



ejected material



material ionized

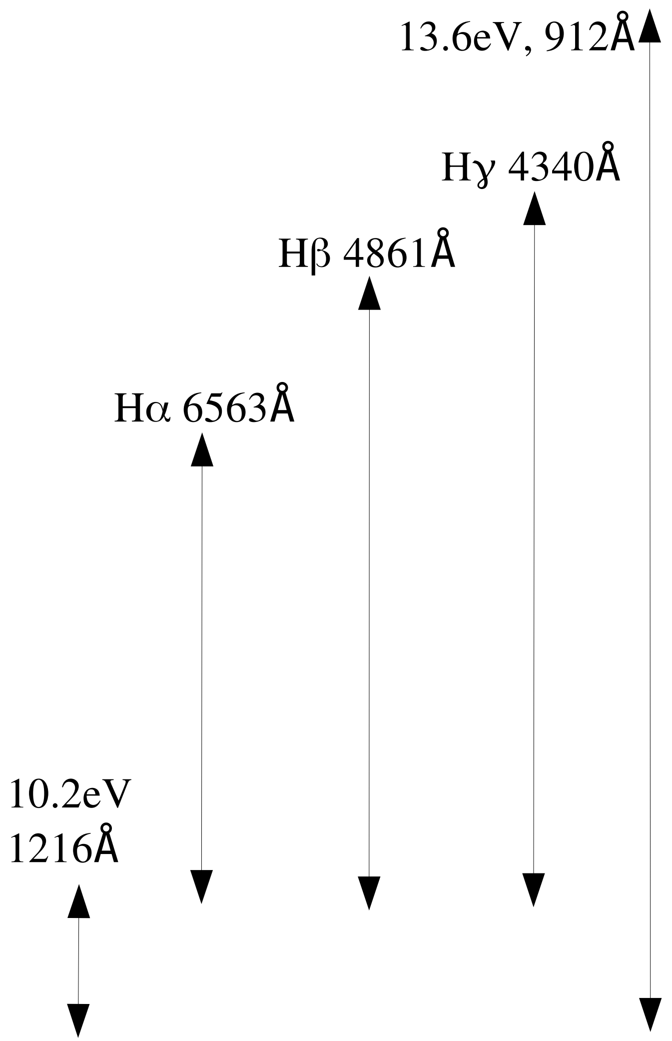
We will cover how HII regions form, what their emitted spectrum is like, and how they serve as useful probes of galaxies. Although we will specifically apply our methods to HII regions, almost all that we derive will be applicable to planetary nebulae as well as many other types of ionized nebulae.

The Plan:
A. Scope of the problem --- inputs needed
B. Photoionization ⇔ recombination equilibrium
C. Thermal equilibrium
D. Emitted radiation
- Free-free (Brehmsstrahlung)
- Recombination lines
- Fine structure lines

E. Remote diagnostics
- Temperatures
- Densities
- Abundances

$A \sim 10^4$ to 10^8 s^{-1} ⇒ lifetimes ~ 10^{-4} to 10^{-8} s. Given that the mean lifetime of an H atom against photo-ionization is 10^6 s, this implies that HI is typically in its ground state & thus ionized by λ<912Å or v>50 km/s

13.6eV	⇔	912Å	⇔	1.6 10^5 K
1eV	⇔	1.2μm	⇔	1.2 10^4 K



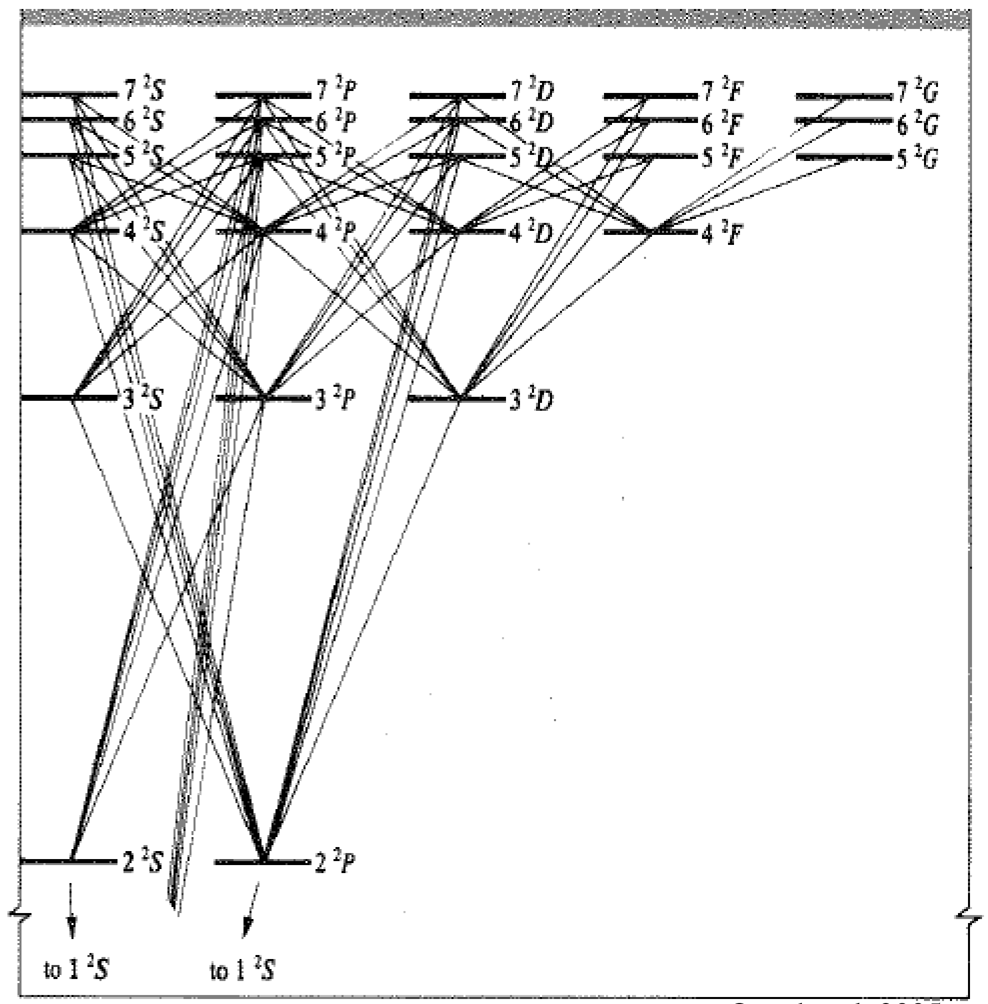


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FIGURE 2.1
Partial energy-level diagram of H I, limited to $n \leq 7$ and $L \leq$ G. Permitted radiative transitions to levels $n \leq 4$ are indicated by solid lines.

Radiatively-Excited Regions

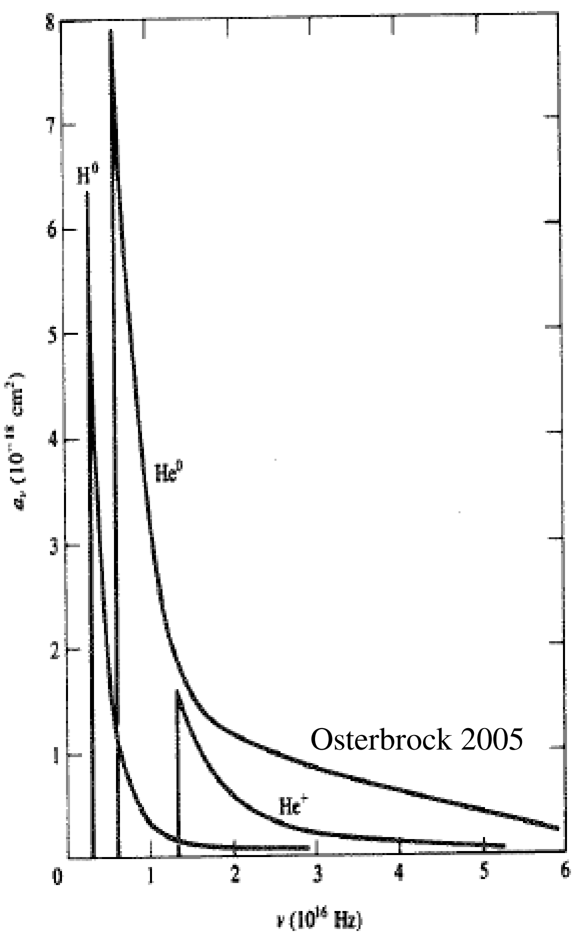
<u>Inputs Needed</u>

'Nebular Properties'

1. Stellar Properties
   - Atmospheric emission — F_ν
   - radius — R_*
2. Density Structure — $n_H(r,\theta,\phi)$
3. Impurity Abundances — n_x/n_H

'Atomic Properties'

1. Photo-ionization cross-sections — $a_\nu(\chi^i)$ or σ_{bf}
2. Recombination Rates — $\alpha_n(\chi^i)$ or β_n
3. Collision Strengths — Ω_{ij}
4. Transition Probabilities — A_{ij}



