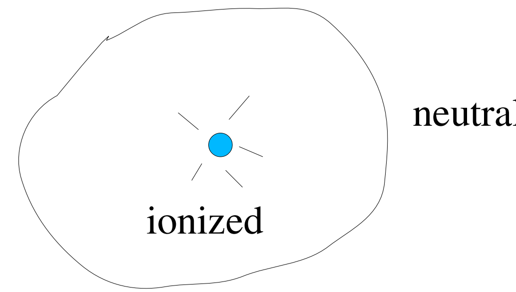
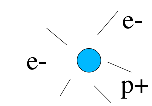
Interstellar Shocks and Dynamics of the ISM

Stars provide momentum to interstellar gas via three main mechanisms:

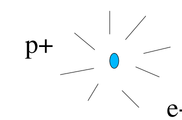
Stars photo-ionize neutral gas (HII regions).
Pressure differences result in nebular expansion.



Massive stars emit stellar winds
Outward stellar radiation momentum converted to stellar gas momentum through absorption by ions like thrice-ionized carbon (CIV).



Supernovae explosions send out ~½ of a star's mass into the ISM.
Radiation and neutrinos also sent radially outwards.



The APOD image of the Galactic Center in the radio:
The wavelength used implies it is tracing ______________________________.

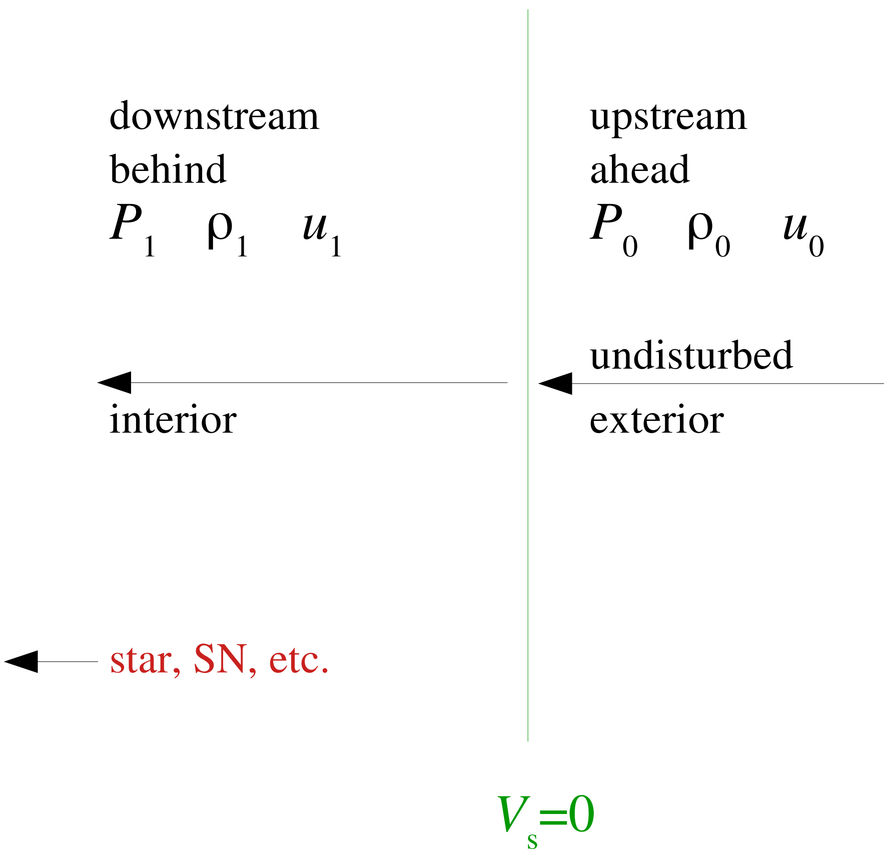
How can you identify the SN remnants?

Why are there so many SN remnants?

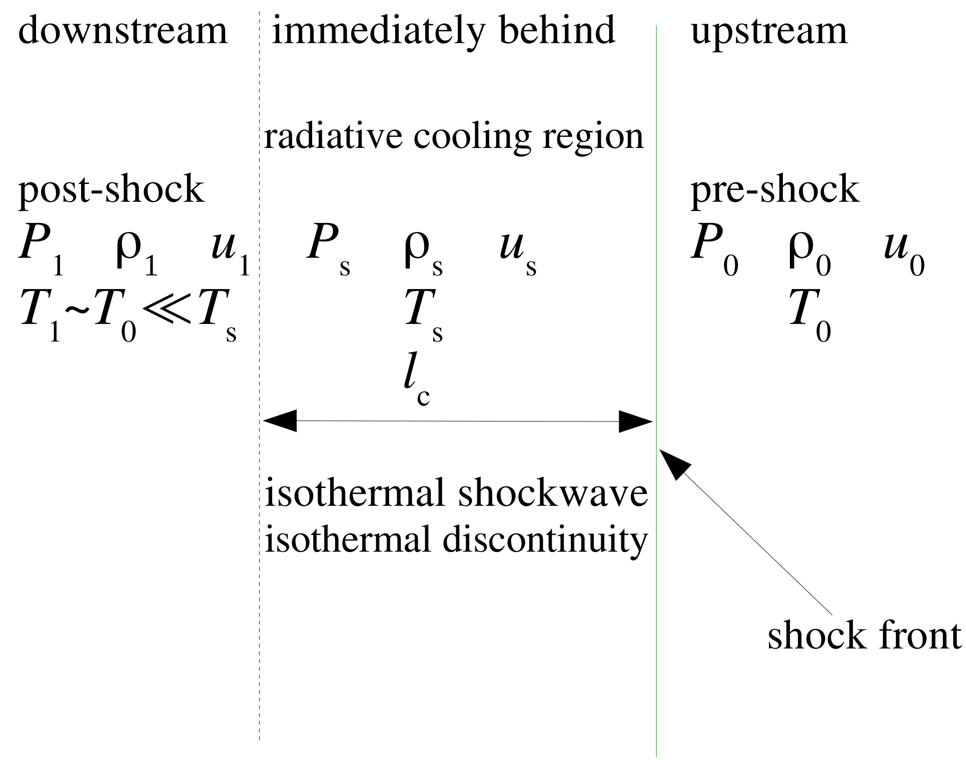
How might we differentiate HII regions from SN remnants?

