Do the following problems and be prepared to discuss them in class.

1. Emission Line Profiles

Emission lines are sometimes modeled with a Lorentzian profile, $I(\nu) \propto \frac{\gamma}{(\nu-\nu_0)^2+(\gamma/4\pi)^2}$, and sometimes with a Gaussian profile, $I(\nu) \propto \exp[-(\nu-\nu_0)^2/2\delta^2]$.

a) Show that the full width of the line profile at half-maximum (FWHM) is given by $\gamma/2\pi$ for a Lorentzian and by 2.355δ for a Gaussian.

b) Show that half of the total energy radiated in a (Lorentzian) emission line comes from the FWHM bandwidth centered on the line center.

2. Doppler Broadening

Calculate the Doppler FWHM for a gas of H atoms radiating at a wavelength of 100 nm at a temperature of 300 K. Show that collisional broadening (or just think collisional timescale) in such a gas will not become comparable until the number density is approximately 10^{27} m^{-3} (use a geometric cross-section for H atom collisions).

3. Emission Line Wavelength

Show that the wavelength of the $J = 1 \rightarrow 0$ rotational transition in ^{12}C^{32}S is approximately 6 mm given that the C–S equilibrium separation is 1.535×10^{-10} m.

4. The Multi-Phase Milky Way

a) Give two reasons why the Galactic plane is hardly visible at optical wavelengths while

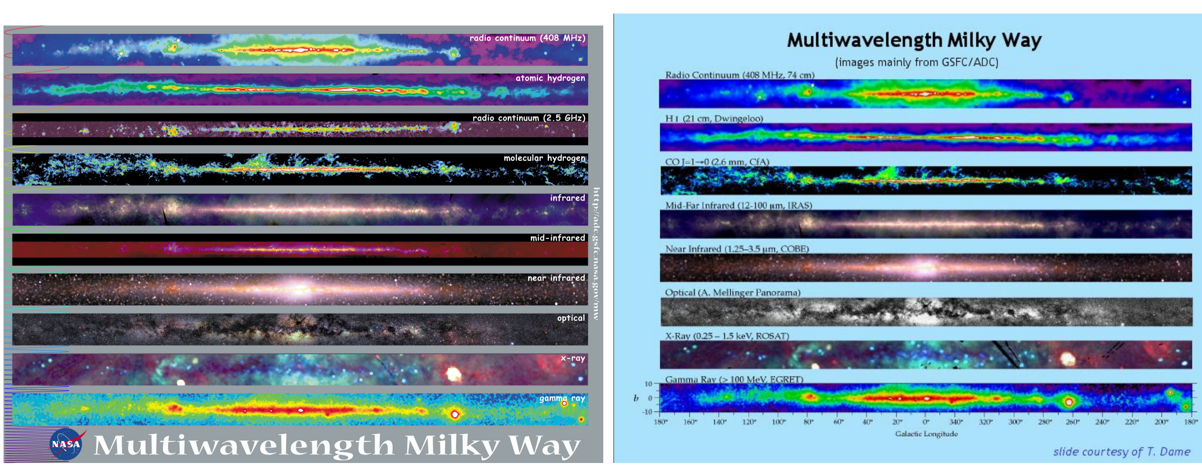

Fig. 1.— Multi-wavelength portraits of the Milky Way

it is very prominent at near-infrared through far-infrared wavelengths.
b) Explain why the mid-plane of the Galaxy is dominated by relatively hard X-ray emission (1.5 keV), while emission at 0.25 keV dominates at higher latitudes.
c) Why is the diffuse γ-ray emission an excellent tracer of interstellar gas?
d) Decribe and explain the appearance of the supernova remnant, Cas A ($l = 112^\circ$), at the various wavelengths.
e) At $(l, b) = (310, 0)^\circ$, a discrete object is visible at certain wavelengths. What kind of object might this be? Explain your answer.
f). Explain why the Crab pulsar is visible (at $l = 185^\circ$) in the radio, X-ray, and γ-ray maps.

5. The Milky Way in X-Rays
The effective cross section for absorption of X-rays by hydrogen atoms is $1.7\times10^{-27}\left(\frac{\text{keV}}{E}\right)^3$ m^2, where E is the energy of the X-rays. X-ray astronomers find that one can "see" (in the sense of the optical depth being less than 1) all the way through the center of the Galaxy and out the other side, a distance of $\sim$ 25 kpc, at X-ray energies of 1.0 keV and above. Derive from this observation the number density of hydrogen atoms in the Galaxy.

6. The Milky Way's Magnetic Field
A simple argument which gives an estimate of the magnetic field in the Galaxy is that it must have an energy density comparable to that of the cosmic rays, in order to confine them to the Galaxy (if they are not confined, but rather fill all of intergalactic space, then it is very difficult to come up with an energy source for them). For a cosmic ray energy density of 0.1 eV cm^{-3}, what is the magnetic field?

7. The Rayleigh-Jeans Law and the Planck Function
a) At a temperature of 100 K, for what range of wavelengths is the Rayleigh-Jeans law a good approximation (to better than 1%) to the Planck function?
b) At 21 cm, for what range of temperatures is the Rayleigh-Jeans law a good approximation (to better than 1%) to the Planck function?

8. Radio Brightness Temperatures
Suppose you observe a background continuum radio source (such as a quasar) of brightness temperature T_0 which is behind two different absorbing clouds with source temperatures T_1, T_2 and optical depths τ_1, τ_2. Write down an expression for the brightness temperature T_b observed at the Earth as a function of the five parameters $T_0, T_1, T_2, \tau_1, \tau_2$.