

This test is closed-note and closed-book. No written, printed, or recorded material is permitted. Calculators are permitted but computers are not. No collaboration, consultation, or communication with other people (other than the administrator) is allowed by any means, including but not limited to verbal, written, or electronic methods. Sharing of calculators is prohibited. If you have a question about the test, please raise your hand. For multiple choice, you may choose two answers, and if one is correct, receive half credit, etc. For full credit on written problems, show the thought process from basic equations to results.

$$V_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad a_{rad} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}$$

$$x_1 = x_0 + v_0 t + \frac{1}{2} a t^2 \quad v_1 = v_0 + a t \quad v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$\rho_{water} = 1000 \text{ kg/m}^3 \quad V_{sphere} = \frac{4}{3} \pi R^3$$

$$\rho_{ice} = 920 \text{ kg/m}^3 \quad V_{cylinder} = \text{Area} \times \text{Length}$$

$$2.2 \text{ lbs} = 1 \text{ kg} \quad 1 \text{ Calorie} = 4200 \text{ J}$$

$$1 \text{ mi} = 5280 \text{ ft} = 1709 \text{ m}$$

$$x_{quadratic} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Work/Energy  $\Sigma \vec{F} = m\vec{a}$   $F_{spring} = -kx$   $F_f = \mu F_n$   $W_{grav} = -\Delta U$

$$W = \vec{F} \cdot \vec{s} \quad W = \Delta K \quad K = \frac{1}{2} m v^2 \quad U_s = \frac{1}{2} k x^2 \quad U_g = mgy \quad P = \frac{\Delta W}{\Delta t} = Fv$$

$$F = -\frac{dU}{dx}$$

Momentum/Impulse  $p = mv$   $J = \Delta(mv) = Ft$   $X_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

Angular Motion  $\theta_1 = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$   $\omega_1 = \omega_0 + \alpha t$   $\omega_1^2 = \omega_0^2 + 2\alpha(\theta_1 - \theta_0)$   $s = r\theta$   $v = r\omega$   $a_{tan} = r\alpha$

$$\Sigma \vec{\tau} = I\vec{\alpha} \quad I = \Sigma_i m_i r_i^2 \quad I_{parallel} = I_{cm} + Md^2 \quad \vec{\tau} = \vec{r} \times \vec{F} = rF \sin \phi \quad W = \Delta K = \tau \Delta \theta$$

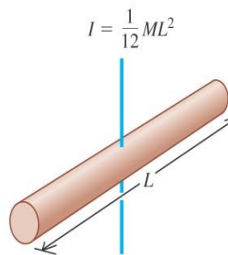
$$K_{rot} = \frac{1}{2} I \omega^2 \quad K_{total} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2 \quad \vec{L} = \vec{r} \times \vec{p} = r m v = I \omega \quad \Delta L = \tau \Delta t$$

Gravity:  $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$   $M_{Earth} = 5.97 \times 10^{24} \text{ kg}$

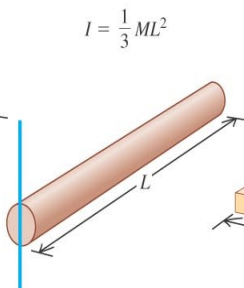
$$F_g = \frac{GM_1 M_2}{r^2} \quad U_g = \frac{-GM_1 M_2}{r} \quad P^2 = \frac{4\pi^2 a^3}{GM} \quad V_{circular} = \sqrt{\frac{GM}{r}} \quad R_{Earth} = 6300 \text{ km}$$

Fluids  $P = \frac{dF}{dA}$   $p_2 - p_1 = -\rho g (y_2 - y_1)$   $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$   $p_1 + \rho g y_1 + 1/2 \rho v_1^2 = p_2 + \rho g y_2 + 1/2 \rho v_2^2$