Schematic description of shock front

Shock reference frame



Schematic of radiating shock, with a close-up view of the region immediately behind the shock



There are three important conservation equations that form the basis for shock physics:

i)

ii)

iii)

Important conclusions are that

u_1/u_0= ρ_1/ρ_0= P_1= T_1=

Sometimes it is preferred to work in the observed rest frame. If v_0 and v_1 are the observed upstream and downstream velocities, and the shock speed $V_s \gg v_0$, then

u_0 v_0 V_s u_1 v_1 V_s P_1= T_1=

What's up with the parameter μ?

Following Carroll & Ostlie (2nd edition) p.291, $n = N/V = \rho/m_{avg}$ $\mu = m_{avg}/m_H$

$\rightarrow$ $P_{gas} = nkT = \rho kT/\mu m_H$

Note that Tielens' μ is my μm_H: on p. 398 he defines μ as the mean mass per species and thus $P=\rho kT/\mu$ whereas I use $P=\rho kT/\mu m_H$.

for a neutral gas $1/\mu_n \sim X + \frac{1}{4}Y + \langle 1/A \rangle_n Z$ (= 1 for just H where Y=Z=)

for an ionized gas $1/\mu_i \sim 2X + \frac{3}{4}Y + \langle \frac{1}{2}+1/A \rangle_i Z$ (= 2 for just H where Y=Z=)

where X = [total mass of hydrogen] / [total gas mass]
Y = [total mass of helium] / [total gas mass]
Z =
A =

Examples:

neutral He: μ_n=

ionized He: μ_i=

If $v_1 = \frac{3}{4}V_s$, i.e., the (fixed frame) downstream velocity is three-quarters of the shock velocity, then

Q: What is the implication for the downstream velocity, in the shock reference frame?

Q: How about the implication for the upstream velocity in the shock reference frame (for a typically strong, fast shock)?

Q: How do these implications jibe with the situation inferred by the diagram on Slide 2?

Q: What is quantitatively required for a *strong* shock to occur?

Deriving the Sound Speed (adapted from Shu)

Compressibility enables a gas to transmit signals. The speed at which these signals propagate is referred to as the sound speed. Consider an observer who moves with an acoustic wave; let u ($\sim c_{sound}$) be the fluid velocity of the gas relative to the observer. Mass and momentum conservation yield

$\phi = \rho u \quad = \text{constant}$ mass flux

$\zeta = P + \rho u^2 \quad = \text{constant}$ momentum flux

Q: ρu^2 is also known as 'ram pressure'. How does ram pressure differ from kinetic pressure P?

Take differentials of the above two equations, e.g.,

$\rho \mathrm{d}u + \qquad = 0$

$\mathrm{d}P + \qquad = 0$

Use these two equations to show that $\mathrm{d}P = u^2 \mathrm{d}\rho$. For a small amplitude wave, the total fluid velocity u is almost entirely due to the motion of our observer at speed c_{sound}; therefore

$$c^2_{sound} = \mathrm{d}P/\mathrm{d}\rho$$

Ignoring electromagnetic radiation, viscosity, thermal conduction, ... this motion takes place with no loss of heat ($T\,\mathrm{d}s = 0$). Hence, the differential change in internal energy per unit mass $\mathrm{d}(3kT/2m)$ equals the differential rate of doing work on the gas by the pressure $-P\mathrm{d}(\rho^{-1})$. Show that

$c^2_{sound} = \gamma\, P/\rho = 5/3\, P/\rho$ for a monatomic gas.

Q: What is γ for a diatomic gas?

Q: Is terrestrial air less or more 'springy' than a monatomic gas? Why?

Ram Pressure (adapted from Shu)

Imagine a wind of mass density ρ_0 and velocity component v which blows perpendicularly on a disk of interstellar gas with area A. In terms of ρ_0, v, and A, what is the momentum transferred per unit time to the disk?

$dp/dt =$

Suppose the mean surface density of the interstellar gas is μ_{gas}, and that this gas is attracted by the gravity of a disk of stars of mean surface density μ_*. The gravitational field of the latter is _______. The gravitational force of attraction on the entire disk of interstellar gas is then

$dp/dt_{gas} = F_{gas} = m_{gas}g =$

Ram pressure stripping will result if the momentum transferred per unit time exceeds the gravitational attraction of the stellar disk for the gaseous disk. What constraint (inequality) does this place on ρ_0?

$\rho_0 >$

Solve ρ_0 for μ_* and μ_{gas} being 10^{11} and 10^9 $M_\odot$ spread out over a disk of 10 kpc, and a typical galaxy cluster speed of 1000 km/s. How would the total intracluster gas mass compare to the total mass of 1000 galaxies, each of 10^{11} $M_\odot$, assuming the cluster has a 1 Mpc radius?

$M_{intracluster} = 4/3\, \pi\, R^3\, \rho_0 =$

Following, but in addition to, what is laid out in Chapter 11 of Tielens

The continuity equation gives us that the compression factor $\rho_1/\rho_0 = u_0/u_1$. For a radiationless shock it is shown that this is 4 for 'strong' shocks in a monatomic gas (i.e., where the Mach number M, the ratio of the flow speed to the sound speed, is large). More generically,

$\beta^2(\gamma+1)/(\gamma-1) - 2\beta(M^{-2}+\gamma)/(\gamma-1) + 2M^{-2}/(\gamma-1) + 1 = 0$ where $\beta = u_1/u_0$ and $M=u_0/c_s$ where $c_s = (\gamma P_0/\rho_0)^{1/2}$

Solving the quadratic for $M \gg 0$ yields the approximate solutions $u_1=u_0$ (no shock) or $u_1=\frac{1}{4}u_0$ (and thus $\rho_1/\rho_0 = 4$). Using the Mach number we can slickly express the following ratios:

$$P_1/P_0 = 2\gamma/(\gamma+1)\, M^2 - (\gamma-1)/(\gamma+1)$$

$$\rho_1/\rho_0 = (\gamma+1)\, M^2 / ((\gamma-1)M^2+2)$$ the 'compression factor'

What is the possible range for the Mach number M?

a) 0 to ∞
b) 1 to ∞
c) 0 to 1
d) 1/∞ to 0

What is the ratio of pre-shock and post-shock pressures for M=1 (assume monatomic)? First try to answer from a conceptual standpoint, and then compute the ratio quantitatively.
a) 1:1
b) 2:1
c) 1:2
d) 4:1
e) 1:4

What is the ratio $\rho_0:\rho_1$ of pre-shock and post-shock densities for $M\rightarrow\infty$, for a monatomic gas?

a) 0:1
b) 1:0
c) 1:1
d) 4:1
e) 1:4

When you include the effects of magnetic fields, it gets a little trickier.

