

Physics 1210 Exam 3
14 April 2011

This test is closed-note and closed-book. No written, printed, or recorded material is permitted. Calculators are permitted but computers are not. No collaboration, consultation, or communication with other people (other than the administrator) is allowed by any means, including but not limited to verbal, written, or electronic methods. Sharing of calculators is prohibited. If you have a question about the test, please raise your hand. For multiple choice, you may choose two answers, and if one is correct, receive half credit, etc. For full credit on written problems, show the full thought process from basic equations to final results.

$$V_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \quad a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad a_{rad} = \frac{v^2}{R} = \frac{4\pi R^2}{T^2}$$

2.2 lbs = 1 kg
2.2 lbs = 1 kg
1 mi = 5280 ft = 1760 m
1 Calorie = 4200 J
1 Ton = 2000 lbs

$$x_1 = x_0 + v_0 t + \frac{1}{2} a t^2 \quad v_1 = v_0 + a t \quad v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$\Sigma \vec{F} = m \vec{a} \quad F_{spring} = -kx \quad F_f = \mu F_n$$

$$V_{sphere} = \frac{4}{3} \pi R^3$$

Work/Energy $W = \vec{F} \cdot \vec{s} \quad W = \Delta K \quad U_s = \frac{1}{2} kx^2 \quad U_g = mgy \quad P = \frac{\Delta W}{\Delta t} = Fv \quad W_{grav} = -\Delta U \quad F = -\frac{dU}{dx}$

Momentum/Impulse $p = mv \quad J = \Delta(mv) = Ft \quad X_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i}$

Angular Motion $\theta_1 = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \quad \omega_1 = \omega_0 + \alpha t \quad \omega_1^2 = \omega_0^2 + 2\alpha(\theta_1 - \theta_0)$

$$\omega = \frac{d\theta}{dt} \quad s = r\theta$$

$$\alpha = \frac{d\omega}{dt} \quad v = r\omega$$

$$a_{rad} = \omega^2 r$$

$$I = \Sigma_i m_i r_i^2 \quad \vec{\tau} = \vec{r} \times \vec{F} = rF \sin \phi \quad \Sigma \vec{\tau} = I \vec{\alpha} \quad W = \Delta K = \tau \Delta \theta$$

$$\text{Power}_{rot} = \tau \omega$$

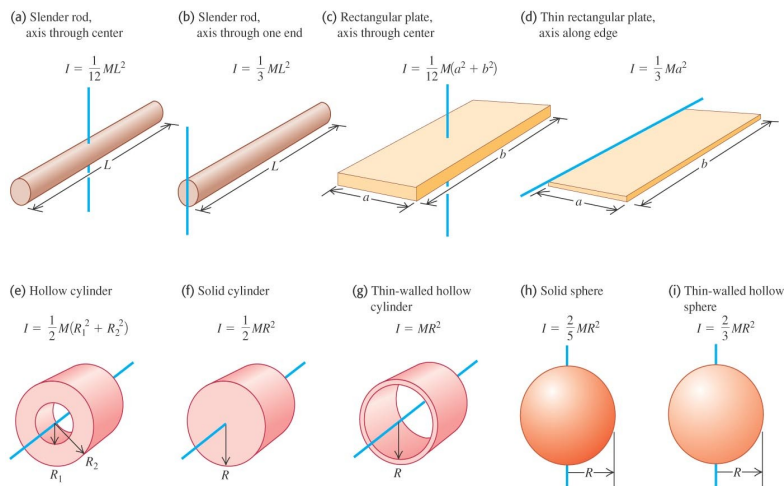
$$K_{rot} = \frac{1}{2} I \omega^2 \quad K_{total} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$\vec{L} = \vec{r} \times \vec{p} = r m v = I \omega \quad \Delta L = \tau \Delta t$$

$$I_{parallel} = I_{cm} + M d^2$$

Gravity: $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

$$F_g = \frac{G M_1 M_2}{r^2} \quad U_g = \frac{G M_1 M_2}{r}$$



Periodic Motion

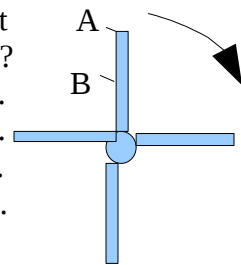
$$f = \frac{1}{T} \quad \omega = 2\pi f \quad \text{pendulum: } \omega = \sqrt{\frac{g}{l}}$$

$$\omega = \sqrt{\frac{k}{m}} \quad \text{physical pendulum: } \omega = \sqrt{\frac{mgd}{I}}$$

$$x = A \cos(\omega t + \phi) \quad v = -\omega A \sin(\omega t + \phi) \quad a = -\omega^2 A \cos(\omega t + \phi)$$

$$\text{damped motion: } x = A e^{(-b/2m)t} \cos(\omega' t) \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

- C 1. (7 pts) A giant windmill with 4 blades of length L is turning in the wind. Point A is at the tip of the blade while point B is halfway out from the axis of rotation. Which is true?
- The angular speed of A is twice that of B, but they have the same linear speed.
 - The angular speed of B is twice that of A, but they have the same linear speed.
 - The linear speed of A is twice that of B, but they have the same angular speed.
 - The linear speed of B is twice that of A, but they have the same angular speed.
 - They both have the same angular and linear speeds.



