Bayesian Hierarchical Modeling of Binary Star Orbits

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Total Amount Requested: $24,989

Motivation

Most stars are born in binary or multiple systems [10]. They can evolve and interact to produce a variety of astrophysical phenomena and transients, including novae, thermonuclear supernovae, gamma-ray bursts, and sources of gravitational waves. Several $30M - $100M wide-field spectroscopic surveys, including APOGEE, GALAH, RAVE, LAMOST, and DESI, are currently measuring the atmospheric properties and radial velocities (RVs) of millions of stars. A substantial fraction of their targets are observed 2 - 5 times, allowing for the detection of spectroscopic binaries whereby the RV of the primary (brighter) star changes between epochs during its orbit. A few epochs are sufficient to distinguish single stars from close binaries that exhibit substantial RV shifts, but insufficient to measure their individual orbital parameters: period P, eccentricity e, and mass ratio $q = M_2/M_1$. To recover the underlying period distribution from the sparsely sampled RV observations, previous studies have assumed simplistic functional forms, e.g., a power-law $f_{\rm logP} \propto (\log P)^{\Pi}$, and that the parameter probability distributions are independent, i.e., $f(P, e, q) = f_{\rm P}(P) \times f_{\rm e}(e) \times f_{\rm q}(q)$. However, even normal binaries containing main-sequence (MS) stars like our Sun exhibit highly correlated parameter distributions [6; 7; 4]. Moreover, some classes of evolved binaries are expected to have complex orbital period distributions. In particular, for MS stars orbiting stellar remnant cores called white dwarfs, the period distribution is predicted to be bimodal [8; 2]. The precise curvature of bimodality and its dependence on eccentricity and mass ratio heavily depend on the as-of-yet unconstrained physical processes of binary mass transfer. **We propose to model binary orbital period distributions from sparsely sampled spectroscopic RV measurements by employing non-parametric Bayesian hierarchical methods that accounts for the period-dependent eccentricity and mass-ratio distributions.**

Methods

We will forward model a suite of artificial binary populations and then recover the underlying distributions using Bayesian techniques. In order to measure the period distribution in a relatively model-independent fashion, we will divide $f_{\rm logP}$ into $N_{\rm logP}$ logarithmic period bins. Each period bin will also have their own eccentricity and mass-ratio distributions. We will adopt physically motivated Bayesian priors, e.g., short-period binaries tidally circularize toward negligible eccentricities. As $N_{\rm logP}$ increases, the difference between neighboring bins must decrease, and so we explore our ability to recover the underlying distributions as a function of $N_{\rm logP}$ for different sample sizes.

Previous studies have modeled the binary distributions from spectroscopic observations based on a single parameter $\Delta{\rm RV}_{\rm max}$, i.e., the maximum RV difference between any two measurements of a system [1; 5]. While binaries with larger $\Delta{\rm RV}_{\rm max}$ tend to have larger velocity amplitudes and thus shorter orbital periods, this method ignores the timespan between observations and the total number of RV measurements for each system. We will therefore measure the Bayesian posterior distributions of P, e, and q for each binary via the Markov Chain Monte Carlo sampler Joker [11], which incorporates all of the RV data. We will then fit the underlying distributions based on this ensemble of individual Bayesian posteriors, hence the hierarchical nature of this problem. We will consider different cadences, timespans, numbers, and uncertainties of RV measurements that mimic the actual spectroscopic surveys.

Short-term Objectives with this Seed Grant

The major goal of this seed grant is to demonstrate that non-parametric orbital period distributions can be recovered from sparsely sampled RV observations by incorporating all of the data, not just ΔRV_{max}, while also also allowing for potential correlations between parameter distributions. Given our team's background, we are uniquely poised to solve this long-standing problem. PI Moe is an expert on multiplicity statistics and measuring the intrinsic distributions of binary parameters [6; 7; 4; 5; 10]. Co-I Myers incorporates data mining, statistical analysis, and machine learning across a variety of astrophysical problems, including modeling binary eccentricity distributions from spectroscopic RV observations [3]. As a statistics professor, Co-I Robinson's expertise on sampling methods and hierarchical models is essential for translating the sampling and cadence of individual stellar measurements to the overall population of close binaries via Bayesian hierarchical techniques.

During this summer 2024, we will start with simplistic period, eccentricity, and mass-ratio distributions as we develop our Bayesian hierarchical model that incorporates all of the RV data. We will then extend to more complex period distributions and correlated parameter distributions to determine how degenerate the recovered distributions are as a function of $N_{\rm logP}$, sample size, and measurement uncertainties. To assess the accuracy and robustness of our methods, we will compare the rms residuals between the input and recovered distributions across both the 1D phase space $f_{\rm logP}$ and the full 3D parameter space $f(P,e,q)$. The following spring we will publish our methods and results in the Astrophysical Journal as we prepare to submit federal grant proposals to apply our techniques to actual spectroscopic data.

Future Plans for External Funding & Collaborations

As mentioned, several large-scale spectroscopic surveys have already each spent \$30M - \$100M building the infrastructure and collecting the data, much of which is now publicly available. Federal agencies such as NSF, NASA, and DOE are continuously awarding \$100k - \$500k grants to analyze their data and make new scientific discoveries. PI Moe has previously collaborated on three different NSF AAG grants that have yielded four high-impact publications (417 citations total) focused on binary statistics with APOGEE [1; 7; 4; 5]. PI Myers is a team member of the DESI collaboration and has received multiple DOE grants to perform various statistical measurements and characterization of DESI data [9]. In November 2025, after our methods paper is published, we will apply for the NSF AAG and DOE grants to apply our pipeline to the APOGEE and DESI observations, respectively. We will also explore computationally focused grants, including NSF CISE and NASA EPSCoR, where this seed grant would highly benefit PI Moe in pursuing these funding sources that are new to him. We will expand our team to our current collaborators working on stellar astrophysics with APOGEE and DESI data. We will also reach out to other UW personnel, such as those in the School of Computing, who could actively contribute to this computational project.

With our larger federal grant, we plan to achieve two important goals. First, we will measure the density of normal MS binaries in the 3D phase space $f(P,e,q)$ for different primary masses and spectral types, which will constrain formation models of binary stars and serve as initial conditions in population synthesis studies of binary evolution. Although some regions of this parameter space are well characterized, other pockets still elude us but can be ascertained with the APOGEE or DESI data. Second, we will measure $f(P,e,q)$ for binaries containing one MS star and one white dwarf, which will constrain physical models of binary mass transfer. Understanding the physical processes of white dwarf binary evolution will also provide insight into the progenitors of thermonuclear (Type Ia) supernovae and binary white dwarf mergers, which produce low-frequency gravitational waves that will be detectable by NASA's upcoming \$1B LISA spacecraft.

Budget

PI Moe: We request 1.0 month ($9,852) of summer salary for PI Moe to be funded by this seed grant. UW charges a 40.9% fringe benefit rate for faculty, and so we request a total of $\$9{,}852 \times 1.409$ = $13,881 for PI Moe.

Co-I Robinson: We request 0.3 months ($4,417) of summer salary for Co-I Robinson, corresponding to $\$4{,}417 \times 1.409$ = $6,224 including fringe.

Co-I Myers: We request 0.3 months ($3,466) of summer salary for Co-I Myers, corresponding to $\$3{,}466 \times 1.409$ = $4,884 including fringe.

Total: The total requested budget is $24,989, just shy of the $25,000 allowed.

References

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