<u>Outputs</u>

1. Ionization equilibrium
2. Thermal equilibrium
3. Emitted spectrum

TABLE 2.1

Recombination coefficients[a] $\alpha_{n\ ^2L}$ for H

$\alpha_{n\ ^2L}$	T = 5000° K	10,000° K	20,000° K
$\alpha_{1\ ^2S}$	2.28×10^{-13}	1.58×10^{-13}	1.08×10^{-13}
$\alpha_{2\ ^2S}$	3.37×10^{-14}	2.34×10^{-14}	1.60×10^{-14}
$\alpha_{2\ ^2P}$	8.33×10^{-14}	5.35×10^{-14}	3.24×10^{-14}
$\alpha_{3\ ^2S}$	1.13×10^{-14}	7.81×10^{-15}	5.29×10^{-15}
$\alpha_{3\ ^2P}$	3.17×10^{-14}	2.04×10^{-14}	1.23×10^{-14}
$\alpha_{3\ ^2D}$	3.03×10^{-14}	1.73×10^{-14}	9.09×10^{-15}
$\alpha_{4\ ^2S}$	5.23×10^{-15}	3.59×10^{-15}	2.40×10^{-15}
$\alpha_{4\ ^2P}$	1.51×10^{-14}	9.66×10^{-15}	5.81×10^{-15}
$\alpha_{4\ ^2D}$	1.90×10^{-14}	1.08×10^{-14}	5.68×10^{-15}
$\alpha_{4\ ^2F}$	1.09×10^{-14}	5.54×10^{-15}	2.56×10^{-15}
$\alpha_{10\ ^2S}$	4.33×10^{-16}	2.84×10^{-16}	1.80×10^{-16}
$\alpha_{10\ ^2G}$	2.02×10^{-15}	9.28×10^{-16}	3.91×10^{-16}
$\alpha_{10\ ^2M}$	2.7×10^{-17}	1.0×10^{-17}	$4. \times 10^{-18}$
α_A	6.82×10^{-13}	4.18×10^{-13}	2.51×10^{-13}
α_B	4.54×10^{-13}	2.59×10^{-13}	2.52×10^{-13}

[a] In cm^3 sec^{-1}.

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Photo-ionization

excess energy $h\nu - h\nu_0$

$h\nu_0$ = threshhold energy

We characterize this process by a cross-section

a_ν = photo-ionization absorption cross-section due to a photon of frequency ν.

The photo-ionization rate per target = $\int d\nu \;\; 4\pi J_\nu a_\nu / h\nu$ from ν_0 to ∞.

$d\nu \;\; 4\pi J_\nu / h\nu$ = # of photons per unit area in interval $\nu, \nu + d\nu$

The quantum mechanical calculation for the transition probability is like that for photon-induced (stimulated) bound-bound transitions, except that the final state is in the continuum. For a bound-free transition from state n and l (see Rybicki & Lightman 1985, and references therein):

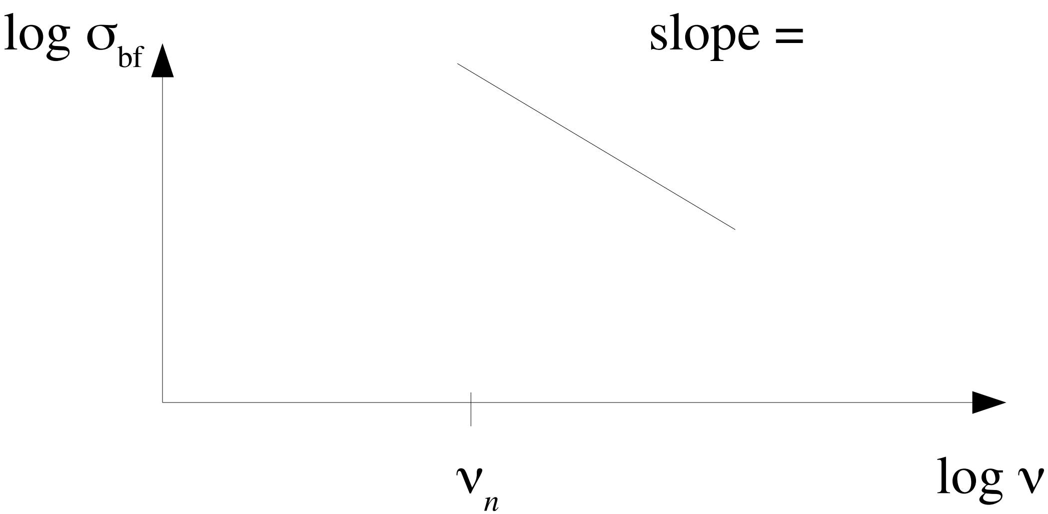
$$\sigma_{bf} = 512\pi^7 m e^{10} Z^4 / (3\sqrt{3} c h^6 n^5) \;\; g(\nu, n, l, Z) / (2\pi\nu)^3$$

where g is the bound-free Gaunt (quantum-mechanical correction) factor. If χ_n is the ionization potential for the n^{th} energy level, σ_{bf} is zero for $\nu < \nu_n$ where

$$\nu_n \equiv \chi_n / h = \alpha_{fs}^2 m c^2 Z^2 / 2 h n^2$$

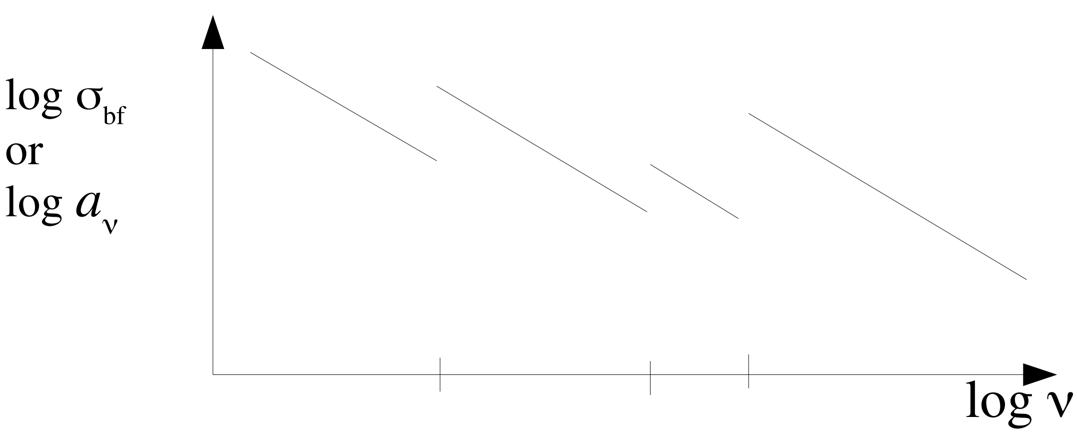
And σ_{bf} rises abruptly for $\nu=\nu_n$, then decreases as ν^{-3}. Near threshold $g\sim1$ (to within 20%). Note we can also write:

$$\sigma_{bf} = 64\pi n g/(3\sqrt{3}Z^2)\ \alpha_{fs} a_0^{\ 2}\ (\nu_n/\nu)^3$$



(see figure on p. 4 of these notes)

The absorption coefficient is $n_n\sigma_{bf} = a_\nu$ where n_n = atomic density of absorbing level. For a number of transitions originating from different levels we have a series of trends:



These different threshold n correspond to ionization from different levels.

The life cycle of Joe Electron within an HII region

i) HI+$h\nu$ (<912Å) → $p^+ + e^{-\prime}$ *Photoionization*

$E_{e^{-\prime}} = h\nu - h\nu_0$ ⇒ kinetic energy injected into gas

ii) $e^{-\prime} + e^- \rightarrow e^{-\prime\prime} + e^{-\prime\prime}$ *Thermalization*

Maxwellian velocity distribution at 10^4 K

iii) $e^{-\prime} + O^+ \rightarrow O^{+\prime} + e^{-\prime\prime\prime}$ *collisional excitation* & cooling photon

$O^{+\prime} \rightarrow O^+ + h\nu$

iv) $e^{-\prime} + p^+ \rightarrow$ HI' *Recombination*

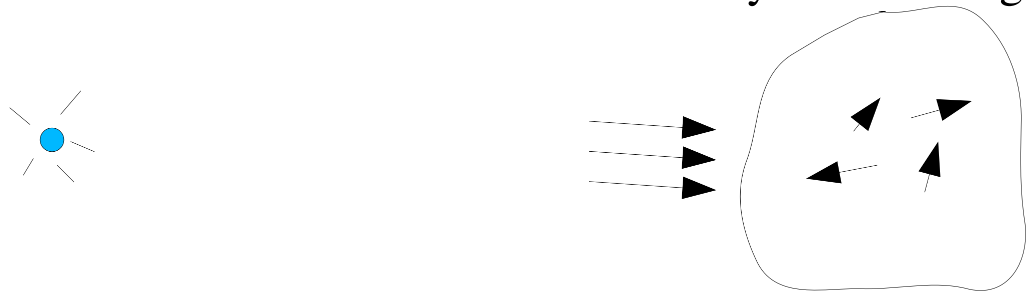
v) HI' → HI + multiple $h\nu$'s Recombination line emission

if i) = iv), then ionization equilibrium

simplifications:

1) HI in ground 1^2S state (A large)
2) electrons with Maxwellian distribution ($\sigma_{elastic} >> \sigma_{inelastic}$)

