Do the following **nine problems** and be prepared to discuss them in class.

1. Mean Free Path

Estimate the mean free path (in units of A.U.) for atom-atom collisions in a typical H I cloud (number density 3×10^7 m^{-3} and diameter 5 pc). Can the internal motions in such clouds be investigated by continuum hydrodynamics? *Continuum hydrodynamics* implies that the mean free path is much smaller than the medium's characteristic lengthscale.

2. Sound Speed

Show that the sound crossing time of an H II region of density $n_{\mathrm{e-}} = n_{\mathrm{H+}} = 10^8$ m^{-3} excited by a star producing 5×10^{48} UV photons s^{-1} is $\sim 2 \times 10^5$ yr. See Tielens p. 399 for a description of the sound speed: $c_s^2 = \gamma P/\rho$ where $\gamma = C_p/C_V$. Assume a standard H II region electron temperature of 8000 K.

3. Strömgren spheres

The Strömgren spheres around two stars emitting 10^{47} and 10^{49} UV photons per second are observed to be in pressure equilibrium with their surroundings. Show that the ratio of the interstellar gas densities around the two stars is 1:10 if the Strömgren spheres have identical radii.

4. Free-Fall Timescale

Suppose that the same evil Martians who destroyed the Mars Polar Lander, having realized that they are vastly more powerful than any beings in the Solar System, decide to suck all of the thermal energy out of the Sun to use for destructive purposes. The Sun will thus experience a "free-fall" collapse.

a) Show that the time it will take for the destabilized Sun to collapse to a point is

$$t_{\mathrm{ff}} = \sqrt{\frac{3\pi}{32G\rho}}$$

where ρ is the density of the Sun before collapse.

b) Calculate the free-fall collapse time (in units of years) for an H I cloud (number density 3×10^7 m^{-3}) and for a giant molecular cloud (number density 10^9 m^{-3}).

c) At what temperature (in K) must a molecular hydrogen cloud of radius 0.8 pc and density 10^{10} m^{-3} be in order to avoid collapse?

For reference, see Tielens p. 80, Dyson & Williams ch. 8, or Carroll & Ostlie ch. 12.

5. Cloud Collapse

As long as a collapsing cloud is transparent, it can radiate away any heat generated by compression, and remain cool. As the density increases the critical mass goes down, and the cloud may fragment into smaller pieces. The fragmentation stops only when the cloud becomes opaque and starts to heat up, raising the pressure, and raising the critical mass. One can estimate the mass of the smallest fragment by noting that the luminosity of a fragment of radius R cannot exceed that from a blackbody of temperature T. In fact, it equals that value when the optical depth of the cloud first reaches unity. In order for fragmentation to continue, the rate of gravitational energy release, which is about equal to $GM^2R^{-1}t_{\rm ff}^{-1}$, must be less than the luminosity. Using this condition and the condition that the mass of a fragment is always the critical mass for the current density, show that the mass of the smallest fragment $M_{\rm frag}$ is proportional to $T^{1/4}$ and that, for a pure molecular hydrogen cloud at a temperature $T = 10$ K, $M_{\rm frag} \approx 0.01 M_\odot$.

6. Hydrogen Emission Lines

Suppose that an electron recombines into the n=5, l=4 level of hydrogen. What is the probability that an Hα photon will be emitted during the radiative cascade starting from $(n, l) = (5, 4)$? Hint: See Tielens Table 2.2. If you're unsure how to compute the probability, I will also accept an answer that outlines all the possible ways an electron may cascade down from n=5, l=4. Also, by "radiative cascade", I mean that each transition involves the electron moving down in energy level n.

7. Disk Angular Momentum

The total mass of H I in the Large and Small Magellanic Cloud system is probably $\sim 10^9 M_\odot$. Show that this gas probably has more angular momentum about the Galactic center than all the gas in the Milky Way.

8. Accretion Disks

It might be thought that matter which spirals from radius r_1 to radius $r_2 < r_1$ through a rotating accretion disk would gain the total (kinetic plus potential) orbital-energy difference associated with those two radii. If the central body has mass M and the spiraling matter has mass m,

a) show that the orbital energy difference is

$$\Delta E = \frac{GMm}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

b) If mass m is transferred in time t to give a steady accretion rate $\dot{m} = m/t$, argue that the luminosity of the accretion disk between radii r_1 and r_2 is given by

$$L = \frac{GM\dot{m}}{2}\left(\frac{1}{r_2} - \frac{1}{r_1}\right).$$

c) Calculate L for a typical accretion disk where $M = 1M_\odot$ (a white dwarf), $\dot{m} = 10^{-8}\,M_\odot\,\text{yr}^{-1}$ (mass transfer from a red dwarf star), $r_1 = 10^{11}$ cm, and $r_2 = 10^9$ cm.
d) What fraction of this luminosity comes from energy released between just $r = 2\times10^9$ cm and $r_2 = 1\times10^9$ cm?
Suppose that the upper and lower faces of the accretion disk radiate like blackbodies with a local effective temperature T_{eff}.
e) Using the parameters L, r_1, and r_2, determine an expression that will allow you to calculate the effective temperature for the disk.
f) Compute T_{eff} for the above example. At what wavelength does this emission peak?
g) What is the average T_{eff} for the part of the accretion disk between just $r = 2\times10^9$ cm and $r_2 = 1\times10^9$ cm? At what wavelength does this emission peak?