The Rankine-Hugoniot jump (conservation) conditions are

$\phi = \rho u$	mass flux
$\zeta = P + \rho u^2 + B^2/8\pi$	momentum flux
$\xi = \tfrac{1}{2}u^2 + \gamma/(\gamma\text{-}1)\ P/\rho + B^2/4\pi\rho$	specific total energy (cgs)

If the magnetic field $\boldsymbol{B}$ is strong enough, 'magnetic waves rather than sound waves may carry the information from one region to another. The Alfvén velocity is defined as

$$\boldsymbol{v}_{\mathrm{A}} = \boldsymbol{B}/(4\pi\rho)^{1/2}\ . \quad \text{(cgs)}$$

When the wave vector is aligned with the magnetic field, the magnetic Alfvén waves are transverse oscillations of the magnetic field, like waves in a skipping rope, and so these do not help to equalize the pressure. In this case, as in the magnetic-field free case, the pressure adjustment (a compression wave) is carried by longitudinal sound waves. However, when the direction of the wave vector is perpendicular to the magnetic field direction, information is carried by longitudinal magnetosonic waves at the magnetosonic sound speed'

$v_{\mathrm{ms}} = (c_{\mathrm{s}}^{\,2}+v_{\mathrm{A}}^{\,2})^{1/2}$ where c_{s} is the sound speed in the ionized plasma

(Dopita & Sutherland 2003). In fact, Alfvén waves can "warn" upstream ions and electrons about an approaching density discontinuity.

If the magnetic field is frozen in the flow, the maximum compression factor is

$\rho_1/\rho_0 = 2^{1/2}\, M_{\mathrm{A}}$ where M_{A} is the Alfvén Mach number

The following is excerpted from Carroll & Ostlie 2017; Harwit 1998; Dopita & Sutherland 2003.

The inclusion of magnetic fields in the study of plasmas is called Magnetohydrodynamics (MHD). Hydromagnetic shocks are particularly important where collisions between particles are rare: the magnetic fields are the main conveyors of pressure throughout the medium. Pressure equilibrium in such conditions is achieved through interaction between particles via ______________________. The speed at which information on pressure differences is conveyed hydromagnetically is the Alfvén speed; in magnetized plasmas it replaces the speed of sound.

The presence of a magnetic field allows for transverse waves that propagate along the magnetic field lines, due to the restoring force of their tension. Recall that the establishment of a magnetic field requires energy (e.g., moving charges). Think of the magnetic energy density ($B^2/2\mu_0$ in SI units) within magnetic fields as created by this process.

If a volume V of plasma is compressed perpendicular to the magnetic field lines, the density of field lines increases (the field increases its strength). Since the mechanical work done to compress the field lines is $W = \int P \, dV$, this implies the presence of a magnetic pressure.

When a magnetic field line gets displaced perpendicular to the field, the pressure in the direction of the displacement _________ while the pressure in the opposite direction _________. This pressure change tends to push back on the compressing force (a restoring force). This is analogous to the oscillations that occur in a string when a portion of a taut string is displaced: the tension pulls in the opposite direction of the 'pluck'. As with the resulting transverse wave on a string, a disturbance in the magnetic field can also propagate down the field line. It travels at the Alfvén speed.

Drivers of Interstellar Shocks (adapted from Dopita & Sutherland)

An interstellar shock requires a compressive piston, and various astrophysical phenomena provide natural pistons:

- Supernovae
- Radiatively-driven stellar winds
- Outflows from newly-born low-mass stars
- Outflows from the nuclei of active galaxies (relativistic flows)
- Expanding Strömgren spheres – See Tielens Section 12.2.3

Theory and observation references: Dyson & Williams 1997 and Lozinskaya 1992

Basic scenario:

A high pressure gas expands supersonically and encounters a stationary gas. The expanding gas pushes on the stationary gas, driving a shock into it. The formerly stationary gas is heated, increasing its pressure, and starts moving 'outwards'. However, the energy to do this must come from the kinetic energy of the expanding medium, so this medium abruptly slows down. The only way this can happen is through another shock, an 'inner' or 'reverse' shock (as opposed to the original shock or 'blast wave'). The velocities of these two shocks are not in general the same, and hence, their temperatures differ. However, their pressures must be the same. Between the two gases, there must therefore be a jump in temperature accompanied by a jump in density such that nT remains constant. The gas on both sides of such a discontinuity is moving out at the same velocity.

Drivers of Interstellar Shocks

Another option is to have a single shock (either forward or reverse) together with a rarefaction wave, e.g., Bicknell & Wagner 2002 (PASA, 19, 129; figure below).

Note there are also instances of slow ($V_s \leq$ 50 km/s) shocks in weakly ionized molecular gas, where the energy transfer between the ionized and neutral species may require a timescale longer than the cooling timescale. Here, the flow variables change continuously, and such shocks are known as *continuous* or *C-shocks* (Draine 1980).