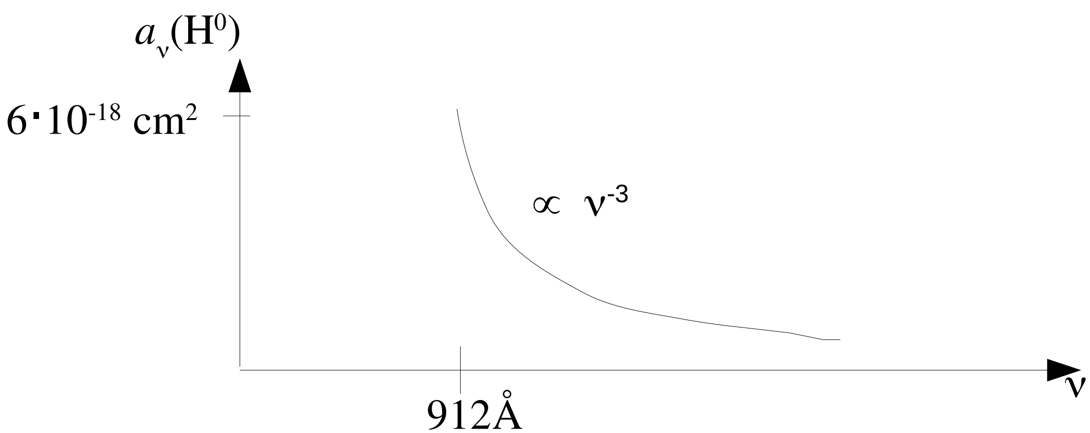
Radiatively-Excited Regions



Consider a volume element of the nebula. In equilibrium,

\# of photo-ionizations per second = # of recombinations per second

The H^0 photo-ionization cross-section



Q: Why is the cross-section not drawn for wavelengths longer than 912Å?

Photo-ionization equilibrium for Hydrogen

$$n(H^0) \int d\nu \;\; 4\pi J_\nu a_\nu(H^0)/h\nu = n_p n_e \beta_A(H^0)$$

$\beta_A(H^0)$: recombination coefficient to all levels of atomic H

where J_ν consists of two parts: $J_\nu = J_\nu^* + J_\nu^{neb}$ — stellar plus nebular

and $4\pi J_\nu^* = e^{-\tau\_\nu} F_\nu R_*^2/r^2$

r = distance from star of radius R_*; $\tau_\nu = \int n(H^0,r)\, a_\nu(H^0)\, dr$

Beware that some texts have defined F_ν different by a factor of π.

Let's use some rough numbers in the equilibrium equation to see what fraction of the hydrogen we expect to be neutral. For T_*=40000 K,

$N_{LyC} = \int d\nu\ L_\nu / h\nu$ from ν_0 to ∞ where $L_\nu = 4\pi R_*^2 F_\nu$

$= 5 \cdot 10^{48}$ photons/s # of Lyman continuum photons

⇒ @ 1 pc the photo-ionization rate is: ($r \sim 3 \cdot 10^{18}$ cm and $a_\nu(H^0) \sim 6 \cdot 10^{-18}$ cm^2)

$\int d\nu\ 4\pi J_\nu a_\nu(H^0)/h\nu \sim a_\nu(H^0)/4\pi r^2 \int d\nu\ L_\nu/h\nu \sim 6 \cdot 10^{-18} / 4\pi\, (3 \cdot 10^{18})^2\ 5 \cdot 10^{48} \sim 3 \cdot 10^{-7}\ s^{-1}$

⇒ lifetime of $H^0 \sim (3 \cdot 10^{-7}\ s^{-1})^{-1} \sim 3 \cdot 10^6$ s

If $\beta_A(H^0) \sim 4 \cdot 10^{-13}$ cm^3 s^{-1}, x = fraction of ionized hydrogen, and $n_p = n_e = xn(H)$:

⇒ $(1-x)n(H) \int d\nu\ 4\pi J_\nu a_\nu(H^0)/h\nu = n_e n_p\, \beta_A(H^0) \Rightarrow (1-x)\, 3 \cdot 10^{-7} = 4 \cdot 10^{-13}\, x^2$

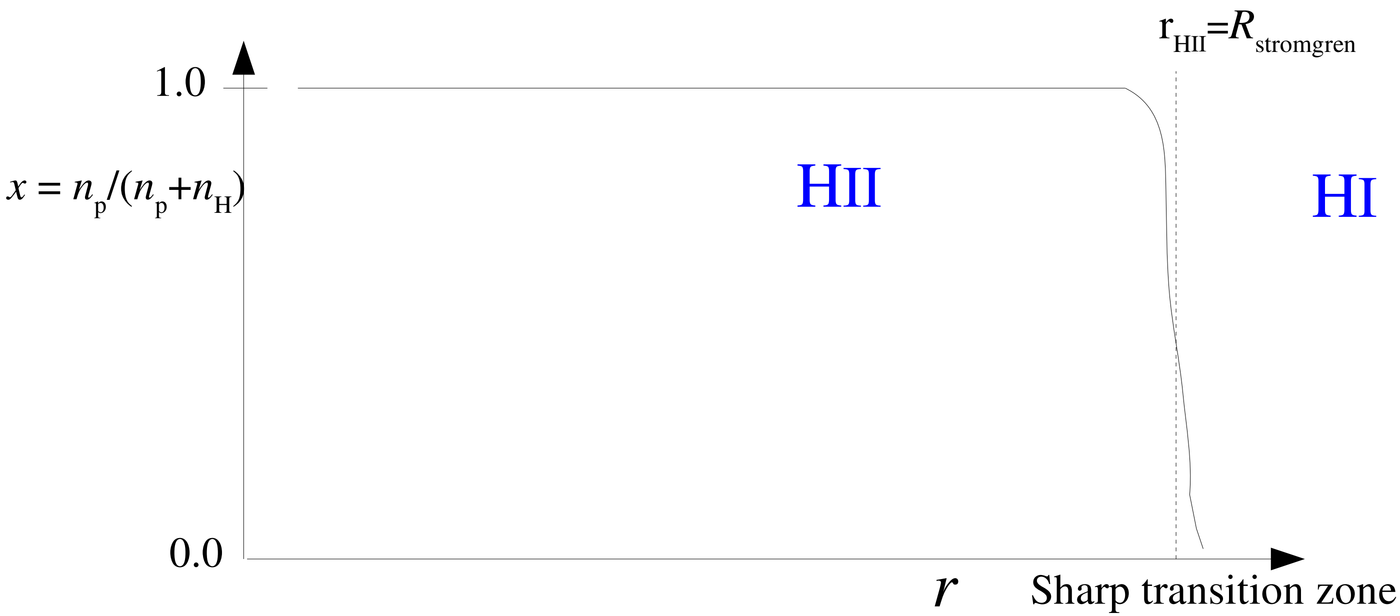
or $1-x \sim 10^{-6}$

⇒ Hydrogen mostly _________.

How, if the lifetime of neutral H is so long?

Mean free path of a 13.6eV photon is $l \sim (n_{H0} a_\nu(H^0))^{-1} \sim (10^3\ cm^{-3}\ 6\cdot10^{-18}\ cm^2)^{-1} \sim 5\cdot10^{-5}$ pc !

The HII region looks like (see also Tielens Figures 7.2, 7.3, 7.4):



The HII region then consists of an almost completely ionized zone terminated by a sharp transition.

Q: An idealized Strömgren sphere has a constant density, temperature, and radius. It has *just been created* within a large HI cloud with properties typical of the cold neutral ISM. Suppose the Strömgren radius is infinitely sharp: the ionization fraction drops from 1.0 to 0.0 at $R_{Stromgren}$. Estimate the pressures inside and outside the sphere. Comment.

HII region sizes (pp. 228-231 of Tielens)

N_{Lyc}	= # Lyman continuum photons from central star
$\beta_A(T_e)$	= recombination rate coefficient to all levels of atomic hydrogen
$\beta_B(T_e)$	= recombination rate coefficient to all $n \geqslant 2$ levels of atomic hydrogen (see Slide 14)
R_S	= Strömgren radius, the size of the HII region
$\langle x \rangle$	= average degree of ionization
n_H	= pre-ionization density of ISM (= n_{e-} = n_{H+} after ionization)
T_{e-}	= temperature

Balancing the rate of photo-ionizations with the rate of recombinations yields

$$N_{\mathrm{Lyc}} = 4\pi/3\, R_S^{3}\, n_H^{2}\, \beta_B(T_e)\, \langle x \rangle^2$$

$$R_S = (3N_{\mathrm{Lyc}}/4\pi n_H^2\, \beta_B)^{1/3} \sim 1.2\, (10^3\mathrm{cm}^{-3}/n_H)^{2/3}\, (N_{\mathrm{Lyc}}/5\mathrm{x}10^{49}\mathrm{photons/s})^{1/3}\ \mathrm{pc}$$

see Table 7.1 on p. 231 of Tielens for different Strömgren radii. A single O star typically will yield a ~ 1 pc HII region for a density of 10^3cm^{-3}.

