

This test is closed-note and closed-book. No written, printed, or recorded material is permitted. Calculators are permitted but computers are not. No collaboration, consultation, or communication with other people (other than the administrator) is allowed by any means, including but not limited to verbal, written, or electronic methods. Sharing of calculators is prohibited. If you have a question about the test, please raise your hand. For multiple choice, you may choose two answers, and if one is correct, receive half credit, etc. For full credit on written problems, show the thought process from basic equations to results.

$$V_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$x_1 = x_0 + v_0 t + \frac{1}{2} a t^2 \quad v_1 = v_0 + a t \quad v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$\rho_{water} = 1000 \text{ kg/m}^3 \quad V_{sphere} = \frac{4}{3} \pi R^3$$

$$\rho_{ice} = 920 \text{ kg/m}^3 \quad V_{cylinder} = \text{Area} \times \text{Length}$$

$$2.2 \text{ lbs} = 1 \text{ kg} \quad 1 \text{ Calorie} = 4200 \text{ J}$$

$$1 \text{ mi} = 5280 \text{ ft} = 1709 \text{ m}$$

$$x_{quadratic} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Work/Energy $\Sigma \vec{F} = m\vec{a}$ $F_{spring} = -kx$ $F_f = \mu F_n$ $W_{grav} = -\Delta U$

$$W = \vec{F} \cdot \vec{s} \quad W = \Delta K \quad K = \frac{1}{2} m v^2 \quad U_s = \frac{1}{2} k x^2 \quad U_g = mgy \quad P = \frac{\Delta W}{\Delta t} = Fv$$

$$F = -\frac{dU}{dx}$$

Momentum/Impulse $p = mv$ $J = \Delta(mv) = Ft$ $X_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

Angular Motion $\theta_1 = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$ $\omega_1 = \omega_0 + \alpha t$ $\omega_1^2 = \omega_0^2 + 2\alpha(\theta_1 - \theta_0)$ $s = r\theta$ $v = r\omega$ $a_{tan} = r\alpha$

$$\Sigma \vec{\tau} = I \vec{\alpha} \quad I = \Sigma_i m_i r_i^2 \quad I_{parallel} = I_{cm} + Md^2 \quad \vec{\tau} = \vec{r} \times \vec{F} = rF \sin \phi \quad W = \Delta K = \tau \Delta \theta \quad \alpha = \frac{d\omega}{dt} \quad a_{rad} = \omega^2 r$$

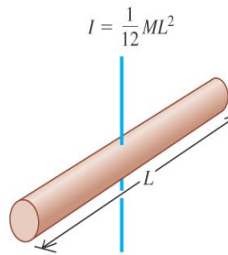
$$K_{rot} = \frac{1}{2} I \omega^2 \quad K_{total} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 \quad \vec{L} = \vec{r} \times \vec{p} = r m v = I \omega \quad \Delta L = \tau \Delta t$$

Gravity: $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ $M_{Earth} = 5.97 \times 10^{24} \text{ kg}$

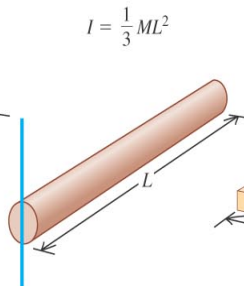
$$F_g = \frac{GM_1 M_2}{r^2} \quad U_g = \frac{-GM_1 M_2}{r} \quad P^2 = \frac{4\pi^2 a^3}{GM} \quad V_{circular} = \sqrt{\frac{GM}{r}} \quad R_{Earth} = 6300 \text{ km}$$

Fluids $P = \frac{dF}{dA}$ $p_2 - p_1 = -\rho g (y_2 - y_1)$ $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$ $p_1 + \rho g y_1 + 1/2 \rho v_1^2 = p_2 + \rho g y_2 + 1/2 \rho v_2^2$

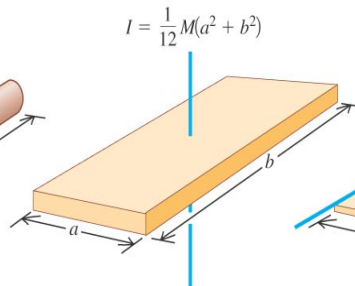
(a) Slender rod, axis through center



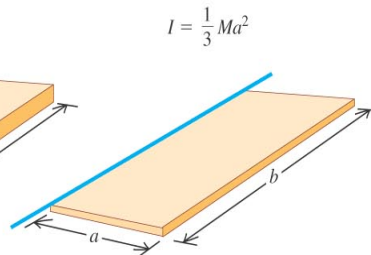
(b) Slender rod, axis through one end



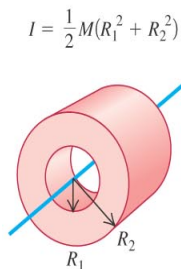
(c) Rectangular plate, axis through center



