Do the following problems and be prepared to discuss them in class.

1. Cooling Time

Define a cooling time t_c by $t_c = -T/(\mathrm{d}T/\mathrm{d}t)$. Consider a 200 K cloud for which $n(\mathrm{H}) = 10^8\ \mathrm{m}^{-3}$, $n(\mathrm{e}^-) = 10^5\ \mathrm{m}^{-3}$, and $n(\mathrm{C}^+) = 4 \cdot 10^4\ \mathrm{m}^{-3}$. Suppose that cooling occurs solely by the excitation of transitions in C^+ by collisions with electrons:

$\Lambda_{\mathrm{e}^-,\mathrm{C}^+} = 8 \cdot 10^{-33}\ \frac{n(\mathrm{e}^-)}{\mathrm{m}^{-3}} \frac{n(\mathrm{C}^+)}{\mathrm{m}^{-3}} \sqrt{\frac{\mathrm{K}}{T}}\ e^{-(92\mathrm{K}/T)}\ \mathrm{J\ m^{-3}\ s^{-1}}$

Show that the cooling time, t_c, is of order 10^4 years.

2. Cooling Rate

Show that $n(\mathrm{H}_2) \sim 10^7\ \mathrm{m}^{-3}$ is necessary in a cloud of density $n = 10^8\ \mathrm{m}^{-3}$ at a temperature of 100 K to give a cooling rate due to H_2 equal to that due to C^+ and e^-, if $n(\mathrm{e}^-) = n(\mathrm{C}^+) = 10^4\ \mathrm{m}^{-3}$. Assume that $\Lambda_{\mathrm{H}_2} = 3 \cdot 10^{-33}\ \frac{n(\mathrm{H}_2)}{\mathrm{m}^{-3}}\ \mathrm{J\ m^{-3}\ s^{-1}}$ and that the cooling due to C^+ is still the equation provided in Problem 1.

3. Heating Rate

Define α as the recombination rate coefficient for $\mathrm{X}^+ + \mathrm{e}^- \rightarrow \mathrm{X} + h\nu$. Also, define E as the excess energy released in the counterpart photoionization process.

a) Show that the heating rate due to photoionization of atom X is $\alpha n(\mathrm{e}^-) n(\mathrm{X}^+) E$.

b) Show that the heating due to C atoms in the cloud of Problem #1 is $1.3 \cdot 10^{-26}\ \mathrm{J\ m^{-3}\ s^{-1}}$. Use $\alpha = 10^{-17}\ \mathrm{m^3\ s^{-1}}$ and see your class notes for the typical value of E in this case.

c) At which temperature T is the heating and cooling due to C^+ approximately balanced *for any cloud*?

4. Heating and Cooling

Determine the fraction of hydrogen in molecular form, $f = 2n(\mathrm{H}_2)/n$, to maintain a cloud temperature of 100 K by H_2 cooling in a cloud heated by photoelectric emission grains at a rate of $10^{-28}\ \mathrm{J\ m^{-3}\ s^{-1}}$. Hint: see class notes and Problem #2 above.

5. Reaction Rate Coefficients

The molecule AB is made in the following ways: A+B→AB+$h\nu$ and A+BC→AB+C with respective rate coefficients of k_1 and k_2. AB is lost in the reaction AB+D→A+BD (rate coefficient k_3) and in photodissociation AB→A+B (rate β). After reviewing Section 4.1 of the text, show that an expression for the equilibrium abundance of AB in terms of other abundances is

$n(\mathrm{AB}) = n(\mathrm{A})[k_1 n(\mathrm{B}) + k_2 n(\mathrm{BC})]/[k_3 n(\mathrm{D}) + \beta]$.

6. Steady State

Suppose that the chemistries of H_2, H_2^+, and H_3^+ are described by

$$\begin{array}{rcll} \mathrm{H_2} + \text{cosmic ray} & \rightarrow & \mathrm{H_2^+} + \mathrm{e^-} & \xi = 10^{-17}\ \mathrm{s^{-1}} \\ \mathrm{H_2} + \mathrm{H_2^+} & \rightarrow & \mathrm{H_3^+} + \mathrm{H} & k_1 = 10^{-15}\ \mathrm{m^3\ s^{-1}} \\ \mathrm{H_3^+} + \mathrm{CO} & \rightarrow & \mathrm{HCO^+} + \mathrm{H_2} & k_2 = 10^{-15}\ \mathrm{m^3\ s^{-1}} \end{array}$$

If $n(\mathrm{CO}) = 10^{-4} n(\mathrm{H_2})$ and $n(\mathrm{H_2}) = 10^{10}\ \mathrm{m^{-3}}$, find the steady state densities of H_2^+ and H_3^+. Hint: Section 4.1 of the text.

7. Steady State

For the chemistry of Problem #6,

a) Write down the differential equation that determines the time dependence of H_2^+.

b) Show that $n(\mathrm{H_2^+})$ approaches steady state on a timescale $[k_1 n(\mathrm{H_2})]^{-1}$.

c) If the typical lifetime of a cloud is 10^6 years, is steady state a good approximation for the abundance of H_2^+?

8. Emission and absorption: cooling and heating

a) Why isn't the absorption of a photon by an atom or molecule in an interstellar cloud necessarily a heating event? It deposits energy into the cloud, but what else must happen for it to heat the cloud?

b) Why isn't the emission of a photon by an atom or molecule in an interstellar cloud necessarily a cooling event?