Thermalization of ejected electrons

The initial distribution of ejected electrons is dependent on the incident photon spectrum $J_\nu a_\nu(H^0)/h\nu$, but the e^--e^- elastic scattering cross-section is large:

$\sigma_{e-} \sim 4\pi(e^2/mv^2)^2 \sim 10^{-13}$ cm^2 (in cgs; $v \sim 5 \cdot 10^5$ m/s for HII region temperatures)

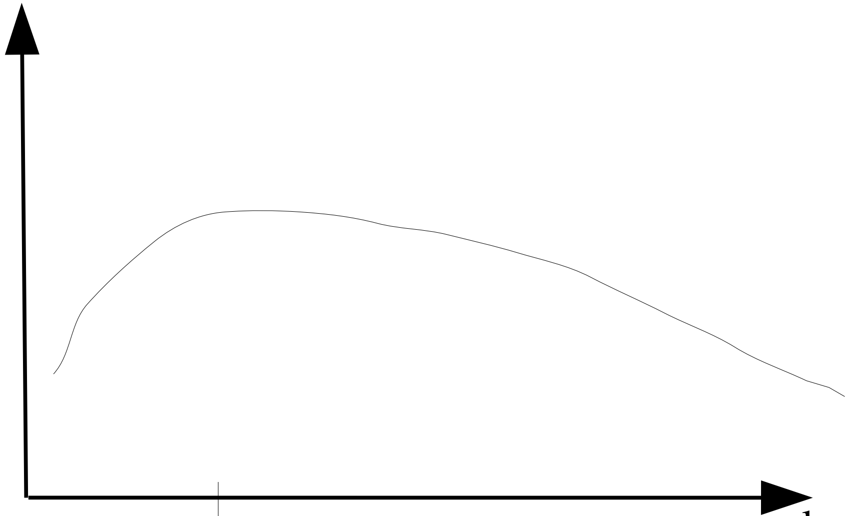
⇒ tend to set up a Maxwell-Boltzmann distribution since other cross-sections are much smaller

β_{rec} = recombination coefficient
$\sim 10^{-12} - 10^{-13}$ cm^3 s^{-1}
and $\sigma_{rec} \sim 10^{-20} - 10^{-21}$ cm^2

⇒ comparisons of these two cross-sections implies that collisions happen many times before recombination.

Q: What simple concept allows us to derive the above formula for σ_{e-}? In other words, what is fundamental to preventing free electrons in a plasma from interacting and exchanging KE?

Radio Emission from HII Regions (Tielens pp. 255-256; Figure 7.12)



For Bremsstrahlung from a nebula of size L and density n, the opacity in the radio is

$\tau \propto T_e^{-1.35}\, \nu^{-2.1}\, n^2 L$ (see Tielens Eqn 7.68, Fig 7.12)

Recall that radiative transfer through a medium yields

$$I_\nu = I_{\nu,0}\exp(-\tau_\nu) + B_\nu(T)(1-\exp(-\tau_\nu))$$

and the Rayleigh-Jeans approximation for blackbody emission is

$$B_\nu(T) \approx 2\nu^2 k T_b / c^2$$

which in the absence of a background source leads to

$$I_\nu \approx B_\nu(T)(1-\exp(-\tau_\nu)) \propto \nu^2 T_b(1-\exp(-\tau_\nu))$$

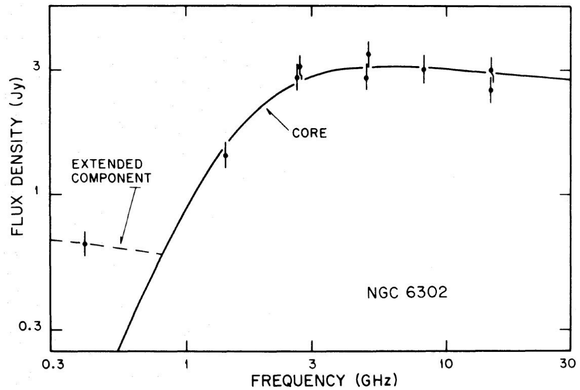
In the limiting cases

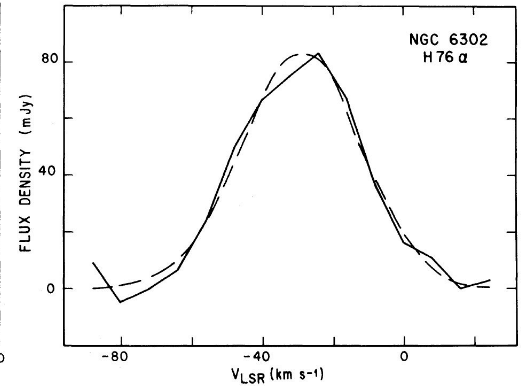
$I_\nu \approx \qquad \propto \qquad$ for $\tau \ll 1$

$I_\nu \approx \qquad \propto \qquad$ for $\tau \gg 1$

[General review & discussion]

NGC6302 Butterfly Nebula
1 kpc away; surface T >250k





- Radio frequency spectra of nebulae (Tielens Figure 7.12)
    - What produces the continuum emission: recombination, free-free, $n_i \rightarrow n_j$, or some combination?
    - What produces the line emission: recombination, free-free, $n_i \rightarrow n_j$, or some combination?
    - (free electrons exhibit a *continuum* of energies and recombine into *specific* energy levels)

    Which typically drives radio emission: continuum or discrete line processes?

- 'On-the-Spot-Approximation' p.230 Tielens; p.20 Osterbrock; p.67 Dyson & Williams

    In balancing the ionizations and recombinations, the recombinations are summed from n=2 to ∞ (i.e., not from n=1 to ∞). Why can we ignore recombinations to the ground state?

    In Osterbrock parlance: hν~I_H and therefore the diffuse radiation has large a_ν and correspondingly _____ mean free paths...

General discussion

- If you observe an HII region, would you expect to see more Hα (n=3→2) photons, or more Lyα (n=2→1) photons? Why?

- When you peer through an eyepiece, why is almost all the H emission seen from an HII region coming from neutral H even though HII regions are almost completely ionized?

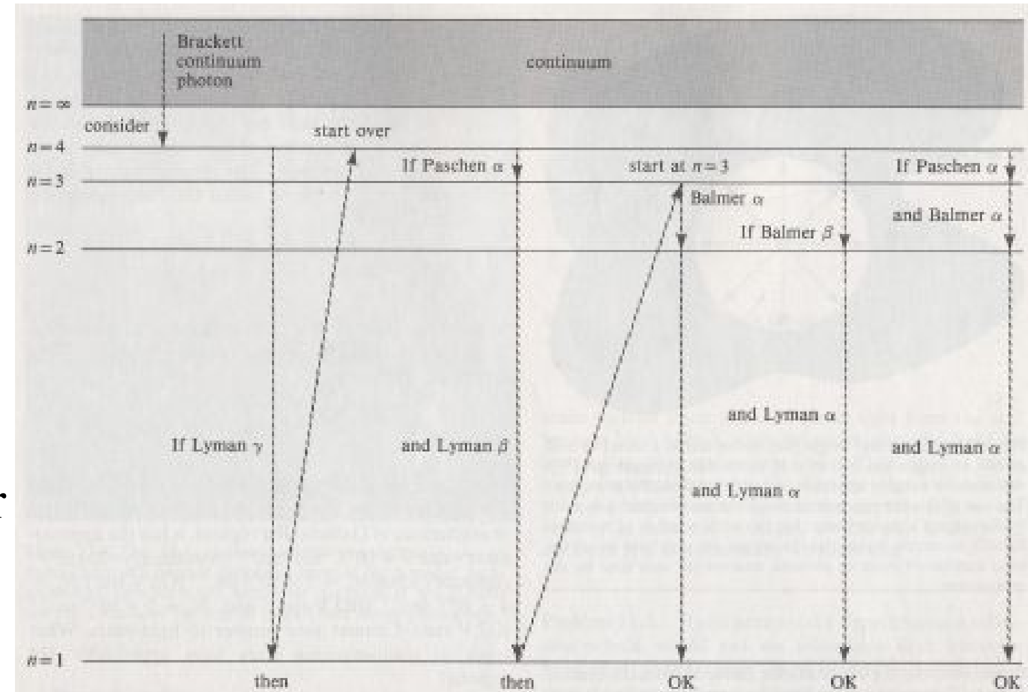
- What is the distinction between the 'direct' and the 'diffuse' radiation field in an HII region? What does this have to do with its optical thickness or thinness?

- What's 'forbidden' about forbidden lines? Do you see them only in low-density regions?

Radiatively-Excited Regions

General review & discussion (adapted from Shu, *The Physical Universe*)

Nebulae are so rarefied that ~all atoms are in their ground states. At 10^4 K most collisions are too weak to excite an e^- in H. However, since N, O, O^+ are just as easily ionized as H, and since the first excited states of their ions N^+, O^+, O^{++} are near the ground states, they can be collisionally-excited.



At terrestrial densities these excited states are immediately triggered back down by collisions (without radiating). In space, radiative de-excitations actually have a chance to occur due to the low densities; early astronomers did not understand the [O III]4959+5007 doublet and attributed them to an unknown element dubbed 'nebulium', a la the 'helium' discovery from solar observations.