9. Ultraluminous Infrared Galaxies

ULIRGs are dusty galaxies that are very luminous in the infrared. The left column of Figure 1 shows continuum images at 2.15 μm, whereas the right column shows wavelength-position diagrams of the Paschenα spectral line ($n = 4 \rightarrow 3$ transition of Hydrogen).
a) Compute the redshift z of the two sources in IRAS 0152+52.
b) Notice that the upper source in IRAS 0152+52 exhibits a narrow, S-shaped curve spanning about $10''$ in the wavelength-position diagram. How fast is the inferred rotation?
c) How massive must this upper source be in order to produce such rotation?
d) Since the Paschenα line represents an electronic transition of Hydrogen, and such transitions in the Milky Way are frequently seen near gaseous regions filled with young stars, strong Paschenα lines are usually an indication of active star-forming regions. However, another spectral line associated with star-forming regions, the Balmerα line ($n = 3 \rightarrow 2$ Hydrogen transition), is typically much stronger than the Paschenα line in the Milky Way. Give a simple and practical argument as to why the Paschenα line could be more useful than the Balmerα line for observing ULIRGs.

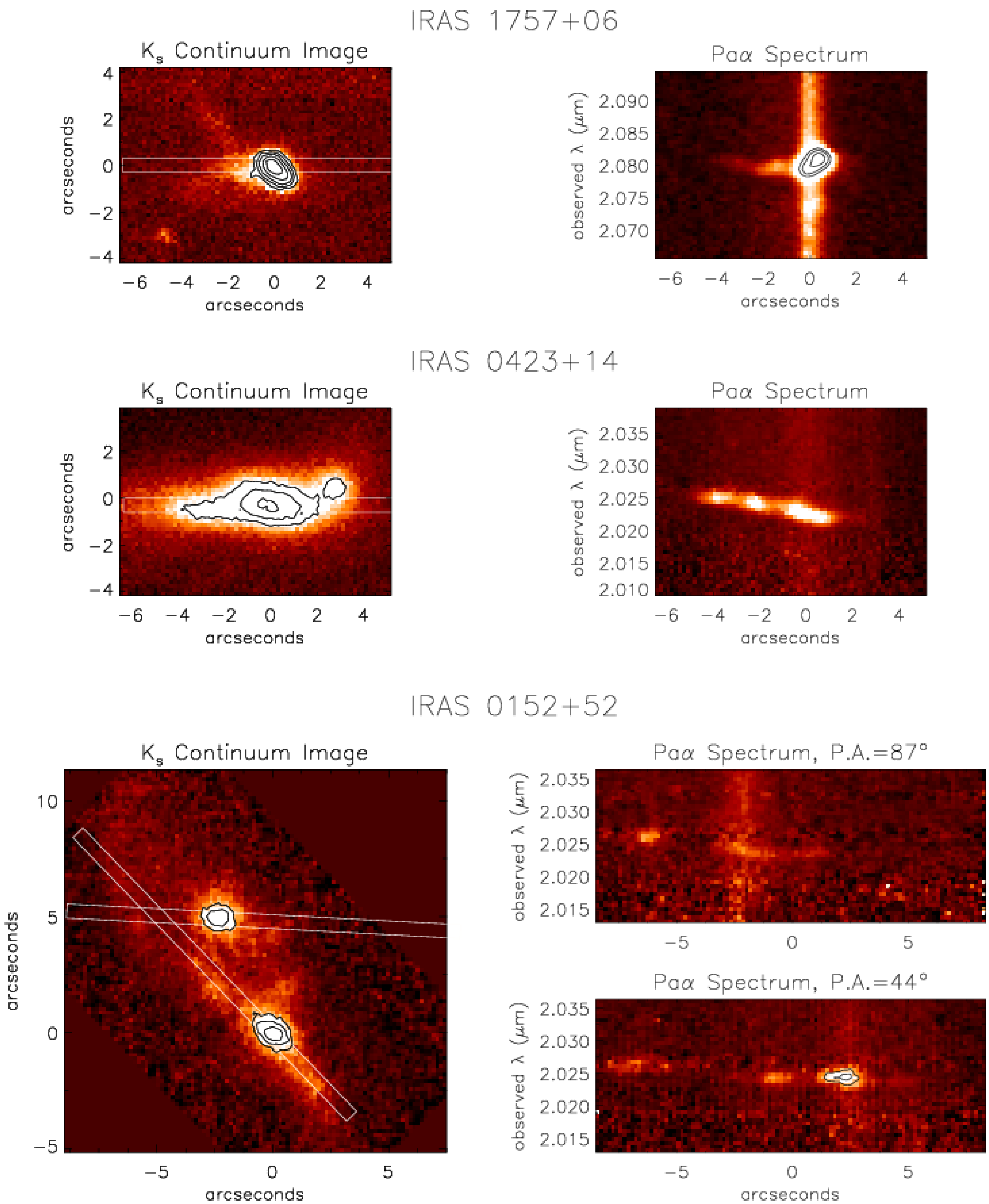

Fig. 1.— Near-infrared integral field spectroscopy of three ULIRGs.