

This test is closed-note and closed-book. No written, printed, or recorded material is permitted. Calculators are permitted but computers are not. No collaboration, consultation, or communication with other people (other than the administrator) is allowed by any means, including but not limited to verbal, written, or electronic methods. Sharing of calculators is prohibited. If you have a question about the test, please raise your hand. For multiple choice, you may choose two answers, and if one is correct, receive half credit, etc. For full credit on written problems, show the thought process from basic equations to results.

$$V_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$x_1 = x_0 + v_0 t + \frac{1}{2} a t^2 \quad v_1 = v_0 + a t \quad v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$\rho_{water} = 1000 \text{ kg/m}^3 \quad V_{sphere} = \frac{4}{3} \pi R^3$$

$$\rho_{ice} = 920 \text{ kg/m}^3 \quad V_{cylinder} = \text{Area} \times \text{Length}$$

$$2.2 \text{ lbs} = 1 \text{ kg} \quad 1 \text{ Calorie} = 4200 \text{ J}$$

$$1 \text{ mi} = 5280 \text{ ft} = 1709 \text{ m}$$

$$x_{quadratic} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Work/Energy $\Sigma \vec{F} = m\vec{a}$ $F_{spring} = -kx$ $F_f = \mu F_n$ $W_{grav} = -\Delta U$

$$W = \vec{F} \cdot \vec{s} \quad W = \Delta K \quad K = \frac{1}{2} m v^2 \quad U_s = \frac{1}{2} k x^2 \quad U_g = mgy \quad P = \frac{\Delta W}{\Delta t} = Fv$$

$$F = -\frac{dU}{dx}$$

Momentum/Impulse $p = mv$ $J = \Delta(mv) = Ft$ $X_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

Angular Motion $\theta_1 = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$ $\omega_1 = \omega_0 + \alpha t$ $\omega_1^2 = \omega_0^2 + 2\alpha(\theta_1 - \theta_0)$ $s = r\theta$ $v = r\omega$ $a_{tan} = r\alpha$ $a_{rad} = \omega^2 r$

$$\Sigma \vec{\tau} = I \vec{\alpha} \quad I = \Sigma_i m_i r_i^2 \quad I_{parallel} = I_{cm} + Md^2 \quad \vec{\tau} = \vec{r} \times \vec{F} = rF \sin \phi \quad W = \Delta K = \tau \Delta \theta$$

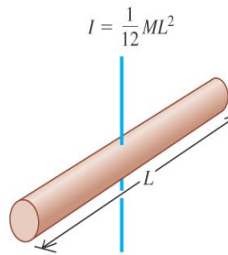
$$K_{rot} = \frac{1}{2} I \omega^2 \quad K_{total} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 \quad \vec{L} = \vec{r} \times \vec{p} = r m v = I \omega \quad \Delta L = \tau \Delta t$$

Gravity: $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ $M_{Earth} = 5.97 \times 10^{24} \text{ kg}$

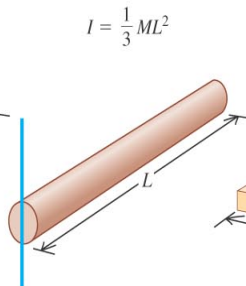
$$F_g = \frac{GM_1 M_2}{r^2} \quad U_g = \frac{-GM_1 M_2}{r} \quad P^2 = \frac{4\pi^2 a^3}{GM} \quad V_{circular} = \sqrt{\frac{GM}{r}} \quad R_{Earth} = 6300 \text{ km}$$

Fluids $P = \frac{dF}{dA}$ $p_2 - p_1 = -\rho g (y_2 - y_1)$ $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ $p_1 + \rho g y_1 + 1/2 \rho v_1^2 = p_2 + \rho g y_2 + 1/2 \rho v_2^2$

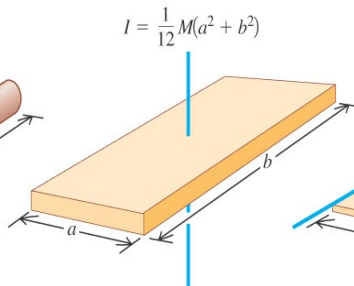
(a) Slender rod, axis through center



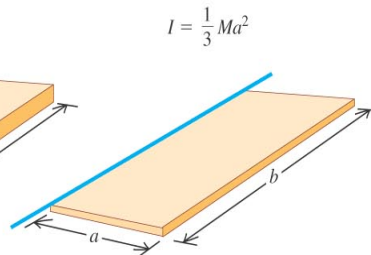
(b) Slender rod, axis through one end



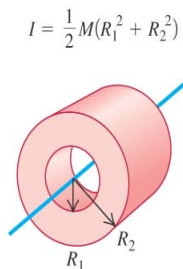
(c) Rectangular plate, axis through center