Problem

The number of Balmer-line photons emitted per unit time in an HII region is equal to the total number of H recombinations—each recombination produces one Balmer photon. The luminosity L_H in all the Balmer lines is $L_H \propto$ ______________________________

where V is the volume of the HII region and $\beta_B(T) \sim 2.6 \cdot 10^{-19}$ m^3 s^{-1}.

Consider collisionally-excited transitions between an upper and lower level of an atom such as O^{++}, with energy separation E_{21}. We'll calculate n_2 if given n_1 (e.g., the ground state).

If the medium remains optically thin to photons of energy E_{21}, upward transitions from 1 to 2 take place via inelastic collisions with thermal e^-s at a rate per unit volume of ____________

where $\gamma_{12}(T)$ is the collisional excitation coefficient when the e^- velocity distribution is characterized by a temperature T. Downward transitions from 2 to 1 can take place either by the spontaneous radiation of photons of energy E_{21} or by *superelastic* collisions with thermal e^-s. The total downward rate per unit volume is given by _________________________

where A_{21} is Einstein's coefficient for spontaneous emission and $\gamma_{21}(T)$ is the collisional de-excitation coefficient. If we ignore transitions from higher states (say, level 3) into and from levels 1 and 2 (to keep this simple), provide an expression for the steady-state population for levels 1 and 2: ____

Upward collisional transitions have an energy threshold: e^-s must have kinetic energies greater than E_{21} before they can cause a transition $1 \rightarrow 2$. Downward collisional transitions have no such threshold. Thus, the fraction of e^-s which contributes to γ_{12} is smaller than that which contributes to γ_{21}. For a Maxwell-Boltzmann distribution of e^- velocities, the fraction is given by the Boltzmann factor $\exp(-E_{21}/kT)$. In addition, the transition rates γ_{12} and γ_{21} will differ by a factor equal to the number of equivalent states of the final level: g_2 for state 2 and g_1 for state 1. We therefore derive the relationship

$\gamma_{12}/\gamma_{21}=$ ____

where γ_{21} will approximately be the product of thermal e^- speed $(kT/m_{e-})^{½}$ and typical superelastic cross section $\sigma_{21} \sim \hbar^2/m_{e-}kT$ (interpret the latter in terms of the square of the ___________________ of a thermal e^-). Assume γ_{21} can be calculated in this or a more accurate fashion, and show that n_2 can be expressed as

$$n_2 = n_1\, n_{e-}\, \gamma_{21} /(A_{21}+n_{e-}\gamma_{21})\; g_2/g_1 \exp(-E_{21}/kT)$$

What condition does n_{e-} have to satisfy in order for Boltzmann's Law to apply,

$$n_2 = n_1\, g_2/g_1 \exp(-E_{21}/kT) \text{ ?}$$

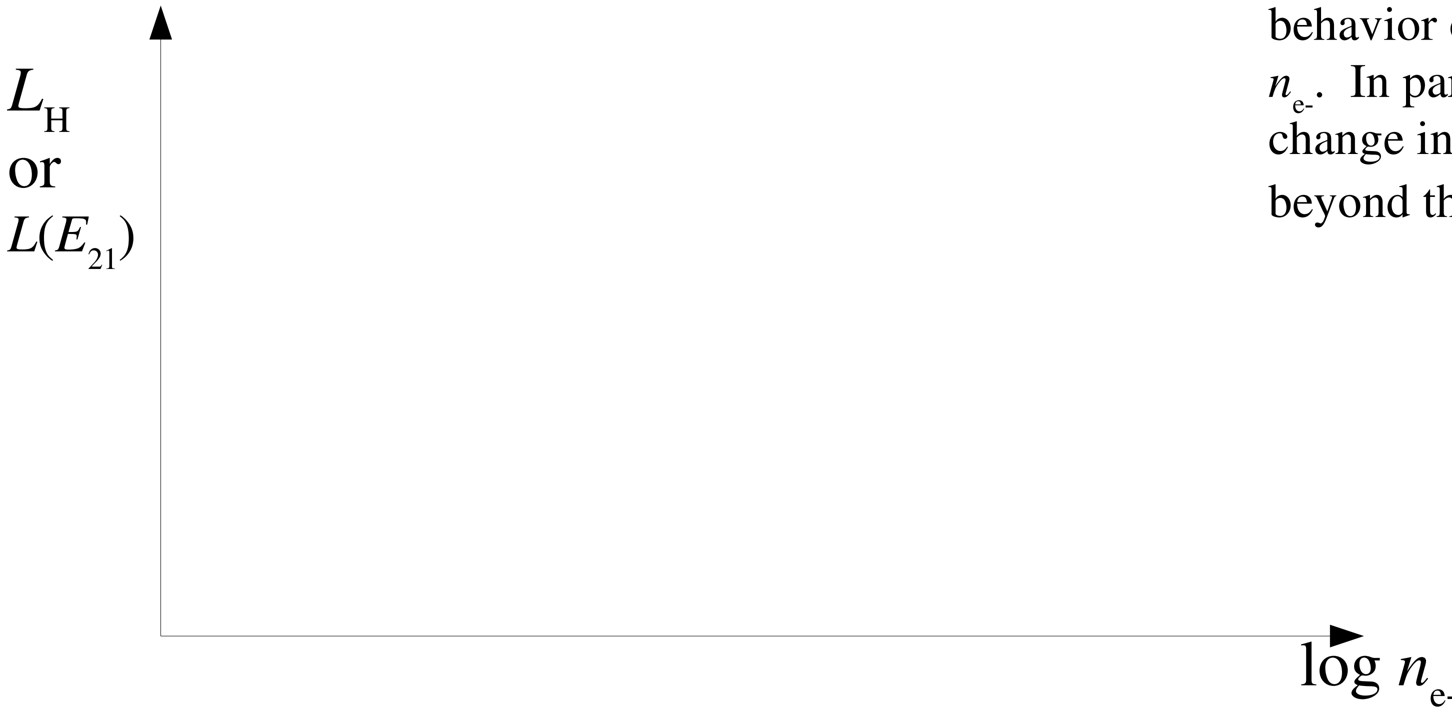
Interpret your answer physically. It won't be satisfied exactly, but explain under what conditions it would be a very good approximation.

If forbidden-line atoms occupy the entire volume V of the HII region, the luminosity $L(E_{21})$ of photons with energy E_{21} must be proportional to the total number of downward radiative transitions per unit time, which is ________________.

Thus show

$$L(E_{21}) \propto n_1\, V\;\; g_2/g_1 \exp(-E_{21}/kT)\, [n_{e-}\, A_{21}\, \gamma_{21}/(A_{21}+n_{e-}\gamma_{21})]$$

For fixed T, sketch schematically the behavior of $L(E_{21})$ and L_H as functions of n_{e-}. In particular, indicate the qualitative change in behavior of when n_{e-} increases beyond the critical value A_{21}/γ_{21}.



For 'forbidden' optical transitions, A_{21} might have a typical value of $A_{21} \sim 10^{-2}$ s^{-1}, implying a characteristic lifetime A_{21}^{-1} against radiative decay of ~10^2 s, much longer than the lifetime ~10^{-8} s that is characteristic of excited states with permitted transitions. Calculate the critical density for a temperature of 10^4 K.

$n_{cr} \sim$

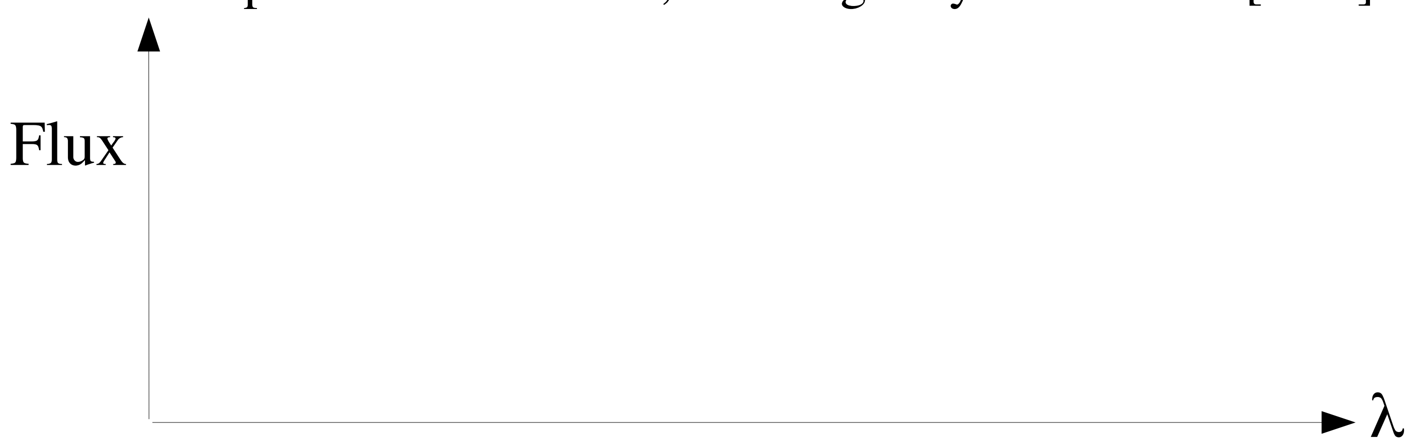
Consider the ratio of the strengths of the forbidden lines to the (permitted) hydrogen lines. Assume that the proportionality coefficients in the expressions for $L(E_{21})$ and L_H have similar orders of magnitude, and show that

