Do the following **eight problems** and be prepared to discuss them in class.

1. Gum Nebula

The Gum Nebula has a radius of ~400 pc and is located ~460 pc from the Sun. It has a charged particle number density of $n_\mathrm{e} + n_\mathrm{p} \sim 3 \times 10^5\ \mathrm{m}^{-3}$ and contains two O stars and of order 10^1 B stars. It also contains the Vela pulsar, a remnant of a supernova that exploded $\sim 10^4$ years ago.

a) Assuming a recombination coefficient of $2.6 \times 10^{-19}\ \mathrm{m}^3\ \mathrm{s}^{-1}$, find the number of O5 V stars, and separately the number of B0 V stars, which would be needed by themselves to keep the nebula ionized.

b) Assuming the gas in the nebula was initially neutral, calculate the amount of energy needed to ionize it. For this part of the problem, assume the nebula to be in the shape of a cylinder 400 pc in radius and 100 pc in length. Compare to the 10^{44} J produced in a typical supernova explosion. Could one average supernova produce the ionization?

c) Calculate the recombination timescale for the nebula. Could the gas remain ionized for the time since the supernova took place?

d) Is the ionization due to the hot stars, the past supernova, or neither?

2. H II Region Ionized Gas Mass

A hot star excites a uniform cloud of hydrogen of density n_1. An identical star illuminates a cloud in which the gas is distributed in small clouds of density n_2 ($n_2 > n_1$). Is the mass of ionized gas the same in both nebulae? Find the ratio M_1/M_2.

3. Photo-Ejected Electrons

Show that the average kinetic energy given to a photo-ejected electron is about kT_* if the star radiates as a blackbody at temperature T_*.

4. Free-Free Emission

The "turn-over" frequency ν_0 of free-free emission for a certain nebula is 1.96×10^8 Hz. The electron density measured in the nebula is $10^8\ \mathrm{m}^{-3}$. Estimate how many stars, each producing 10^{49} UV photons s^{-1}, are required to keep the nebula ionized. Assume that the optical depth in the free-free continuum is $\tau_\lambda = 4.1 \times 10^{-6} (\frac{\lambda}{\mathrm{m}})^{2.1} (\frac{\epsilon}{\mathrm{cm}^{-6}\mathrm{pc}})$ where ϵ is the emission measure. Note: use the generic expression $\epsilon = n^2 L$ and not the scenario-specific expression from Tielens Equation 7.63 (which I believe has a typo).

5. Emission Line Profiles

An observer measures the profiles of the Hα and λ6584 [N II] lines in a certain nebula. The measured half-intensity widths are 0.05 and 0.04 nm, respectively. Find the electron temperature and velocity v of random mass motions (i.e., consider v to be random bulk motions in addition to those caused by the gas temperature). Assume that the total half-intensity width radiated by an ion or atom of mass m and speed v is $\delta\lambda = \frac{2\lambda}{c}\sqrt{\ln 2}\sqrt{2kT_{\rm e}/m + v^2}$.

6. H II Regions in the Radio

Suppose a 500 pc-distant H II region has a radius 0.1 pc, $n_{\rm e}$=10^4 cm^{-3}, and T=10^4 K. At radio wavelengths, the optical depth is given by Tielens Equation 7.68 where EM is assumed to be in units of cm^{-6} pc.

a) At what wavelengths will this H II region be optically thick when looking through the center (e.g., $\tau_\lambda > 3$)?

b) Are these wavelengths observable from the surface of the Earth?

c) What will be the flux in Janskys observed from such an object at the wavelength where $\tau_\lambda = 10$? (calculate the intensity and integrate over the angular area of the object). Is this observable? You will need to look up the sensitivity capabilities of modern radio telescopes.

7. H II Region Radii

Two pure hydrogen H II regions have the same average density n_1, but in one the matter is uniformly distributed whereas in the other it is clumped into small clouds of density n_2 which fill only 20% of the volume. The distribution of the clumps within this volume is homogeneous and isotropic.

a) What is the ratio of the Strömgren radii of the two regions, if illuminated by the same type O star?

b) If both stars suddenly stopped producing hydrogen ionizing photons at $t = 0$, what would be the ratio of the H II regions' recombination times? To simplify things, assume that the recombination rate within each region is constant and equal to the recombination rates just before $t = 0$.

c) Now invoke a more realistic time-dependent recombination rate for $t \geq 0$. Suppose that $n_{\rm e}(t) = n_{\rm e}(t{=}0)e^{-t/\tau}$, where τ is a constant. Determine an expression for the time $t_{1/2}$ where half of the electrons and protons in region #1 have recombined into neutral hydrogen atoms. Hint: Don't assume $0.5n_1(0){=}n_1(0)e^{-t_{1/2}/\tau}$ and thus $t_{1/2}{=}\tau\ln(2)$. Instead, balance the total # of recombinations at $t_{1/2}$, namely $0.5N_1(0)$, with an integral expression that sums up all the recombinations per time from t=0 to t=$t_{1/2}$.

8. H II Region Expansion

When a hot star is born within an H I region, it produces an H II region whereby the boundary between the ionized interior and neutral exterior is defined by the radiative balance between ionizations and recombinations. However, the heated, higher pressure sphere of gas tends to expand into the cooler H I region.

a) If the temperature inside the H II region is 10^4 K and that in the H I region is 100 K, find the ratio of the final Strömgren radius (when the pressures are balanced) to the initial Strömgren radius (before any expansion takes place).

b) Also find the ratio of the masses of the initial and final Strömgren spheres.