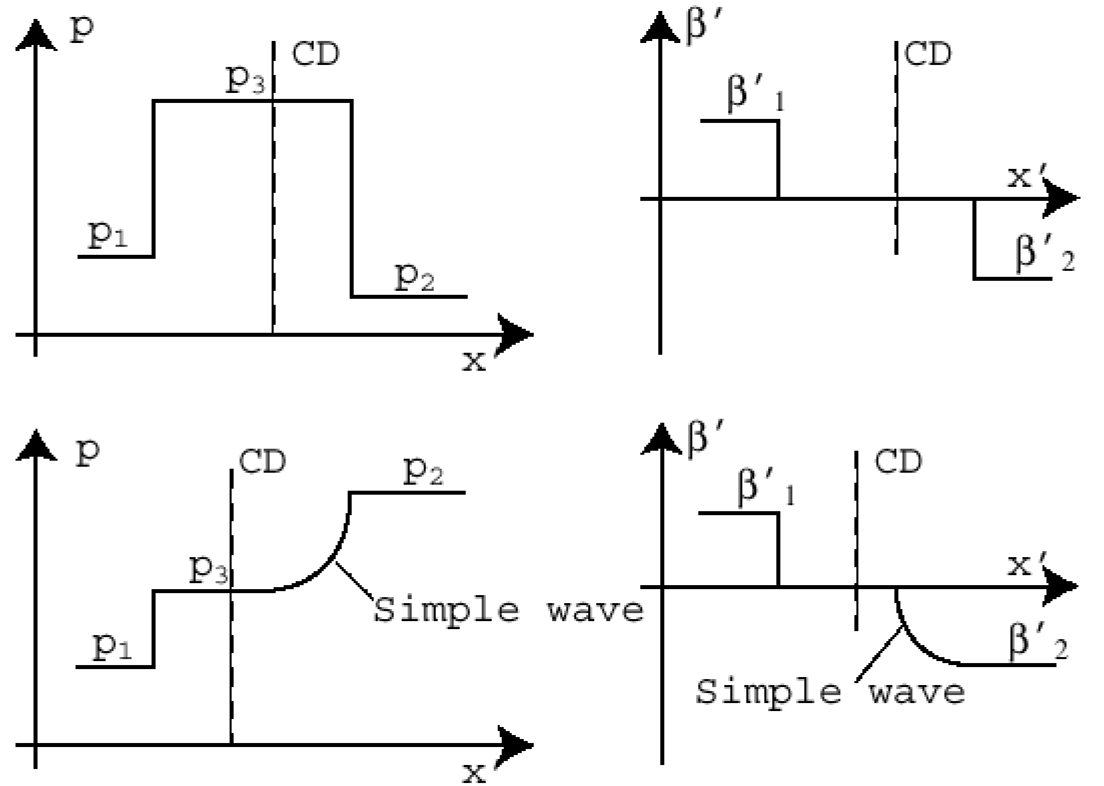
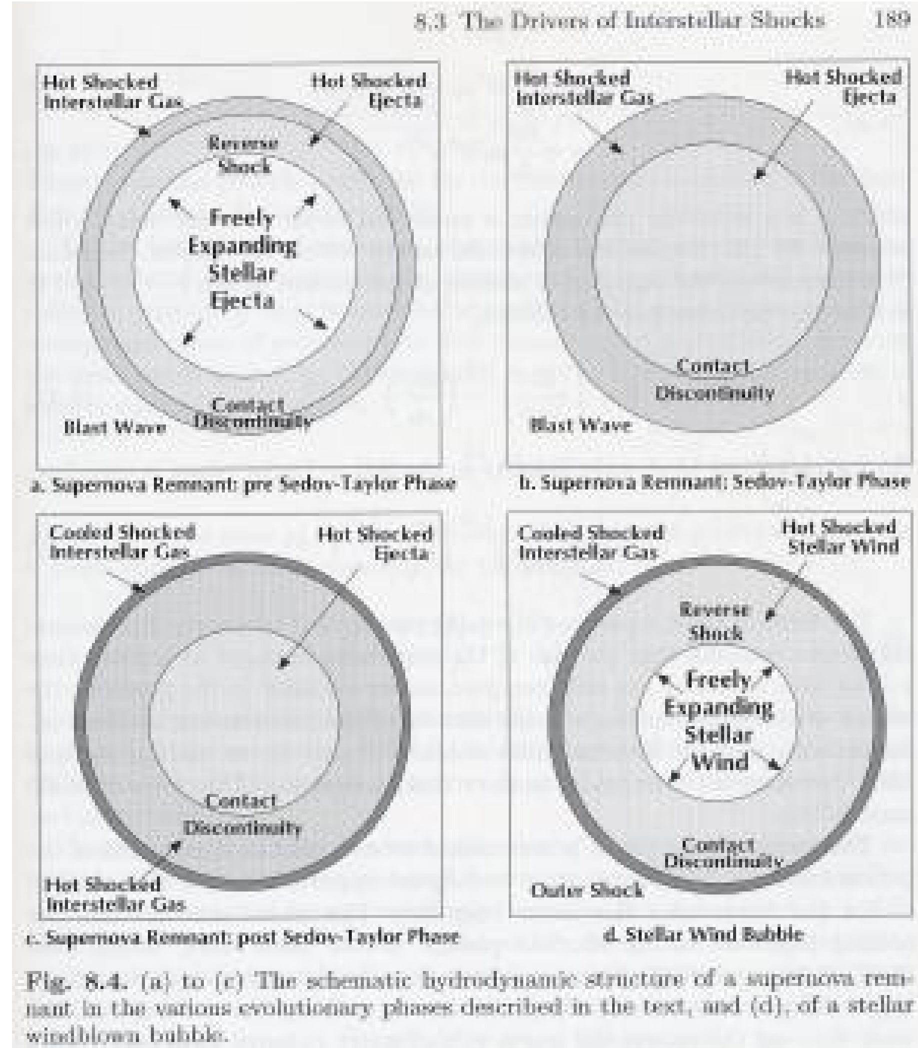


Figure 6: The behaviour of pressure and velocity in a relativistic shock tube. The upper part of the diagram is for the case wherein a forward and reverse shock form; the lower part of the diagram is for the case of a reverse shock plus a relativistic simple wave. The frame of the contact discontinuity is also the rest frame of the emitting plasma.