Radiatively-Excited Regions

$$L(E_{21})/L_H \sim n_1/n_p \; g_2/g_1 \; \exp(-E_{21}/kT) \;\; \gamma_{21}/\beta_B \;\; 1/(1+n_{e-}/n_{cr})$$

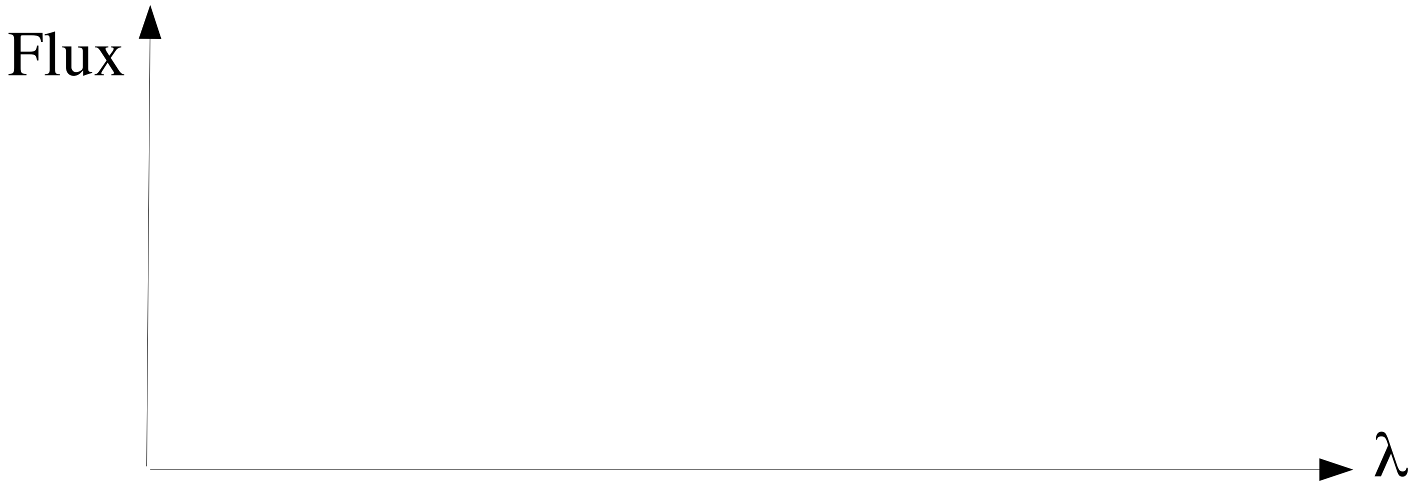
For a heavy element like oxygen ([O III]), $n_1/n_p \; g_2/g_1$ might typically be $\sim 10^{-3}$, and λ_{21} = 5007 Å in the optical regime. For T=10^4 K graph the ratio of line strengths for various values of n_{e-} from 10^9 m^{-3} to 10^{14} m^{-3}.

Consider the following situation. You observe a very low-density region, say a planetary nebula. You take a spectrum and you choose the exposure time to capture 10,000 Hα photons. Roughly sketch what the spectrum would like, showing only the Hα and [O III] 5007Å lines.



Now you use the same telescope, detector, etc., to observe a region of much higher density, but the same temperature. Sketch the spectrum when:

- using the same *exposure time* as before.
- using the same *criterion* as before –10,000 Hα photons.



Telescope time is precious and thus astronomers tend to observe an object only as long as necessary before moving to their next source. The above example explains why there is a common misconception about forbidden lines only appearing in the low density ISM.

Problem

What is the timescale, t_{rec}, over which a free proton in a Strömgren sphere radiatively recombines with an e^-? Assume $n_p = n_e \sim 10^6$ m^{-3} and $T \sim 10^4$ K.

It is often assumed that velocity distributions of particles are Maxwellian. The validity of this assumption rests on the ability of particles to collide elastically with one another and share their kinetic energy. For a Maxwellian to be appropriate, the timescale for a collision must be short compared to other timescales of interest. We will test these assumptions.

What is the timescale $t_{e\text{-}e\text{-}}$ for free e^-s to collide with one another in a Strömgren sphere? Consider collisions occurring at relative velocities typical of those in an e^- gas at temperature $T_{e\text{-}}$. Determine a general expression, and then evaluate for the above numbers.

Repeat the above but for t_{p+p+} for protons at temperature T_p.

Problem (continued)

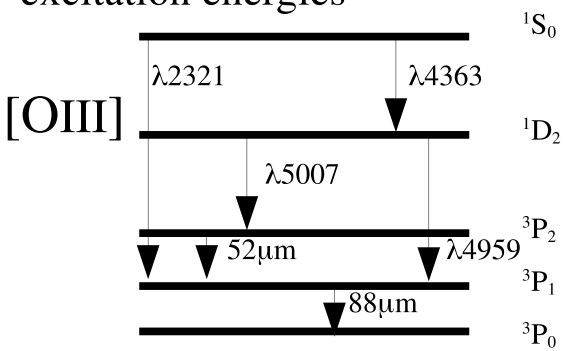
Suppose that initially $T_{e-} > T_{p+}$. What is the timescale over which e⁻s and p⁺s equilibrate to a common kinetic temperature? This is not merely the timescale for a p⁺ to collide with an e⁻. You must consider also the amount of energy exchanged between an e⁻ and p⁺ during each encounter. Estimate, to order-of-magnitude, the time it takes a cold p⁺ to acquire the same kinetic energy as a hot e⁻. Call this time $t_{e\text{-}p+}$. Again, express you answer symbolically and then toss in the original values to produce a number.

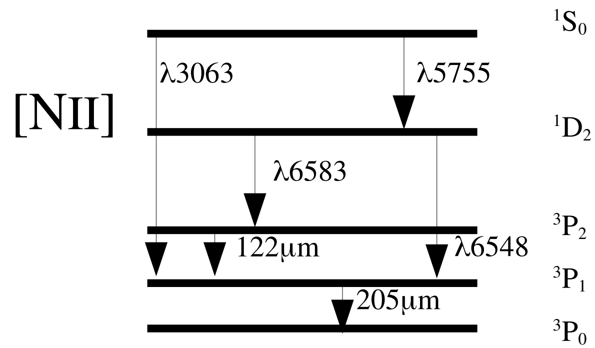
For $T_{e-} \sim T_p \sim 10^4$ K,

$t_{e\text{-}e\text{-}}/t_{rec} \sim$ $t_{p+p+}/t_{rec} \sim$ $t_{e\text{-}p+}/t_{rec} \sim$

Electron Temperature T_e (adapted from Osterbrock & Ferland 2005 ch. 5)

[OIII] and [NII] are good examples of ions that have optical emission lines stemming from significantly different excitation energies





[OIII]: In low n_e limit (ignoring collisional de-excitation), every excitation to ^1D produces λ5007 or λ4959 (2.9:1 odds). Excitations up to ^1S produces λ4363 or λ2321, with relative probabilities again determined by transition probabilities. If λ4363, then λ4959 or λ5007 is produced immediately thereafter.

$$[j_{\lambda4959}+j_{\lambda5007}]/j_{\lambda4363} = \Omega(^3P,^1D)/\Omega(^1D,^1S)\ \ [A_{1D2,3P2}+A_{1D2,3P1}]\ /\ A_{1S0,1D2}\ \ h\nu^*(^3P,^1D)/h\nu(^1D,^1S)\ \ e^{\Delta E(D,S)/kT}$$

where

$$\nu^*(^3P,^1D) = [A_{1D2,3P2}\,\nu(\lambda5007) + A_{1D2,3P1}\,\nu(\lambda4959)]\ /\ [A_{1D2,3P2}+A_{1D2,3P1}],$$

A is the transition probability, and Ω represents "collision strength" (e.g., the γ parameters from Slide 17).