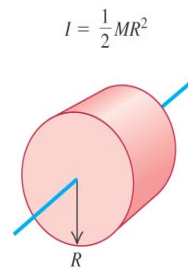
(d) Thin rectangular plate, axis along edge



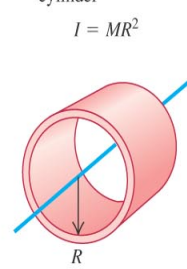
(e) Hollow cylinder



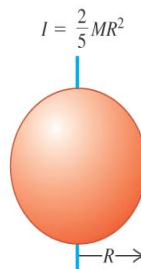
(f) Solid cylinder



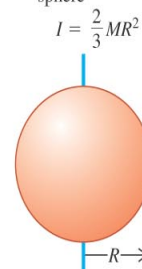
(g) Thin-walled hollow cylinder



(h) Solid sphere



(i) Thin-walled hollow sphere



1. (10 pts) In one experiment a researcher observes that a spinning wheel starting from rest rotates 12 radians in 4 seconds. In a second experiment the wheel starts from rest and rotates 24 radians in 8 seconds. Compared to the first experiment, the angular acceleration in the second experiment is

- A. the same B. twice as large C. four times as large **D. half as large** E. one fourth as large

$$\theta_1 = \frac{1}{2} \alpha_1 t_1^2 \quad \text{so} \quad \alpha_1 = \frac{2\theta_1}{t_1^2} \quad \frac{\alpha_2}{\alpha_1} = \frac{\theta_2}{\theta_1} \frac{t_1^2}{t_2^2} = \frac{24}{12} \frac{4^2}{8^2} = \frac{1}{2}$$

2. (10 pts) A pendulum on a Grandfather clock momentarily comes to a rest when it is 10 degrees from the vertical. The pendulum can be treated as a thin uniform rod that pivots about one end with length 20 cm and mass 0.5 kg. The torque on the rod at this instant is most nearly

- A. 0.086 N m** B. 0.162 C. 0.050 N m D. 0.100 N m E. 0.200 N m

$$\frac{L}{2} M g \sin \theta = \tau$$

3. (10 pts) In a collision involving rotating bodies (satellite?!) freely floating in space which is always true?

- A. kinetic energy is conserved B. linear momentum is conserved
C. rotational kinetic energy is conserved D. angular momentum is conserved
E. A and C **F. B and D** G. A, B, C, and D

4. (10 pts) Salt water has a specific gravity of 1.02. When you take a ship that, in a pure water ocean, has its hull (the hull is the part of the ship below the waterline) submerged to a depth H, the depth of the hull submerged in a salt water ocean is nearest

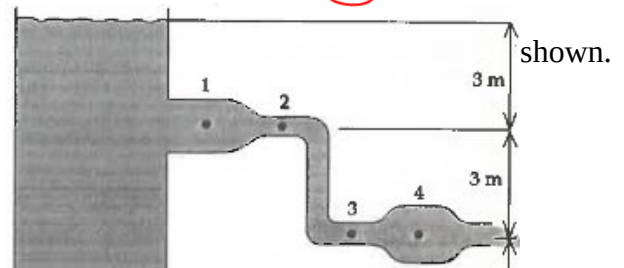
- A. 0.98 H** B. H C. 1.02 H D. 1.02²H E. sqrt(2) H

5. (10 pts) A star (approximated as a solid sphere) when it becomes a neutron star collapses from a radius of R to 1/10 R. It rotates once per day before the collapse; afterward it rotates

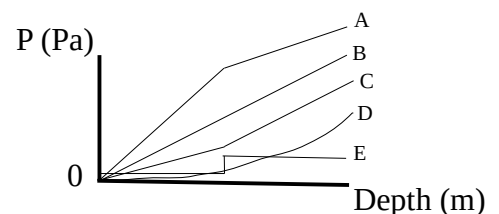
- A. 1/100 times per day B. 1/10 times per day C. once per day D. 10 times per day **E. 100 times per day**

6. (10 pts) Water is discharged from an open tank in the manner shown. At which point is the static pressure, P, the least?