Drivers of Interstellar Shocks

<u>Supernova Explosions</u>

A supernova explosion typically deposits about _______ of kinetic energy into the ISM. The explosion for massive stars is due to the outward pressure exerted by outflowing neutrinos produced by the eventual collapse to a neutron star or black hole. Considering that ordinary matter is extremely optically thin to neutrinos (... can pass through light years of lead ...), this truly emphasizes the extremes in sizescales and densities we typically encounter in astrophysics! For low-mass stars, the process involves thermo-nuclear explosions of stellar material under initially electron-degenerate conditions.



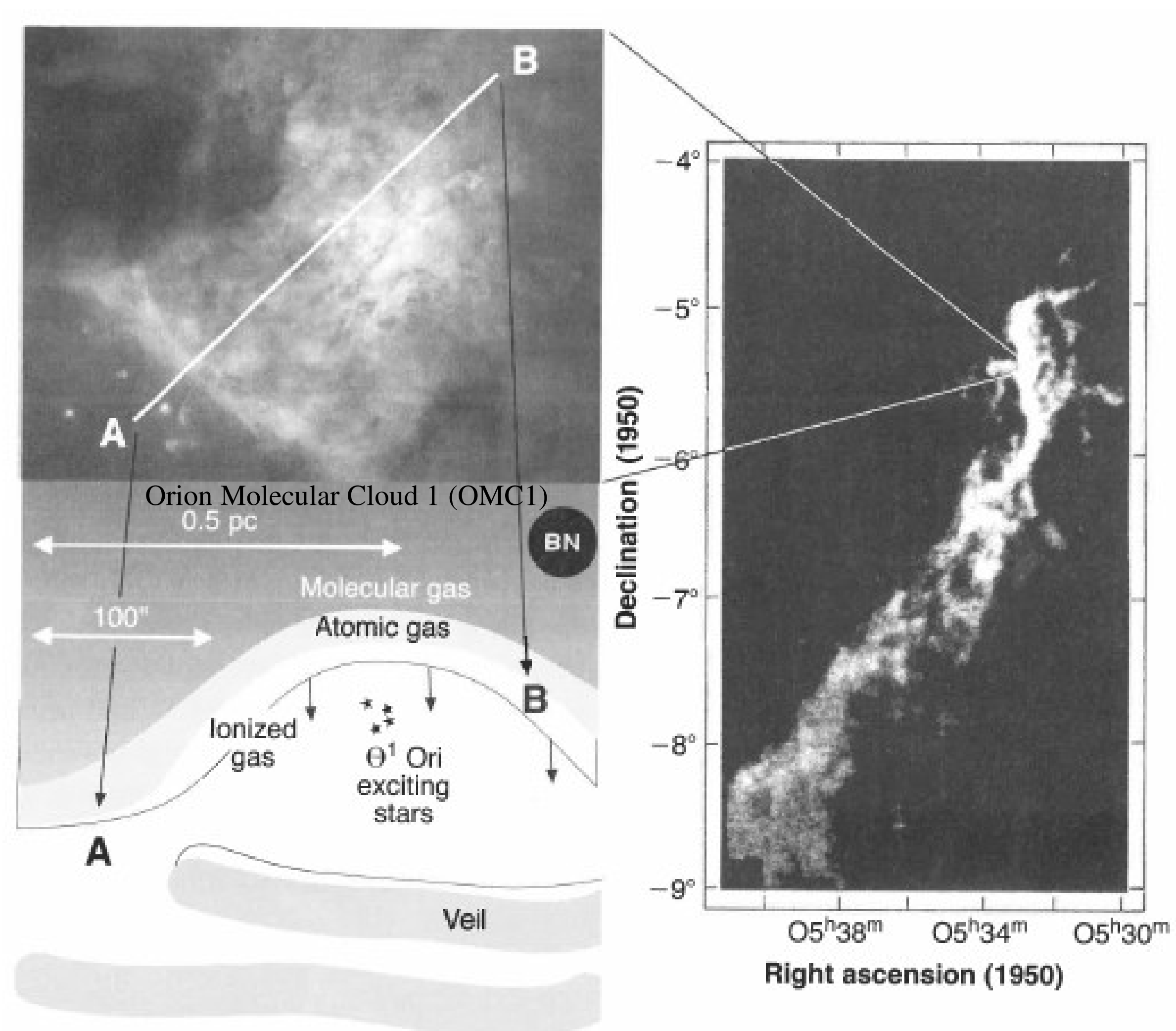
Dopita & Sutherland 2003

The initial blast wave is slowed down as ISM is swept up. When the mass of interstellar gas that has been swept up is quite a bit larger than the mass ejected, the reverse shock has swept back to the center and all of the ejecta have been shocked to high T. This marks the beginning of the Sedov-Taylor phase in which the expanding bubble of hot plasma drives the blast wave outward.

Stellar Wind Bubbles: The Orion Nebula M42, a case study

(O'Dell 2001 review; Osterbrock & Ferland 2005 book; Jensen 2003 notes)

M42 is not the largest, most luminous, nor highest surface brightness HII region. But it is the most studied, with ~100 papers published/year on it.



Osterbrock & Ferland 2005

'M42 is a component of the ridge of giant molecular clouds in the constellation Orion, which are part of the Eridanus superbubble. The long ridge of molecular material extending N-S through the belt of Orion and its sword region is home to several star associations of various young ages, with M42 probably the youngest. Even the host cloud OMC-1 appears to have produced three regions of massive SF, but also nearby are embedded groupings to the NW and SW of the tight clustering of early type stars a.k.a The Trapezium.'

O'Dell 2001

The Orion Nebula M42

The radial velocity of the Orion Nebula, including θ^1 Ori C, with respect to the Sun is 28 km/s. How are the different components listed in Table 1 of O'Dell (2001) moving with respect to the star?

The Orion molecular cloud has a density of ~10^{11} m^{-3} and temperature ~100 K, whereas the ionized region has density ~5 10^9 m^{-3} and temperature ~10^4 K. Given this, can you explain the sequence (and signs) of the radial velocities in Table 1?

