

Do the following problems and be prepared to discuss them in class.

1. Cooling Time

Define a cooling time t_c by $t_c = -T/(dT/dt)$. Consider a 200 K cloud for which $n(\text{H}) = 10^8 \text{ m}^{-3}$, $n(\text{e}^-) = 10^5 \text{ m}^{-3}$, and $n(\text{C}^+) = 4 \cdot 10^4 \text{ m}^{-3}$. Suppose that cooling occurs solely by the excitation of transitions in C^+ by collisions with electrons:

$$\Lambda_{\text{e}^-, \text{C}^+} = 8 \cdot 10^{-33} \frac{n(\text{e}^-) n(\text{C}^+)}{\text{m}^{-3} \text{ m}^{-3}} \sqrt{\frac{\text{K}}{T}} e^{-(92\text{K}/T)} \text{ J m}^{-3} \text{ s}^{-1}$$

Show that the cooling time, t_c , is of order 10^4 years.

2. Cooling Rate

Show that $n(\text{H}_2) \sim 10^7 \text{ m}^{-3}$ is necessary in a cloud of density $n = 10^8 \text{ m}^{-3}$ at a temperature of 100 K to give a cooling rate due to H_2 equal to that due to C^+ and e^- , if $n(\text{e}^-) = n(\text{C}^+) = 10^4 \text{ m}^{-3}$. Assume that $\Lambda_{\text{H}_2} = 3 \cdot 10^{-33} \frac{n(\text{H}_2)}{\text{m}^{-3}} \text{ J m}^{-3} \text{ s}^{-1}$ and that the cooling due to C^+ is still the equation provided in Problem 1.

3. Heating Rate

Define α as the recombination rate coefficient for $\text{X}^+ + \text{e}^- \rightarrow \text{X} + h\nu$. Also, define E as the excess energy released in the counterpart photoionization process.

- Show that the heating rate due to photoionization of atom X is $\alpha n(\text{e}^-) n(\text{X}^+) E$.
- Show that the heating due to C atoms in the cloud of Problem #1 is $1.3 \cdot 10^{-26} \text{ J m}^{-3} \text{ s}^{-1}$. Use $\alpha = 10^{-17} \text{ m}^3 \text{ s}^{-1}$ and see your class notes for the typical value of E in this case.
- At which temperature T is the heating and cooling due to C^+ approximately balanced for any cloud?

4. Heating and Cooling

Determine the fraction of hydrogen in molecular form, $f = 2n(\text{H}_2)/n$, to maintain a cloud temperature of 100 K by H_2 cooling in a cloud heated by photoelectric emission grains at a rate of $10^{-28} \text{ J m}^{-3} \text{ s}^{-1}$. Hint: see class notes and Problem #2 above.

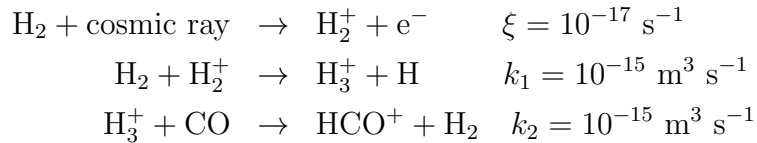
5. Reaction Rate Coefficients

The molecule AB is made in the following ways: $\text{A} + \text{B} \rightarrow \text{AB} + h\nu$ and $\text{A} + \text{BC} \rightarrow \text{AB} + \text{C}$ with respective rate coefficients of k_1 and k_2 . AB is lost in the reaction $\text{AB} + \text{D} \rightarrow \text{A} + \text{BD}$ (rate coefficient k_3) and in photodissociation $\text{AB} \rightarrow \text{A} + \text{B}$ (rate β). After reviewing Section 4.1 of the text, show that an expression for the equilibrium abundance of AB in terms of other abundances is

$$n(\text{AB}) = n(\text{A})[k_1n(\text{B}) + k_2n(\text{BC})]/[k_3n(\text{D}) + \beta].$$

6. Steady State

Suppose that the chemistries of H_2 , H_2^+ , and H_3^+ are described by



If $n(\text{CO}) = 10^{-4}n(\text{H}_2)$ and $n(\text{H}_2) = 10^{10} \text{ m}^{-3}$, find the steady state densities of H_2^+ and H_3^+ .
Hint: Section 4.1 of the text.

7. Steady State

For the chemistry of Problem #6,

- Write down the differential equation that determines the time dependence of H_2^+ .
- Show that $n(\text{H}_2^+)$ approaches steady state on a timescale $[k_1n(\text{H}_2)]^{-1}$.
- If the typical lifetime of a cloud is 10^6 years, is steady state a good approximation for the abundance of H_2^+ ?

8. Emission and absorption: cooling and heating

- Why isn't the absorption of a photon by an atom or molecule in an interstellar cloud necessarily a heating event? It deposits energy into the cloud, but what else must happen for it to heat the cloud?
- Why isn't the emission of a photon by an atom or molecule in an interstellar cloud necessarily a cooling event?