- A. 1 **B. 2** C. 3 D. 4 E. All the same



7. (10 pts) A beaker is filled half with oil, with specific gravity 0.8, and half with water. Which of the pressure versus depth curves best describes this beaker? A. A B. B **C. C** D. D E. E



8. (10 pts) A hollow cylinder of radius 0.2 m and length 4 m floats so that half of its volume is submerged in water. The mass of the cylinder is close to

- A. 80 kg B. 160 kg **C. 250 kg** D. 500 kg E. 800 kg

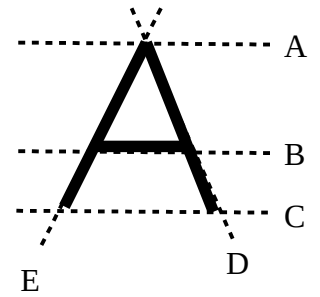
$$F_b - F_w = 0$$

$$\rho V_{\text{cyl}} g - m_c g = 0$$

$$\rho V_{\text{cyl}} = m_c$$

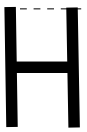
9 (10 pts) Consider this letter A made of three thin rods. About which of the shown axes of rotation (dotted lines) is the moment of inertia the greatest?

A. B. C. D. E.



10. (10 pts) A metal "H" is formed by the two thin rods of mass M and length L connected symmetrically by a rod of mass $\frac{1}{2}M$ and length $\frac{1}{2}L$. It is rotated about an axis shown by the dashed line. The moment of inertia is

A. $\frac{1}{6}ML^2$ B. $\frac{3}{4}ML^2$ C. $\frac{11}{12}ML^2$ D. $\frac{7}{8}ML^2$ E. $\frac{19}{24}ML^2$



$$\frac{1}{3}ML^2 + \frac{1}{3}ML^2 + \frac{1}{2}M\left(\frac{1}{2}L\right)^2 = \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{8}\right)ML^2 = \frac{19}{24}ML^2$$

11. (10 pts) Write a few lines of Matlab code that would plot the position (in meters) of a wheel of radius $R=0.5$ m rolling at angular speed $\omega=0.2$ rad/s as a function of time from $t=0$ to $t=4$ sec. Assume that the wheel starts at the origin at $t=0$.

$$x = v_0 t \quad v_0 = \omega R$$

$$\text{so } x = \omega R t$$

$$\omega = 0.2$$

$$R = 0.5$$

$$t = 0:0.5:4$$

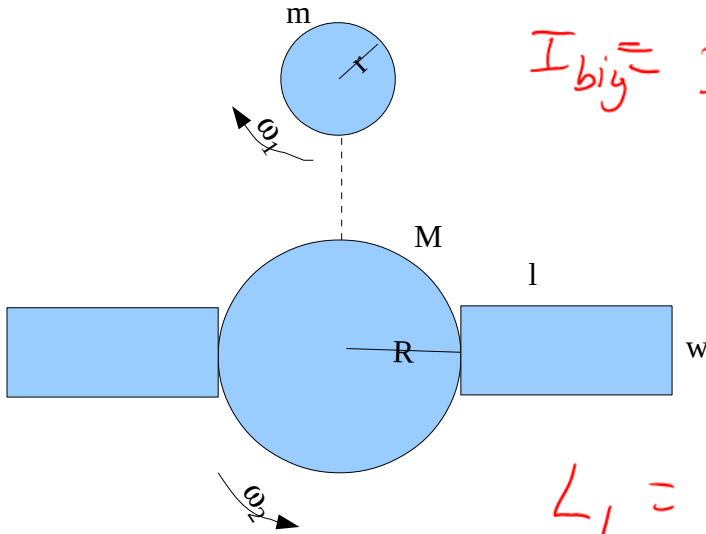
$$x = \omega * R * t$$

$$\text{plot}(t, x)$$

12. (50 pts) An artificial satellite of mass m is shaped like a solid cylinder of length L and radius r is rotating at ω_1 radians per second. A larger satellite is composed of a hollow spherical inner core of radius R and mass M and two solid rectangular solar panels of mass X , width w and length l attached as shown. The larger satellite is initially spinning at ω_2 radians per second about its center of mass with an axis of rotation aligned with that of the smaller satellite. Both axes of rotation are perpendicular to your page. The smaller satellite collides with the larger satellite along the path indicated by the dotted line sticks to the outer surface of the sphere.