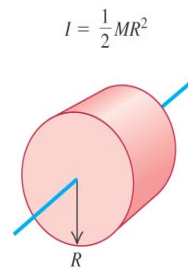
(d) Thin rectangular plate, axis along edge



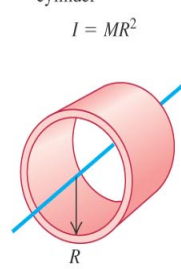
(e) Hollow cylinder



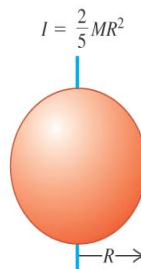
(f) Solid cylinder



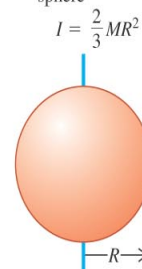
(g) Thin-walled hollow cylinder



(h) Solid sphere



(i) Thin-walled hollow sphere



1. (10 pts) A propeller experiencing a constant torque starts from rest and travels an angular distance θ radians in t seconds. If you let it spin for $2t$ seconds instead it travels

- A. $1/2 \theta$ B. $\sqrt{2} \theta$ C. 2θ D. $5/2 \theta$ E. 4θ

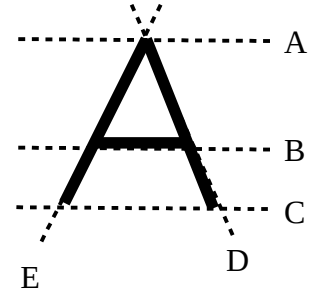
$$\theta_1 = \frac{1}{2} \alpha t^2$$

2. (10 pts) Car A with tires of radius R travels at speed V . A car B with tires radius $2R$ travels at the same speed. The angular velocity of the wheel on car B is

- A. same as A B. twice that of A C. half that of A D. $\sqrt{2}$ that of A E. four times that of A

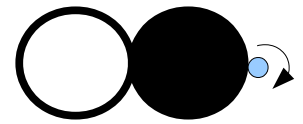
3. (10 pts) Consider this letter A made of three thin rods. About which of the shown axes of rotation (dotted lines) is the moment of inertia the greatest?

- A. B. C. D. E.



4. (10 pts) A hollow cylinder and a solid cylinder, each having mass M and radius R , are welded together along their outer diameters as shown. The structure is rotated about an axis parallel to the symmetry axis of the cylinders. The axis of rotation is shown and lies on the surface of the solid cylinder. What is the moment of inertia?

- A. $2 MR^2$ B. $5/2 MR^2$ C. $6 MR^2$ D. $9/2 MR^2$ E. $11/2 MR^2$



$$I = I_{\text{hollow}} + I_{\text{solid}} = MR^2 + M(3R)^2 + \frac{1}{2}MR^2 + M(R)^2 = 11\frac{1}{2}MR^2$$

5. (10 pts) A torque has fundamental mks units that are the same as

- A. kinetic energy B. work C. momentum D. A and B E. none of the above

6. (10 pts) Write a few lines of Matlab code that would plot the position (in meters) of a wheel of radius $R=0.5$ m rolling at angular speed $\omega=0.2$ rad/s as a function of time from $t=0$ to $t=4$ sec. Assume that the wheel starts at the origin at $t=0$.

$$R = 0.5$$

$$\omega = 0.2$$

$$t = 0:0.5:4$$

$$\theta = \omega * t$$

$$x = \theta * R$$

$$\text{plot}(t, x)$$

7. (10 pts) Salt water has a specific gravity of 1.02. When you take a ship that, in a pure water ocean, has its hull (the hull is the part of the ship below the waterline) submerged to a depth H , the depth of the hull submerged in a salt water ocean is nearest

- A. $0.98 H$ B. H C. $1.02 H$ D. $1.02^2 H$ E. $\sqrt{2} H$

8. (10 pts) In a collision involving rotating bodies (satellite?!) freely floating in space which is always true?

