Do the following **seven** problems and be prepared to discuss them in class.

### 1. Photo-Dissociation Regions

Show that the relation between the intensity of the incident FUV field $G_0$ and the density $n_0$ of a photo-dissociation region is $G_0 \approx 10^2(\frac{n_0}{10^3\text{cm}^{-3}})^{4/3}$.
Hint: start with Tielens Equation 9.1 and derive Equation 9.2 en route to Equation 9.4. Do not merely assume Equation 9.2; derive it (or a close approximation thereof).

### 2. Photo-Dissociation Regions

How deep into PDRs are ionizations by photons equal to ionizations by cosmic rays?
**a)** By equating the photo-ionization rate with the cosmic ray ionization rate $\zeta_{\text{CR}}$ show that this balance occurs at an optical depth of $A_V = \frac{1}{b_i}\ln[\frac{a_i G_0 A_i}{\zeta_{\text{CR}}}]$, where $A_i$ is the abundance of species $i$ relative to Hydrogen, $a_i$ and $b_i$ are coefficients representing the photoionization rate in the average interstellar radiation field and the depth dependence of the radiation field due to dust attenuation, respectively, and $G_0 = 10^4$ is the intensity of the radiation field in "Habing units" of the average interstellar radiation field (i.e., $G_0$ itself is unitless).
**b)** Compute the optical depth for silicon ($A_{\text{Si}} = 1$ per $10^6$ per H) and carbon ($A_{\text{C}} = 150$ per $10^6$ per H). Note that the $b_i$ values in Table 8.1 are unitless (and not $10^{-10}$ s$^{-1}$).

### 3.) The Intensities of [SiII]34.82$\mu$m and [CII]157.7$\mu$m

**a)** Assuming $C^+$ and $Si^+$ are the dominant ionization stages of carbon and silicon, optically thin emission, and excitation by only atomic H collisions, provide expressions for cooling as a function of density and temperature for [SiII]34.82$\mu$m and [CII]157.7$\mu$m. Note: the inclusion of $A_{\text{ul}}$ in Equation 2.53 is a typographical error, and the equation should instead read $n^2\Lambda = \gamma_{\text{lu}} A_j n^2 h\nu_{\text{ul}}$. For estimates on $A_j$ abundances, see class notes from near the beginning of the semester. For a reminder about $\gamma_{\text{lu}}$, see Tielens Equation 2.29 and Slide 17 from Chapter 7 notes.
**b)** Give an expression for the [CII]/[SiII] line intensity ratio.
**c)** Plot the [CII]/[SiII] line intensity ratio for the temperature range of $50 < T < 2000$ K.

### 4. Orion Nebula

See Tielens Table 9.3 (p.343) for the PDRs near NGC 2023 and the Orion Bar. Note that the intensities are in erg cm$^{-2}$ s$^{-1}$ sr$^{-1}$.
**a)** Use the theoretical curve in Figure 9.2 (see the version in the errata!) to estimate the gas density for both regions. Are these densities consistent with observations of the 'clump' or 'interclump' conditions?
**b)** Use the theoretical curves in Figure 9.9 to estimate the gas density and incident FUV

field for both regions. Are these consistent with observations of the 'clump' or 'interclump' conditions?

### 5. Molecular Clouds

Consider an isolated, homogeneous cloud of radius $R$, mass $M$, and one-dimensional velocity dispersion $\sigma$. Observationally, the one-dimensional velocity dispersion scales with the size and mass of the cloud: $\sigma(\mathrm{km\ s^{-1}}) \approx 0.55R^{1/2}$ where $R$ is in pc and $\sigma(\mathrm{km\ s^{-1}}) \approx 0.15M^{1/4}$ where $M$ is in $M_\odot$. It follows that the average density of a molecular cloud is related to its size via $\bar{\rho} = 134R^{-1}\ M_\odot\ \mathrm{pc}^{-3}$. The average density-size relation implies that all molecular clouds have the same column density.

**a)** Use the average density-size relation and Equation 5.96 to compute the visual extinction corresponding to this column.

**b)** Use the virial theorem to show that Equation 10.46 of Tielens is approximately correct. Determine the (small) factor by which the equation is off for a Gaussian linewidth. Note that $\Delta v$ is the Full Width at Half Maximum and $\Delta v \neq \sigma$, but recall that the two parameters are closely related.

### 6. Radiative vs mechanical energy output in massive stars

The nuclear-burning lifetime of massive stars is approximately $\tau = 5(M_*/40M_\odot)^{-0.4}$ Myr, and in this time thermonuclear reactions convert approximately 0.3% of the rest mass energy into radiative energy. Stellar winds are driven by the radiation field and carry a momentum flux $\dot{M}_\mathrm{w}v_\mathrm{w} = \eta L_*/c$. The wind terminal velocity is given by $v_\mathrm{w} = \epsilon(GM_*/R_*)^{\frac{1}{2}}$ where $\epsilon \sim 2$. For a 60 $M_\odot$ main sequence star that ultimately explodes as a supernova,

**a)** What is the total luminous energy released during its lifetime?

**b)** What is the star's average luminosity $L_*$ during its lifetime?

**c)** What is the stellar wind terminal velocity $v_\mathrm{w}$?

**d)** Radiatively-driven wind theory suggests $\eta \sim 3$ as a result of multiple scattering in the wind. Determine an upper limit for $\eta$ in terms of $c$ and $v_w$. Hint: the wind cannot carry away more power than is produced by the star.

**e)** What is the momentum flux of the stellar wind during its lifetime? (use $\eta \sim 3$)

**f)** Calculate the ratio of mechanical-to-luminous energy during the star's lifetime. The 'mechanical' energy is the kinetic energy of the wind.

**g)** Which is larger, the energy deposited by the stellar winds, or the energy produced by the supernova explosion?

### 7. Novae

Problem #8 in Homework #5 could very well describe the periodic mass transfer responsible for periodic novae emission from a binary system. In these systems, the accreting mass slowly builds, until reaching a critical point of violent nuclear burning. Suppose that pure Hydrogen accretes for $10^2$ yr between bursts, and that the nuclear burning, which involves fusing Hydrogen into Helium, lasts for 10 days.

**a)** How much nuclear energy is released during each outburst? Compare this to the total gravitational energy released in the accretion process (use the numbers from Problem #8 in Homework #5c).

**b)** What is the burst luminosity? Compare this to the the accretion disk luminosity.