Do the following **ten problems** and be prepared to discuss them in class.

1. Extinction

Suppose that the interstellar medium contains spherical dust grains with radius 10^{-7} m and uniform number density 10^{-6} m^{-3}, and extinction efficiency $Q_{\rm ext} = 0.5$ at wavelength λ_0. Show that the extinction in magnitudes at wavelength λ_0 for a 1 kpc distant star is ~ 0.5 mag.

2. Grain Growth

How long does it take grains to grow? Suppose at time $t = 0$ the grain radius is $r(0)$, and that the grain grows by the addition of species i (an atom or molecule) which has mass m_i and mean thermal speed $\bar{v}_i$. Let s be the bulk density of the material and ϵ the sticking coefficient.

a) Show that the grain radius as function of time is $r(t) = r(0) + \frac{\epsilon n_i m_i \bar{v}_i}{4s} t$.

b) What assumption is made concerning n_i?

c) How long would it take for a grain of radius 10^{-7} m to grow to a radius of 2×10^{-7} m by adding a mantle of CO? Assume $n_{\rm CO} = 10^6$ m^{-3}, $T = 10$ K, $\epsilon = 1$, and $s_{\rm CO} = 10^3$ kg m^{-3}.

3. Grain Lifetime

Show that the lifetime for interstellar grains by sputtering in supernova blast waves is of order 10^9 yr. Assume a uniform interstellar gas, H-atom density 10^6 m^{-3}, a Galactic supernova rate of one every 30 yr, and that a supernova of energy 5×10^{43} J may set up shock waves that destroy all grains in a mass of gas ≈ 300 $M_\odot$.

4. Grain Heating

Suppose that interstellar grains are heated entirely by UV absorption, the UV flux is 3.7×10^{10} photons m^{-2} s^{-1} nm^{-1}, the bandwidth is 100 nm, and the mean photon energy is 9 eV.

a) If the grain absorbs UV photons with 100% efficiency but cools by radiating like a blackbody peaking in the infrared with an efficiency of 0.1%, show that the grain temperature is ~ 17.5 K.

b) For standard grains (a=100 nm; ρ=2000 kg m^{-3}) of this temperature, estimate the flux of the re-radiated infrared radiation at 100μm in a 20$'$ diameter beam looking toward the Galactic pole. Take the path length to be 100 pc and ISM density to be the standard 10^6 H atom m^{-3}. Express your result in Jy. Assume (but check) that the optical depth at 100μm is $\ll 1$.

5. Irradiation and Grain Speed

A large speed $v_{\rm gr}$ may be obtained by grains if they are blown by radiation pressure through an envelope of cool gas surrounding a star. The radiation acceleration is usually much greater than gravity, and acceleration outwards is impeded only by collisions with the gas. Balancing these two forces for sherical grains of radius a at a distance r gives:

$E(\frac{R_*}{r})^2 Q_{\rm pr} \frac{\pi a^2}{c} \approx n(r)\pi a^2 m_{\rm H} v_{\rm gr}^2$

where E is the photon power per unit area emitted by the star, $n(r)$ is the grain number density at distance r, and $Q_{\rm pr} \approx 1$ is the radiation pressure efficiency of the grain. Show that the grain speed at the radius of the star is ~ 3 km s^{-1} if $T_* = 10^{3.5}$ K and $n(R_*) = 10^{18}$ m^{-3}.

6. Dust Temperature

Suppose two 05 stars are located near the origin of a spherical shell of dust. The inner radius of the shell is approximately 1 A.U. and the outer radius is 5 pc.

a) Plot the dust temperature as a function of distance from the stars. Assume that the dust cloud is optically thin and that the ratio of the optical absorbing coefficient to the IR emissivity coefficient is 10^3.

b) How would your plot quantitatively change if the central heating source were a single O5 star?

c) How would your plot qualitatively change if the dust cloud were optically thicker (but the same size)? i.e., say $\tau \sim 5$ so that the effects of absorption and scattering are enhanced, and thus fewer and fewer photons reach the outskirts as compared to the optically thin case. Note that $Q_{\rm abs}$(UV/optical) is already maxed out near unity, and so boosting the absorption coefficient isn't a possible way to increase τ in this situation.

d) What does it mean for the dust cloud to be optically thicker? In other words, what would have to be the physical difference in the clouds?

e) We assume the dust particles are heated only by the energy directly coming from the central stars. Why is it ok to neglect the effects of the thermally re-radiated energy by neighboring dust particles?