This expression works up to densities of $n_{e-}\sim10^5$ cm^{-3}. Inserting numerical values, and including dependence on n_{e-},

$$[j_{\lambda4959}+j_{\lambda5007}]/j_{\lambda4363} = 7.73\exp(3.29\times10^4/T)\ /\ [1+4.5\times10^{-4}(n_{e-}/T^{1/2})\ ]$$

⇒ see Figure 7.14 Tielens and next slide

Electron Temperature T_e (adapted from Osterbrock & Ferland 2005 ch. 5)

Assumptions:
- isothermal
- optically thin
- low density limit

Features:
- Don't need to know distance to nebula
  →

- Fairly insensitive to extinction
  →

- [OIII] provided traditional measure, but [OIII]4363 difficult to measure, so [OI], [NII], and [SIII] are now preferred

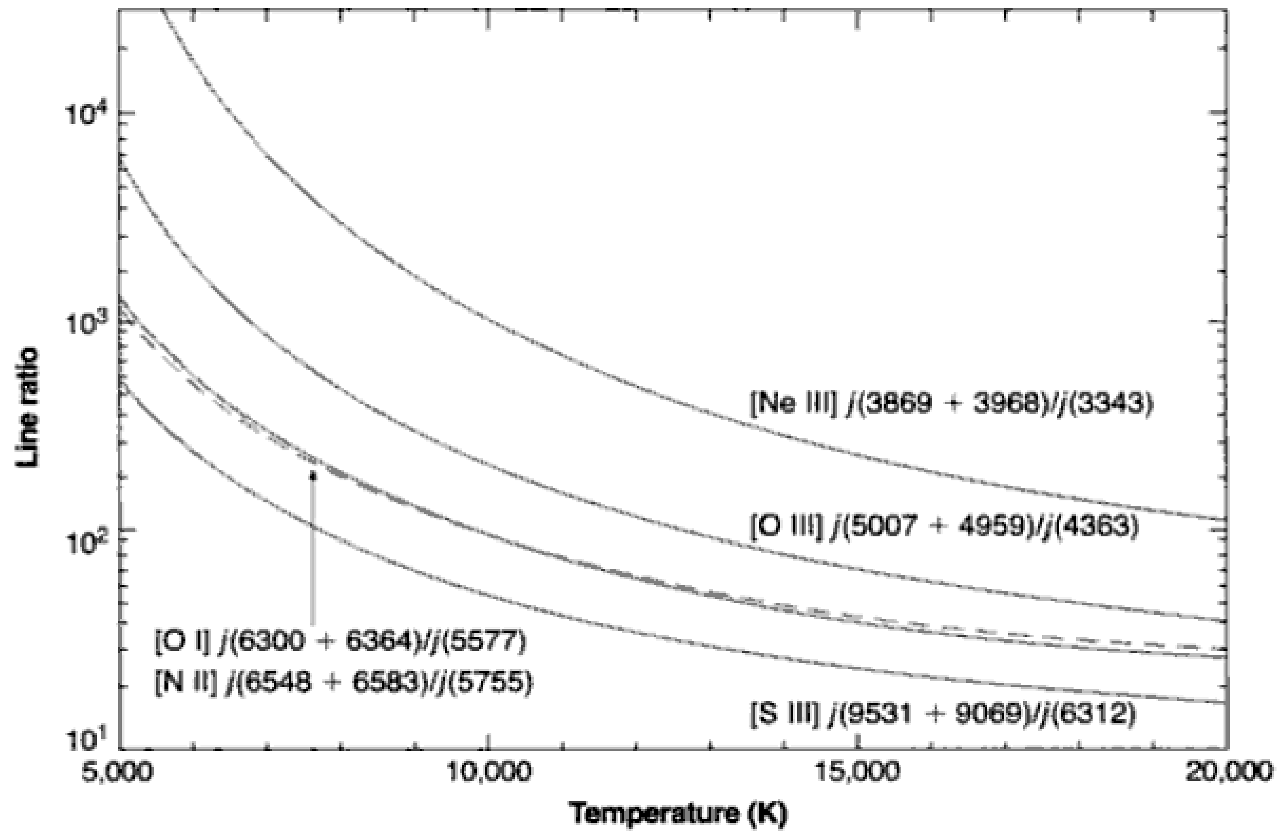


Figure 5.1

Four temperature sensitive forbidden line ratios are shown as a function of the electron temperature. The [O I] (solid line) and [N II] (dashed) ratios are nearly coincident, partially because of their similar excitation potentials. The ratios are shown in the low density limit ($n_e = 1$ cm^{-3}).

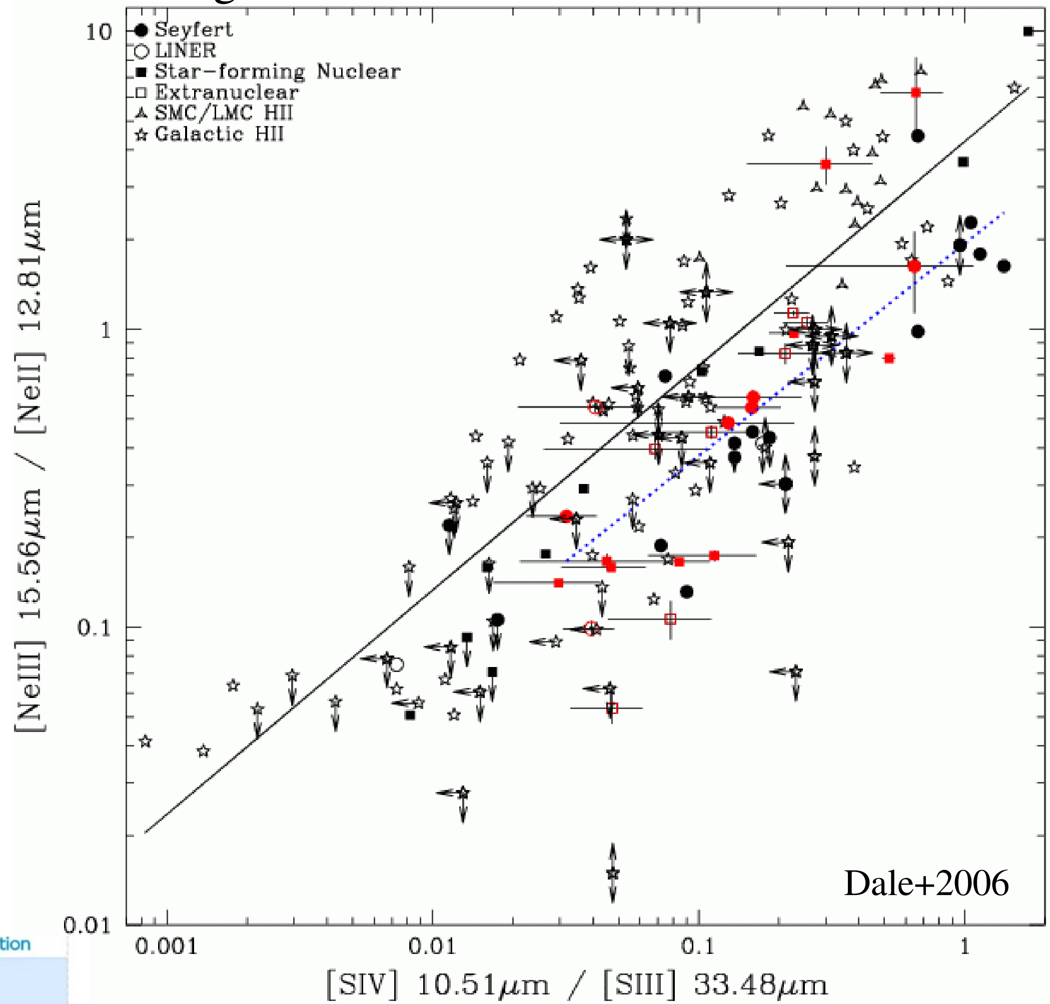
Stellar Effective Temperature

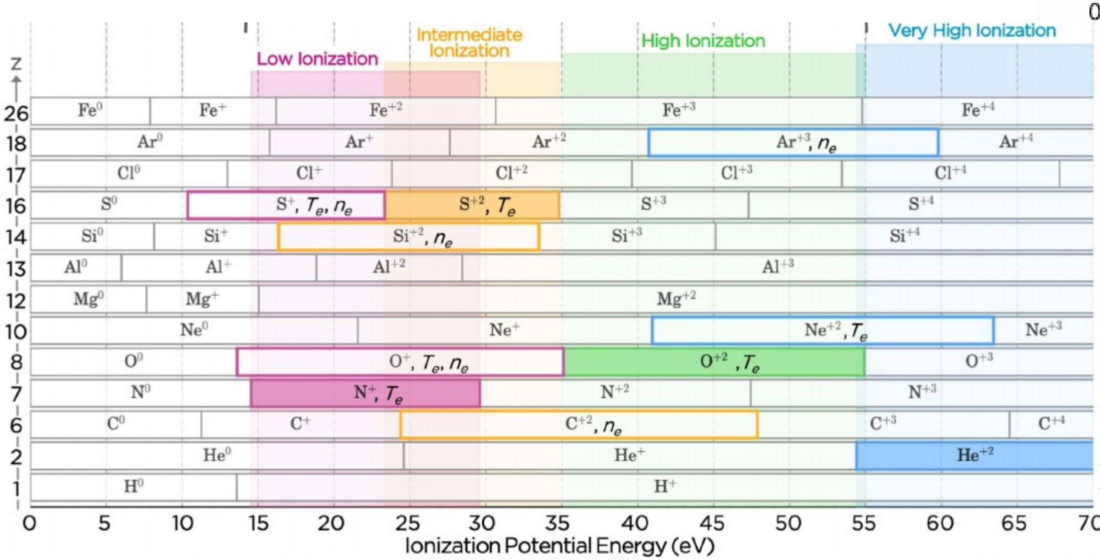
A high-to-low ionization ratio of the same atomic element gives you the “stellar effective temperature” or the radiation field *hardness*.

Can be estimated from “sequential stages of ionization of a single element”, e.g., Ne^{2+}/Ne^{+}

See Tielens Figs 7.15 and 7.16

Low-metallicity galaxies show the hardest radiation fields in the figure at right.