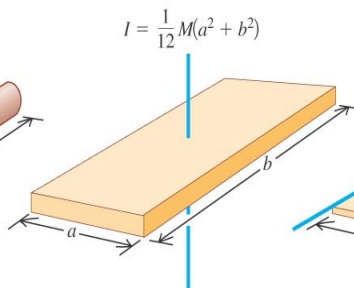
(a) Slender rod, axis through center



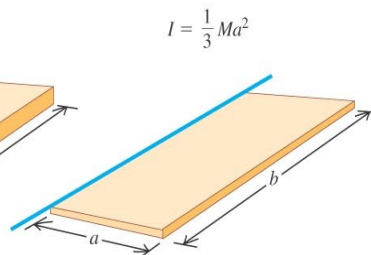
(b) Slender rod, axis through one end



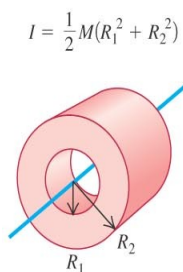
(c) Rectangular plate, axis through center



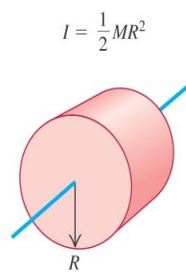
(d) Thin rectangular plate, axis along edge



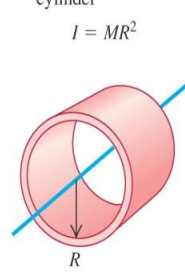
(e) Hollow cylinder



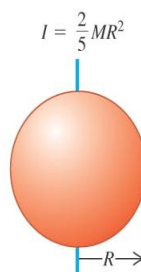
(f) Solid cylinder



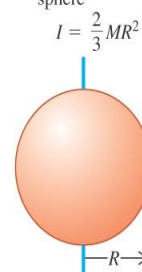
(g) Thin-walled hollow cylinder



(h) Solid sphere



(i) Thin-walled hollow sphere



1. (10 pts) A propeller experiencing a constant torque starts from rest and spins up with a constant acceleration and travels an angular distance  $\theta$  radians in  $t$  seconds. How long does it take to go  $12\theta$ ?

- A.  $\sqrt{2}t$  B.  $2t$  C.  $2\sqrt{2}t$  D.  $12t$  E.  $144t$

2. (10 pts) Car A with tires of radius  $R$  travels at speed  $V$ . Car B with tires radius  $2R$  travels at the same speed. The angular velocity of the wheel on car B is

- A. same as A B. twice that of A C. half that of A D.  $\sqrt{2}$  that of A E. four times that of A

3. (10 pts) Consider this letter L made of two thin rods, one of length  $L$ , mass  $M$  and the other length half  $L$ , mass half  $M$ . The moment of inertia of this object rotated about the dotted line axis is

- A.  $6/24 ML^2$  B.  $7/24 ML^2$  C.  $8/24 ML^2$  D.  $12/24 ML^2$  E.  $25/24 ML^2$

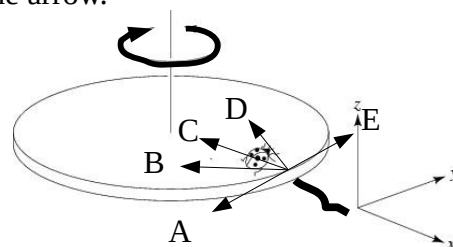
$$M\left(\frac{L}{2}\right)^2 + \frac{1}{2} \left(\frac{1}{2}M\right) \left(\frac{1}{2}L\right)^2 = \frac{1}{4}ML^2 + \frac{1}{24}ML^2 = \frac{7}{24}ML^2$$



4. (10 pts) A ladybug sits on a thin solid disk rotating in the direction of the arrow.

The rate of rotation is decreasing due to a stick rubbing radially on the perimeter of the wheel at the location shown. Big arrows ABCDE lie in the plane of the disk. The arrow that best represents the direction of the frictional force is (circle one)

- A. B. C. D. E.



5. (10 pts) In the same picture, the arrow that best represents the bugs total acceleration vector is (circle one)

- A. B. C. D. E.