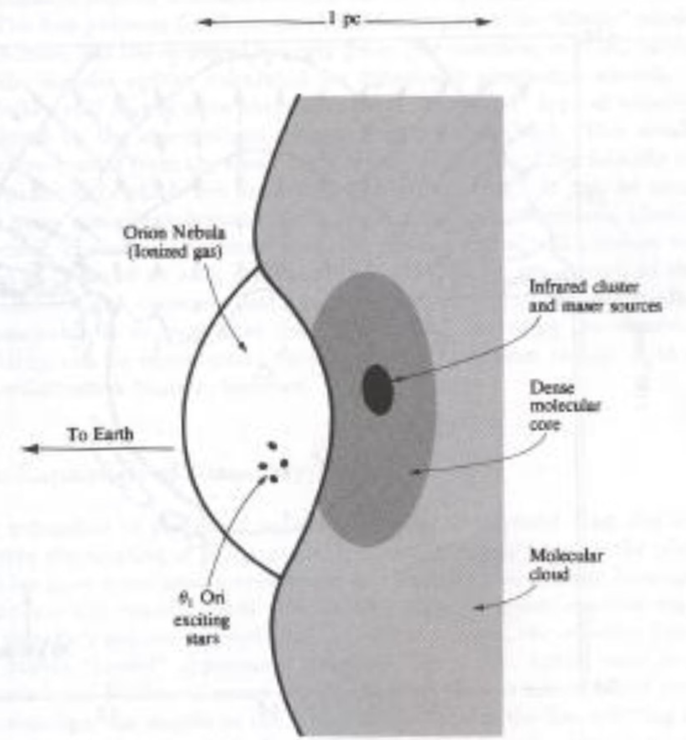


FIGURE 6.4
Schematic sketch of NGC 1976, the Orion Nebula, an ionized blister of gas expanding away from the Orion molecular cloud, into which the ionization front is moving.

Osterbrock & Ferland 2005

TABLE 1 The ionization front behind θ^1 Ori C

	Key ion	Markers	$V_\odot$ (km s^{-1})	Density (cm^{-3})	Depth (pc)
PDR	H^0	CO, C II	28	10^5	?
IF	H^+	[O I], [S II]	25.5	≥6000	10^{-4}
Low ionization	He^0	[O II], [N II]	18.8 ± 1.5	7000	2×10^{-3}
Medium ionization	He^+	[O III], H II, He I, [Cl III]	17.9 ± 1.3	4000	0.06

O'Dell 2001

Stellar Wind Bubbles, continued; Dopita & Sutherland 2003

Hot stars produce fast winds driven by radiation pressure. The outward momentum flux is

$$dM_w/dt\ v_w = \eta L_*/c$$

where $dM_w/dt \sim 10^{-5}\ M_\odot\ \mathrm{yr}^{-1}$ outward mass flux

and $v_w \sim \varepsilon(GM_*/r_*)^{1/2} \sim 1000$ to 4000 km/s the terminal wind velocity

'The wind flows out in free expansion until encountering the ISM. At this point it passes through an adiabatic shock at an inner radius R_{inner}. The pressure P throughout the region between the inner and outer shock can be taken as approximately constant, since the hot gas has a sound speed of order 500 km/s, and therefore the sound-crossing timescale in the hot plasma is much shorter than the dynamical expansion timescale of the bubble, which has an expansion speed in the range 20-100 km/s. The pressure in the hot plasma is given by the rate of change of momentum per unit area of the stellar wind across the inner shock:

$$P = 3\ dM_w/dt\ v_w\ /\ 16\ \pi\ R^2_{inner}.$$

The relatively low expansion velocity of the bubble will ensure that the outer shock is radiative and isothermal at the temperature (~10,000 K) of the pre-shock gas, which is kept ionized by the photons from the central star.'

Review Question (adapted from Harwit 2000)

For cool post-shocks, γ=5/3 for either monatomic or diatomic gases, and the compression attained in an adiabatic shock cannot exceed

$\rho_1/\rho_0 =$

Question:

For a hot shock (T~1000 K), will the compression be larger or smaller than the above value for cool shocks? Give both a conceptual and a quantitative explanation. Does it matter if it is a monatomic or diatomic gas?

Review Question

Since H_2 involves only **two** atoms, why are there **two** degrees of freedom due to vibration for hot molecular gas?

a) Vibration occurs both 'outwards' and 'inwards'

b) Vibration occurs both radially and tangentially (like a dog wagging its tail)

c) Both the kinetic and potential of the oscillator must be taken into account

d) Two particles are vibrating, not one

Review Question

The table below indicates that the simplest molecule with the smallest moment of inertia, molecular hydrogen, requires the highest temperatures to induce rotation. However, since the Equipartition Theory suggests ½kT for every degree of freedom, and since the rotational energy is $\frac{1}{2}I\omega^2$, wouldn't you expect H_2 to require the lowest temperature to rotate?

i.e., isn't $T_{rot} \propto I$?

a) Evil Martians inserted this conspiracy into our textbooks

b) The above reasoning is flawed since it ignores the dependence of ω on I.

c) Moment of inertia has nothing to do with rotational energy

d) Actually, H_2 has a relatively large moment of inertia since the atomic spacing is so large compared to that in the other listed diatomic molecules; i.e., CO, OH, etc have very small atomic spacings, and that offsets their larger atomic masses.

Characteristic Temperatures

Characteristic temperature of vibration of diatomic molecules	
Substance	θ_{vib}(K)
H_2	6140
O_2	2239
N_2	3352
HCl	4150
CO	3080
NO	2690
Cl_2	810

Characteristic temperature of rotation of diatomic molecules	
Substance	θ_{rot}(K)
H_2	85.4
O_2	2.1
N_2	2.9
HCl	15.2
CO	2.8
NO	2.4
Cl_2	0.36

see Sears & Salinger 1986

Application to thermal instabilities (e.g., Field 1965; Balbus 1995; Dopita & Sutherland 2003)

Compressive shocks can trigger collapse, but do not guarantee permanent compression without cooling (Harwit p. 416)

A region with a certain temperature will cool on a timescale τ_{cool} (see p. 401 Tielens). Consider an embedded region with a slightly lower temperature. The medium will be thermally unstable if the temperature contrast between the warmer and cooler regions increases with time. This would happen if the cooling timescale in the cooler region is less than the cooling timescale in the hotter region.