- A. kinetic energy is conserved B. linear momentum is conserved
C. rotational kinetic energy is conserved D. angular momentum is conserved
E. A and C F. B and D G. A, B, C, and D

9. (10 pts) When a fluid flowing through a horizontal pipe enters a smaller diameter portion of the pipe

- A. its speed increases and its pressure increases
B. its speed increases and its pressure decreases
C. its speed decreases and its pressure increases
D. its speed decreases and its pressure decreases
E. it turns into a black hole and sucks in everything with it

10. (10 pts) On a spinning airplane propeller (a thin rod of radius 1 m) the angular speed of a point 0.33 m from the axis of rotation is close to

- A. nine times that at R B. three times that at R **C.** Same as at R D. 1/3 that at R E. 1/9 that at R

11. (10 pts) As a bubble of air rises from a deep undersea vent in the ocean floor to the surface

- A. its radius shrinks B. its radius stays the same **C.** its radius grows D. the bubble dissolves

12. (10 pts) A star (approximated as a solid sphere) when it becomes a neutron star collapses from a radius of R to 1/10 R. It rotates once per day before the collapse; afterward it rotates

- A. 1/100 times per day B. 1/10 times per day C. once per day D. 10 times per day **E.** 100 times per day

13 (40 pts) A small solid cylinder (mass M_1 and radius R_1) is welded together with a larger ^{solid} cylinder (mass M_2 and radius R_2). Around the inner cylinder is wound a massless cord attached to a hanging mass Q. Sitting atop the larger cylinder is a sharp axe blade of mass Z that rests on the outside diameter where there is coefficient of kinetic friction μ between the axe and wheel. As usual, g is the acceleration due to Earth's gravity.

A. Give an expression for the moment of inertia of the combined rotating double cylinder.

$$I_{\text{tot}} = \frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2$$

B. Give an expression for the linear acceleration of Q as the mass drops.

$$\sum \tau = I_T \alpha$$

$$\alpha = \frac{a}{R_1}$$

$$\tau_Q - \tau_f = I_T \frac{a}{R_1}$$

$$QgR_1 - Zg\mu R_2 = I_T \frac{a}{R_1}$$

$$\frac{R_1 g(QR_1 - Z\mu R_2)}{\frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2} = a$$

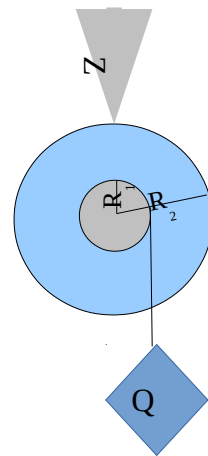
$$\frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2$$

C. Give an expression for the rotational kinetic energy after some time t.

$$K_{\text{rot}} = \frac{1}{2} I \omega^2$$

$$\omega = \omega_0 + \alpha t$$

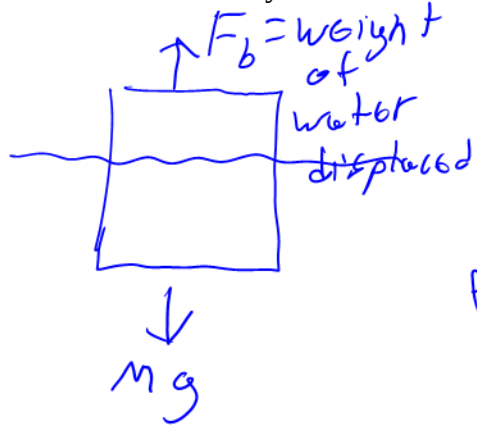
$$K_{\text{rot}} = \frac{1}{2} \left[\frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2 \right] (\alpha t)^2 \quad \omega = \alpha t$$



13. (40 pts) Your team is designing a floating oceanic research station in the shape of a hollow cube. The six walls of the cube are to be 0.08 m thick and 12 m on each side (i.e., square). You want 1/4 of the cube to remain above the water and the other 3/4 to be submerged.

A. What must be the total mass of the station?

B. What is the density of the material that you need to use to make the walls?



$$\sum F_y = ma = 0$$

$$F_b - Mg = 0$$

$$\rho_w V_w g - Mg = 0$$

$$\rho_w V_w = M$$

$$1000 \frac{\text{kg}}{\text{m}^3} \cdot 1296 \text{m}^3 = M$$

$$1.296 \times 10^6 \text{kg} = M$$

$$V_{\text{box}} = (12 \text{m})^3 \\ = 1728 \text{m}^3$$

$$V_w = \frac{3}{4} V_{\text{box}} \\ = 1296 \text{m}^3$$

$$b). \quad \rho = \frac{M_{\text{box}}}{V_{\text{walls}}}$$

$$= \frac{1.296 \times 10^6 \text{kg}}{69.12 \text{m}^3} = 18,750 \frac{\text{kg}}{\text{m}^3}$$

$$V_{\text{walls}} = 6 \text{walls} \times 12 \times 12 \times 0.08 \text{m} \\ = 69.12 \text{m}^3$$