6. (10 pts) In the same picture the direction of the torque is best given by (circle one)

- x y z -x -y -z

7. (10 pts) Imagine now that there is no frictional torque and that the disk initially spins at angular speed  $\omega$ . The disk mass is  $M$  and the bug's mass is also  $M$ . The disk radius is  $R$  and the ladybug initially sits at radius  $R$  but moves in to  $\frac{1}{2}R$ . The new angular speed of the disk is

- A.  $\frac{1}{2}\omega$  B.  $\omega$  C.  $<2\omega$  D.  $2\omega$  E.  $>2\omega$

$$I_1 \omega_1 = I_2 \omega_2$$

$$\left(\frac{1}{2}MR^2 + MR^2\right)\omega_1 = \left[\frac{1}{2}MR^2 + M\left(\frac{1}{2}R\right)^2\right]\omega_2$$

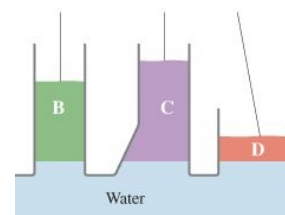
$$\frac{3}{2}\omega_1 = \frac{3}{4}\omega_2 \rightarrow \omega_2 = \frac{12}{6}\omega_1$$

8. (10 pts) In a collision involving rotating bodies (satellites?!) freely floating in space which is always true?

- A. kinetic energy is conserved B. linear momentum is conserved  
C. rotational kinetic energy is conserved D. angular momentum is conserved  
E. A and C F. B and D G. A, B, C, and D

9. (10 pts) A column of three fluids of unknown density float atop a bath of water. Rank the density of the fluids relative to that of water and each other.

- A.  $\rho_w < \rho_C < \rho_B < \rho_D$   
B.  $\rho_w > \rho_C > \rho_B > \rho_D$   
C.  $\rho_w < \rho_D < \rho_B < \rho_C$   
D.  $\rho_w > \rho_D > \rho_B > \rho_C$   
E.  $\rho_w > \rho_C = \rho_B = \rho_D$



10. (10 pts) Mass conservation for incompressible fluids implies that when a blood in an artery flows from a place with a big cross sectional area to a small cross sectional area clogged by fatty buildups it experiences

- A. an increase in speed and a decrease in pressure
- B. an increase in speed and an increase in pressure
- C. a decrease in speed and a decrease in pressure
- D. a decrease in speed and an increase in pressure
- E. an increase in speed and no change in pressure

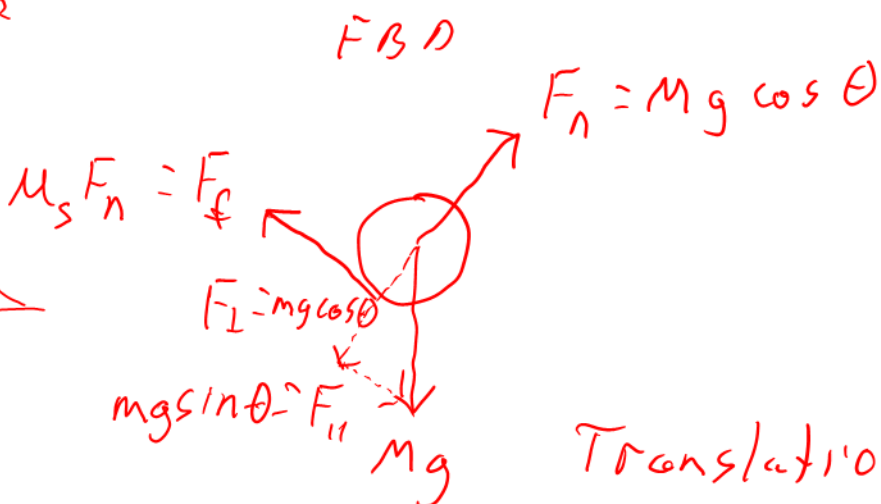
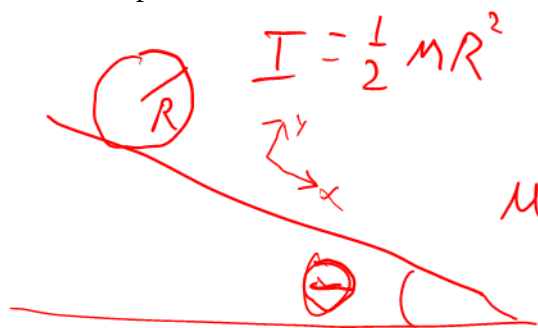
11. (10 pts) When you take a solid sphere submerged fully in water at depth  $D$  to a depth  $2D$  it experiences