If the cooler region is sufficiently small that sound waves can travel across it in less than τ_{cool}, then the pressures in the hotter and cooler region will equalize during the cooling, and the cooling will occur *isobarically* (at constant _________).

On the other hand, if the region is large, the cooling will (at least initially) proceed *isochorically* (at constant _________).

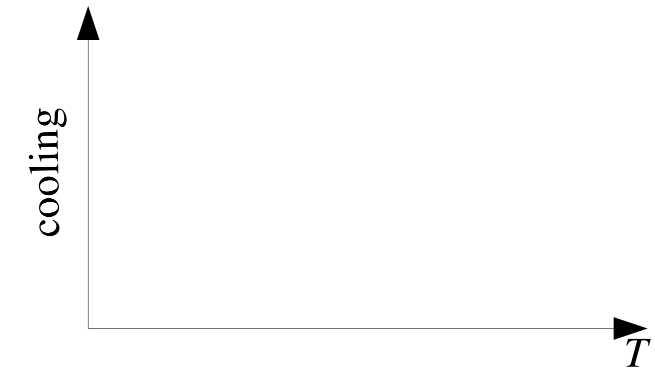
Isobaric cooling is particularly unstable because $\tau_{cool} \propto n^{-1}$ and $n \propto T_e^{-1}$. The cooling timescale in the lower temperature and higher density regions decreases increasingly rapidly during the cooling, and the cooling collapse is said to 'run away'.

Since the sound speed and the cooling timescale both decrease with decreasing temperature, the size of the regions within which isobaric cooling is occurring becomes smaller and smaller, so that the medium tends to break up into a fractal hierarchy of blobs and filaments.

Q: One way to parametrize a shock width is by its 'cooling length'. Exactly how does a shock cool? Would it be different for a monatomic gas than for a diatomic gas?

Q: Why does the gas inside a supernova bubble not cool very much in the early phases of the supernova expansion? In general, how can ISM cooling increase with decreasing temperature?

Plot cooling vs *T* for shocked gas.



Q: The first phase in the expansion of a supernova is called *free expansion*, during which the ejecta expand at a constant velocity. From momentum considerations, the speed will not change significantly until the swept-up mass approximately equals the ejecta mass. For a supernova that ejects 4 solar masses at 5,000 km/s into an HI medium of density 10^7 m^{-3}, find the radius and time at which the free expansion phase ends.

Large Scale Flows in Star-Forming Regions

Describe the transition from the *energy-conserving* phase to the *momentum-conserving* phase of a shock. Is momentum conserved during the *energy-conserving* phase? Is energy conserved during the *momentum-conserving* phase?

Shock velocities decrease with time. As the shock velocity decreases, the shock temperature decreases ($T_s \propto V_s^2$). And as the shock temperature drops, the cooling of the gas can increase. 'The cooling can be quite catastrophic and results in the formation of a thin shell of cool material immediately behind the shock ... [compared to massive stars, for low mass stars with low shock velocities] ... the shocked wind loses all its thermal energy *immediately* as it passes through the shock. Consequently, the adiabatic expansion of shocked wind gas does *not* drive the shell of swept-up gas. Rather, this is driven by the *momentum* in the wind' (D&W pp. 132, 150-1).

In other words, the pressure associated with shocked winds for high mass stars can generate their own momentum, whereas for low mass stars the total momentum in the flow is equal to the sum of all momentum contributions from the stellar wind (it can't create additional momentum).

Argue from basic principles why the shock temperature is proportional to V_s^2.

Q: How could a pure Hydrogen plasma cool?

a) bremsstrahlung

b) it couldn't: astroplasmas always have, at some level, a population of heavier elements and that's how they cool: collisions bump bound electrons up and they cascade back down (e.g., forbidden lines). A pure H plasma couldn't cool because there are no bound electrons to be bumped up by collisions.

c) recombination – speedy electrons are captured and thus lose their kinetic energy via the ensuing radiative cascade down to the n=1 energy level

d) using a very big fan, plus a fine water mist

For gas cooling we've mentioned various possible mechanisms. Let's discuss the recombination option described in choice c above. Suppose a speedy electron was captured in an extremely high energy orbital (e.g., n=1000, say). What happens to its kinetic energy? What is the energy source of the photons emitted via the ensuing cascade?

Planetary Nebulae (Osterbrock 1989)

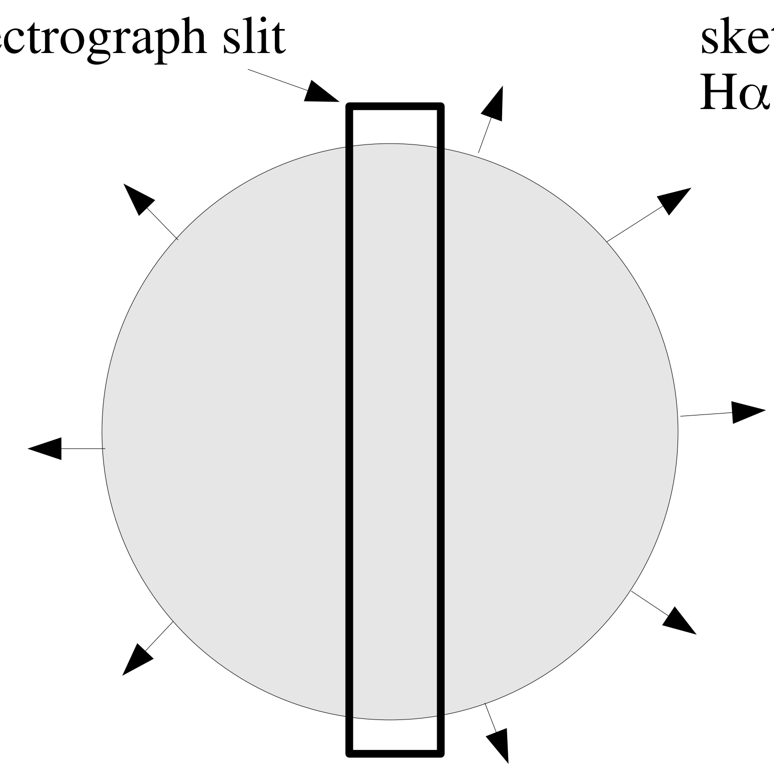
PNe are hot ionized gas clouds that expand into a surrounding near-vacuum. Following the laws of an adiabatic expansion, and assuming initially at rest, the outward velocity is given by

$v_{exp} = 2c_\gamma/(\gamma\text{-}1)$ where c_γ is the adiabatic sound speed.