A. Give an expression for the moment of inertia, I_{big} , of the larger satellite before the collision.

B. Give an expression for the angular speed of the joint mess after the collision. (Assume that $m \ll M$ so that the center of mass of rotation remains the center of the sphere). You may use I_{big} as a variable in your answer.



$$I_{big} = \frac{2}{3}MR^2 + 2I_{panel}$$

$$I_{panel} = \frac{1}{12}X(w^2 + l^2) + X\left(R + \frac{l}{2}\right)^2$$

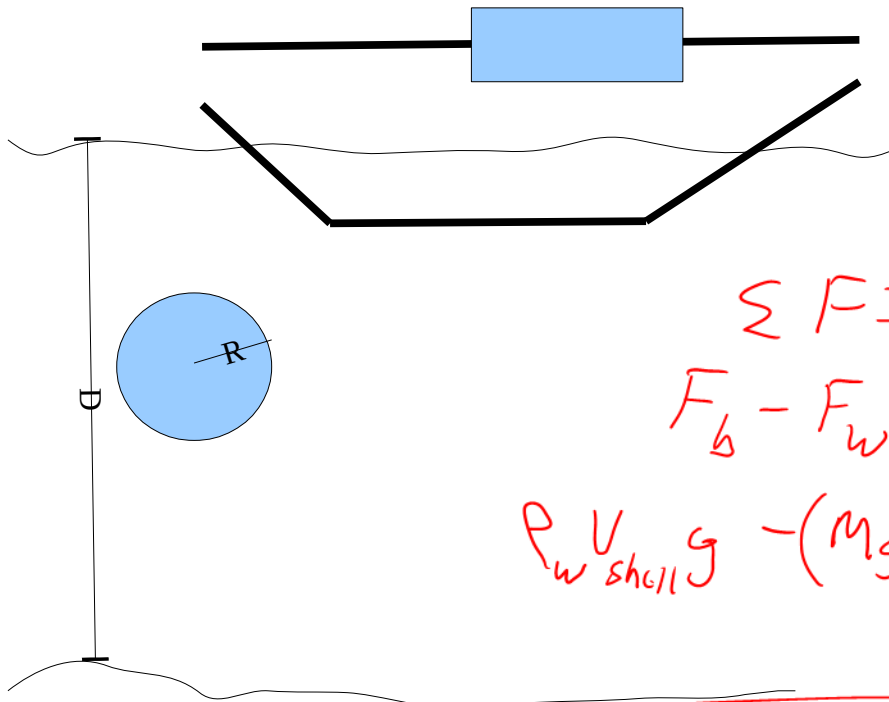
$$L_1 = L_2$$

$$I_{big}\omega_2 + I_{small}\omega_1 = I_{tot}\omega_f$$

$$\frac{I_{big}\omega_2 + \frac{1}{2}mr^2\omega_1}{I_{big}} = \omega_f \quad \text{let } I_{tot} \approx I_{big}$$

13. (40 pts) An underwater storage system for crude oil calls for steel spherical shells of radius R and (empty) mass M to be filled with oil and dropped overboard (starting from rest) a distance D from the surface to the ocean floor. Oil has density ρ_o and water density ρ_w . (You may ignore any drag forces of the water on the shell.)

A. Give an expression for the time, t , required for a shell to reach the ocean floor (assume that R is really small compared to D so that you don't need to account for R in the distance traveled).



$$\text{where } m = \rho_{\text{oil}} V_{\text{shell}} + M$$

$$\Sigma F = m a$$

$$F_b - F_w = m a$$

$$\rho_w V_{\text{shell}} g - (Mg + \rho_{\text{oil}} V_{\text{shell}} g) = (\rho_{\text{oil}} V_{\text{shell}} + M) a$$

$$\frac{g(\rho_w V_{\text{shell}} - M - \rho_{\text{oil}} V_{\text{shell}})}{(\rho_{\text{oil}} V_{\text{shell}} + M)} = a_y$$

$$y = y_0^0 + v_{y0}^0 t + \frac{1}{2} a_y t^2$$

$$D = y = \frac{1}{2} a_y t^2$$

$$t = \sqrt{\frac{2D}{a_y}} \leftarrow$$