- A. twice the pressure and no change in buoyant force
- B. twice the pressure and twice the buoyant force
- C. four times the pressure and twice the buoyant force
- D. four times the pressure and no change in buoyant force
- E. none of these

12. (10 pts) When you turn on the water in a shower and the shower curtain moves inward toward the stream of water this is best described as a consequence of

- A. Pascal's law
- B. Archimedes' principle
- C. Bernoulli's equation
- D. Joule's effect
- E. Voldemort's curse

13. (40 pts) A logging company harvests trees, shaving off the branches and leaving just the trunk of the tree, a solid cylinder with diameter  $D$  and length  $L$  and mass  $M$ . They want to roll the trunks down a slope where the coefficient of static friction is  $\mu_s$  so that they roll without slipping. Find an expression for the maximum angle of the slope  $\theta$  below the horizontal so that the trunks roll without slipping.



Rotation

$$\Sigma \tau = I \alpha$$

and  $a = R \alpha$

$$F_f R = \frac{1}{2} MR^2 \alpha$$

$$\mu_s Mg \cos \theta R = \frac{1}{2} MR^2 \left( \frac{a}{R} \right)$$

$$\mu_s g \cos \theta = \frac{1}{2} a$$

$$2\mu_s g \cos \theta = a$$

Translation

$$\Sigma F = ma$$

$$F_{\parallel} - F_f = ma$$

$$Mg \sin \theta - \mu_s Mg \cos \theta = Ma$$

Eliminate  $a$ , solve for  $\theta$

$$g \sin \theta - \mu_s g \cos \theta = a$$

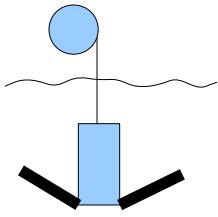
$$g \sin \theta - \mu_s g \cos \theta = 2\mu_s g \cos \theta$$

$$\sin \theta = \mu_s (\cos \theta + 2 \cos \theta)$$

$$\frac{\sin \theta}{\cos \theta} = 3\mu_s = \tan \theta$$

$$\text{atan}(3\mu_s) = \theta$$

14. (40 pts) A boat anchor is suspended by a massless cable into sea water of density  $\rho_w$  at depth  $D$  below the surface. The anchor, made of metal density  $\rho_A$ , is shaped like a rectangular box width  $W$  length  $L$  and height  $H$  with two small cylinders of length  $l$ , radius  $r$ , and mass  $m$  protruding from the bottom. The top of the cable is wrapped around a hollow drum of mass  $Q$  and radius  $R$ . (Assume that  $D$  is much larger than the size of the anchor). Give an expression for the torque that the drum motor must produce in order that the anchor rise with a linear acceleration  $a$ .



Mass of anchor is

$$\rho_A W L H + \rho_A 2\pi r^2 l = M$$

weight of water displaced =  $F_{\text{buoy}} = \rho_w \text{Vol}_{\text{Anchor}} g$

$$= \rho_w g (W L H + 2\pi r^2 l)$$

$$\sum \tau = I \alpha$$

$$\tau_m - T R = Q R^2 \left(\frac{a}{R}\right)$$

$$\sum F = m a$$

$$T + F_b - M g = M a$$

$$\tau_m = Q R a + R T = R(Q a + T)$$

$$T = M a + M g - F_b$$

$$= Q R a + R [M(a+g) - F_b]$$

$$= Q R a + R(a+g)(\rho_A W L H + \rho_A 2\pi r^2 l) - R \rho_w g (W L H + 2\pi r^2 l)$$