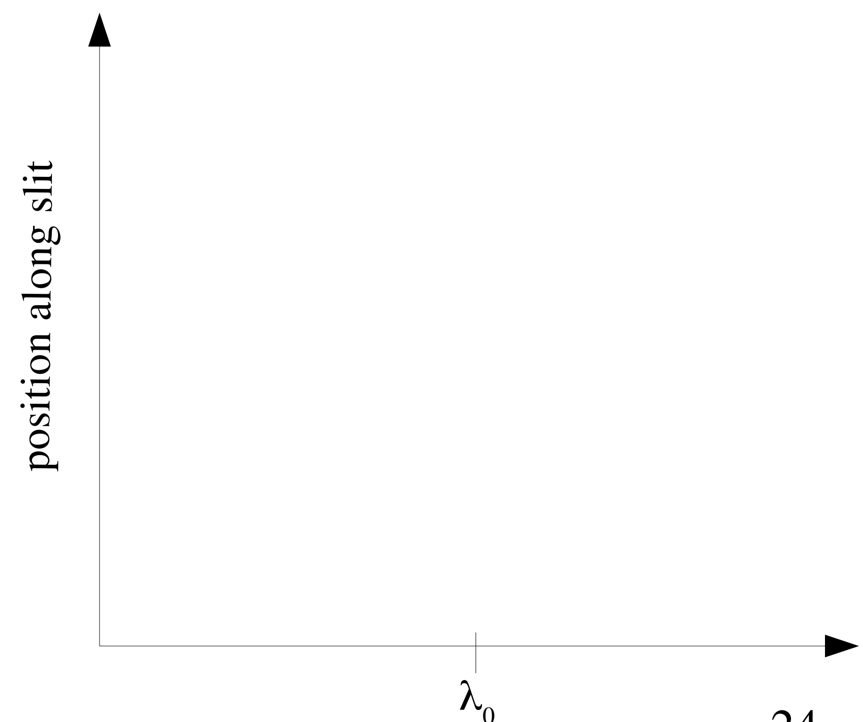
A rarefaction wave moves inwards into the undisturbed gas at c_γ. Thus the outer edge of the expansion, and the inner edge of the rarefaction, at time t have reached radii

$r_{exp} = r_0 + v_{exp}t$ and $r_{rar} = r_0 - c_\gamma t$.

All the gas between these radii is moving inward/outward (circle one) with speed increasing/decreasing from ____ at r_{rar} to _____ at r_{exp}. For a spherical nebula, the inward-running rarefaction wave ultimately reaches the center of the nebula and is reflected, and the gas near the center is then further accelerated outward.

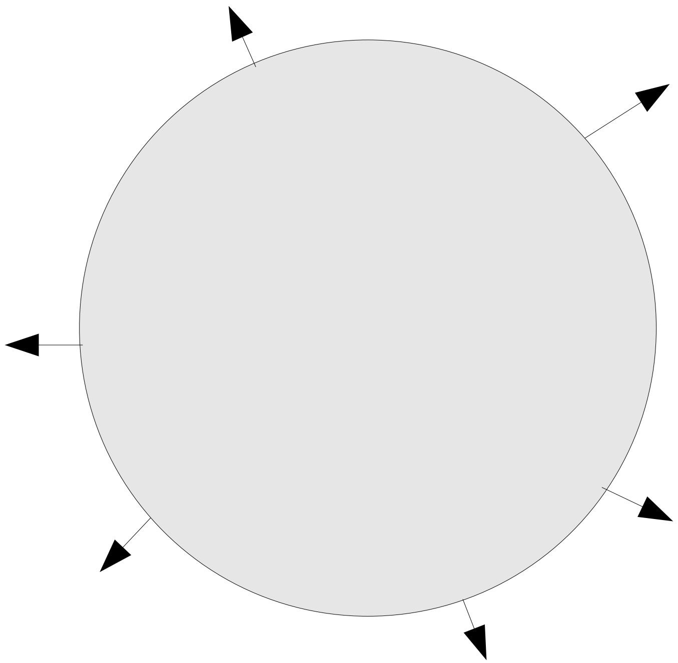


sketch the observed Hα line spectrum



Planetary Nebulae (Osterbrock 1989)

How do you think the ionization trends with expansion velocity?



Final Exam hints

A signal-to-noise ratio of unity (*S/N*=1) is by definition simply noise. The minimum *S/N* to be considered as a detection is 3. A *S/N* of 5 or 10 is a more standard goal in a proposal for emphasizing that the plan is to obtain secure detections.

Rule-of-thumb: increasing the integration time by a factor of β yields an increase in *S* by a factor of β and an increase in *N* by a factor of sqrt(β), and thus an increase in *S/N* by a factor of sqrt(β). Thus, to double the *S/N*, you need to quadruple the integration.
→ This rule-of-thumb doesn't apply for non-linear responses or situations that are limited by 'systematics' (e.g., flat-fielding, readout noise, etc.).

The noise in a spectrum can be estimated by the standard deviation in the continuum level; $\pm1\sigma$ (or a 2σ width) encompasses 68.3% of the data points for a normal distribution.

Example:
Suppose the observatory staff tell you that a source of flux X observed for time Y yields *S/N*=Z → [*f*=X,*t*=Y,*S/N*=Z]
Furthermore, suppose your source flux is B. This translates the above combination to [*f*=B,*t*=Y,*S/N*=sqrt(B/X)Z].
But suppose your targeted *S/N* is C not sqrt(B/X)Z. In that case, we have [*f*=B,*t*=Y(C/(sqrt(B/X)Z))^2,*S/N*=C].

Note: For most galaxy spectra, FIR is ~ ½TIR.