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Electron Density n_{e-} (adapted from Osterbrock & Ferland 2005 ch. 5)

We've seen diagnostics using lines from the same element with

1. different ionizations, and
2. the same ionization but "significantly" different excitation energies

Now we'll explore a third diagnostic that uses lines from the same element, but this time the lines have nearly the same energy levels. If the two very similar levels have different radiative transition probabilities or different collisional de-excitation rates, the relative populations of the two levels will depend on the density.

Low density limit: each collisional excitation is followed by a downward radiative transition. Line ratio is that of statistical weights. For [OII]
$j_{\lambda 3729}/j_{\lambda 3726}=g(^2D_{5/2})/g(^2D_{3/2})=3/2=1.5$

High density limit: collisional excitations & de-excitations predominate, setting up a Boltzmann population. For [OII]:
$j_{\lambda 3729}/j_{\lambda 3726} = g(^2D_{5/2})/g(^2D_{3/2})\ A_{\lambda 3729}/A_{\lambda 3726}$
$= 3/2\ 3.6\text{x}10^{-5}/1.6\text{x}10^{-4} = 0.34$

The transition between these limits occurs near the "critical density."

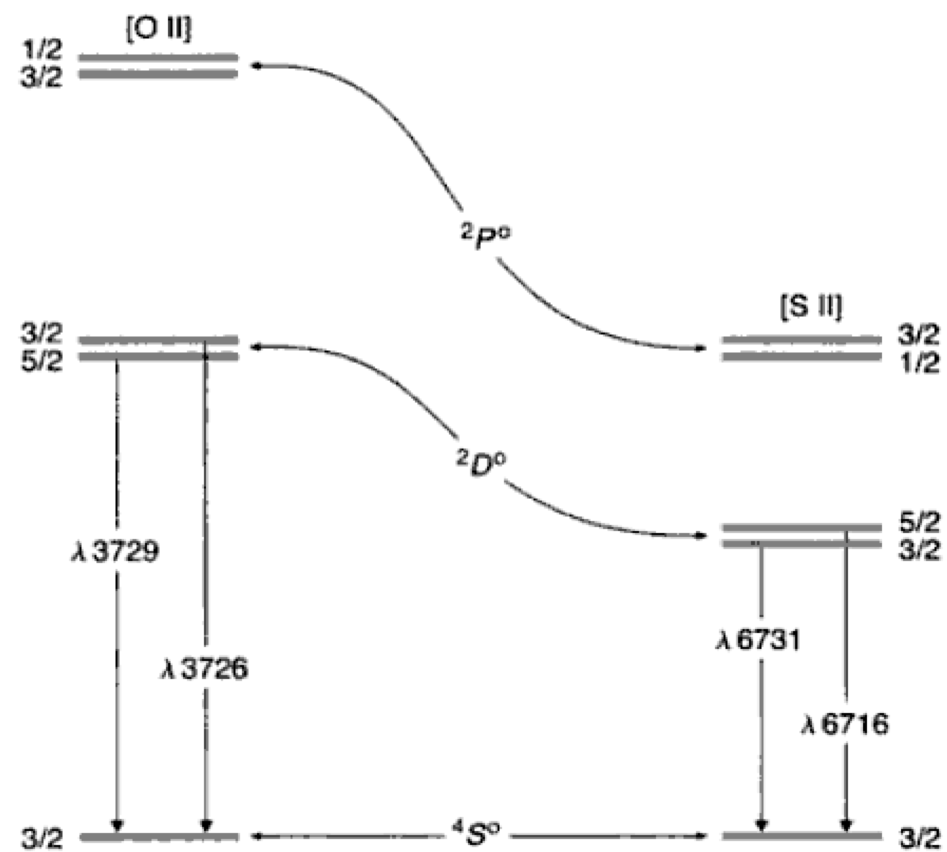

Figure 5.7
Energy-level diagrams of the $2p^3$ ground configuration of [O II] and $3p^3$ ground configuration of [S II].

Electron Density n_{e-} (adapted from Osterbrock & Ferland 2005 ch. 5)

Knowledge of n_{e-} can be fed back into the equation(s) on Slide 24 for deriving the effective e- temperature, since those equations have a slight dependence on n_e.

See Figure 7.13 Tielens for infrared versions (even less sensitive to extinction). Which ratios would be superior for distinguishing between high and low densities?

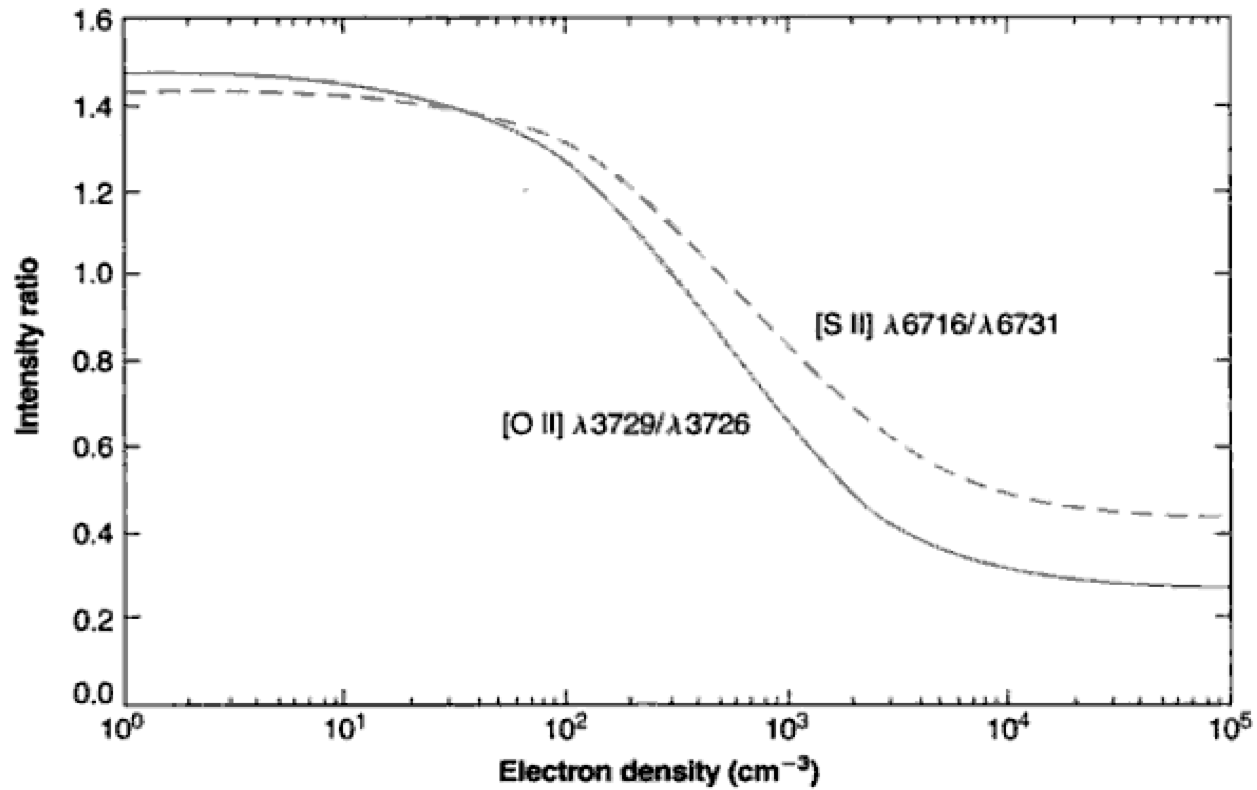

Figure 5.8

Calculated variation of [O II] (*solid line*) and [S II] (*dashed line*) intensity ratios as functions of n_e at T = 10,000 K. At other temperatures the plotted curves are very nearly correct if the horizontal scale is taken to be $n_e(10^4/T)^{1/2}$.

Table 5.2

Electron densities in H II regions

Object	$I(\lambda3729)/I(\lambda3726)$	n_e (cm^{-3})	
NGC 1976 A	0.5	3.0×10^3	near Orion nebula core
NGC 1976 M	1.26	1.4×10^2	outer Orion nebula
M 8 Hourglass	0.67	1.6×10^3	
M 8 Outer	1.26	1.5×10^2	
MGC 281	1.37	70	
NGC 7000	1.38	60	

HII region density typically drops w/radius

Concept Question (adapted from Dopita & Sutherland)

Something like 1 in 10^6 recombinations will occur with n>50; such recombinations will produce lines with radio frequencies less than 50 GHz. For these recombined electrons the radiative transition probabilities to lower excited states are relatively low. Compared to lower n states, are the collisional rates for these electrons relatively high or low? Explain.

The orbiting electron is at a distance r= from the nucleus.
The frequency of transitions between principal quantum numbers m & n is $\nu/\nu_0=n^{-2}-m^{-2}$ where $h\nu_0$=13.6eV for H, for example. Usually only the H and He recombination lines are strong enough to be observed in the radio.

α transitions are for m-n=1, β transitions for m-n=2, etc. e.g., H40α: m= → n= He50γ: m= → n=

radiative transition probability for H$n\alpha$ is $A_{n\alpha}$~$1.167\times10^9/n^6$, and the radiative decay timescale to level n is

τ_{rad} =