7. Dark Globules

Dark globules, with a typical radius of 0.05 pc, contain enough dust to reduce background starlight at optical wavelengths by a factor of 50 or more, when peering through their centers.

a) Calculate the number density of grains in a globule if the radius of the dust particles is 100 nm and the extinction efficiency is $Q = 0.5$.

b) Calculate the total mass (in $M_\odot$) of the globule assuming a typical dust-to-gas mass ratio of 1:100.

8. Grains and Magnetic Fields

Here we will study the way in which way grains align, perpendicular or parallel to the magnetic field.

a) Suppose grains are prolate spheroids with moments of inertia $I_{||}$ and $I_{\perp}$ ($I_{||} < I_{\perp}$). In thermodynamic equilibrium, we expect the rotational energy to be the same on both axes. Show that this means that the angular momenta $J_{||}$ and $J_{\perp}$ are related by the equation

$$J_{||} = J_{\perp} \left(\frac{I_{||}}{I_{\perp}} \right)^{0.5}$$

so that

$$J_{||} < J_{\perp}$$

and the particle therefore tends to rotate on the short axis. If there is a magnetic field present, particles with $\mathbf{J}$ *not* parallel to $\mathbf{B}$ will have a magnetic moment $\mathbf{M}$ induced, which will dissipate energy as the particle rotates. Eventually a minimum of energy will be reached where $\mathbf{J} \parallel \mathbf{B}$. Thus the grains end up perpendicular to the field lines.

b) How fast is the grain angular frequency of rotation ω? Use the equipartition theory to estimate the angular speed ω of a grain (assume spherical symmetry) in a *gas* of temperature T (approximately 100 K in an H I cloud). Assume grains with a=100 nm and ρ=2000 kg m^{-3}. Why should it be the gas temperature, rather than the grain temperature, which is used in this calculation?

9. Dust Color Temperature

Assuming that dust grains in the planetary nebula NGC 7027 emit as blackbodies, the

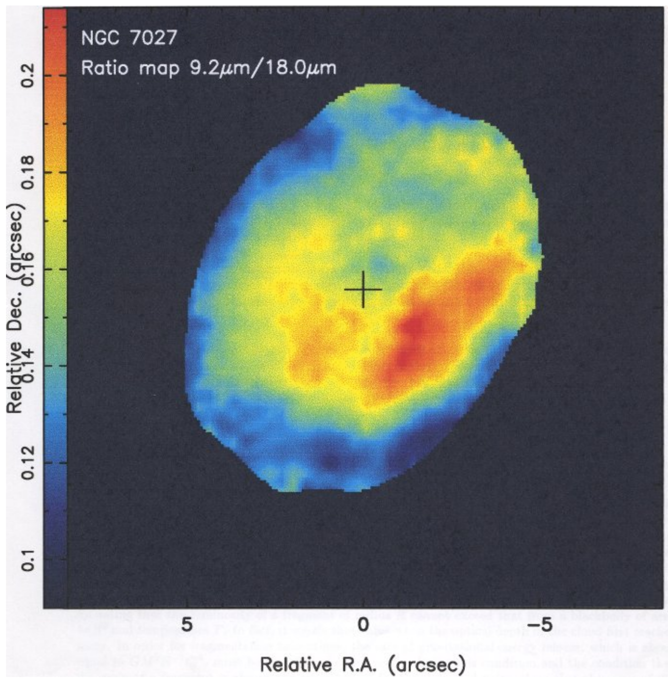

Fig. 1.— $I_\nu(9.2\mu m)/I_\nu(18.0\mu m)$ for NGC 7027. The color bar quantifies this ratio, with intensities in units of Jansky per steradian. The x- and y-axes have the same angular scales.

9.2μm-to-18.0μm ratio can be used to calculate the dust temperature across the nebula.

a) Plot $I_\nu(9.2\mu m)/I_\nu(18.0\mu m)$ as a function of dust temperature (for $0 < T(\mathrm{K}) < 500$). What is the grain temperature in the hottest part of the nebula?

b) Theoretically calculate the expected temperature of a dust grain that lies in the hottest part of the nebula. Assume that the only source of heating is the hot central star (marked by a cross) and that the grains are in thermal equilibrium with the environment. Assume:

Distance to NGC 7027 = 880 pc

Luminosity of central star = $7700 L_\odot$

grain radius = 0.1μm

grain albedo (fraction of incident energy that is reflected) $A = 0.1$

grain emissivity $\epsilon = 0.1$

c) Compare and comment upon the temperatures calculated in parts a) and b).

10. Extinction

Suppose that dust produced extinction $A(\lambda)$ proportional to the frequency of light. What would be the value of R_V?