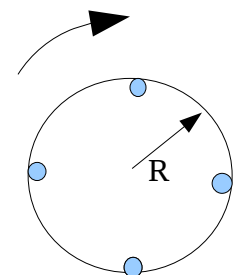
- B 2. (7 pts) You move to a planet where the radius is $3x$ that of Earth, the mass is $6x$ that of Earth, and the length of its day is $5x$ that of Earth. The gravitational acceleration, g_p , on this planet compared to Earth will be

A. $1/3g$ B. $2/3g$ C. $5/6g$ D. $5/3g$ E. $2g$ F. $18/5g$ G. $9g$

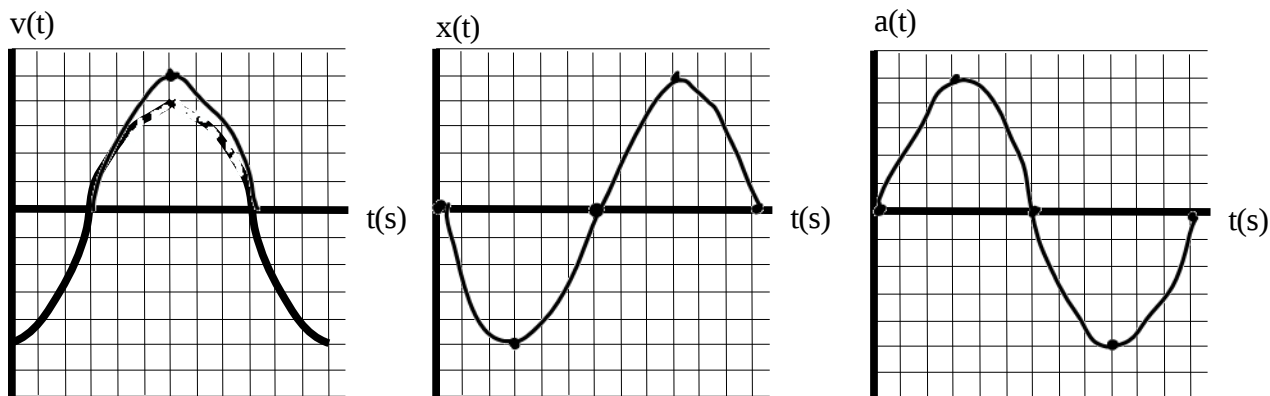
- A 3. (7 pts) Four children of mass M ride at radius R on a magical massless Merry-go-Round that rotates in a horizontal plane at angular speed ω_0 . Shown is a top-down view. Two *more out to $2R$*

and two move in to radius $\frac{1}{2}R$. The new angular speed of the ride is

- $8/17\omega_0$
- $4/13\omega_0$
- $\frac{1}{2}\omega_0$
- $5/13\omega_0$
- ω_0



4. (8 pts) Given the velocity-time graph below for an oscillating object, sketch the position-time and the acceleration-time graph that corresponds.



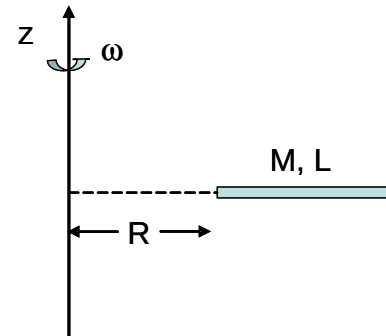
- D 5. (7 pts) A very smooth solid steel ball is rolling down a very smooth steel slope without slipping. Which one of the following statements is NOT true?

- The frictional force could produce a torque for the rolling motion
- The gravitational force could produce a torque for the rolling motion
- The gravitational force does positive work
- The frictional force does negative work
- The total mechanical energy of the ball is conserved

D

6. (7 pts) A uniform rod with length L and mass M is rotating around axis z as the figure shows. Circle the correct moment of inertia of the rod for this rotational motion.

- A. MR^2 B. $M(R+L/2)^2$ C. $MR^2 + 1/3ML^2$
 D. $M(R+L/2)^2 + 1/12ML^2$ E. None of the above



D

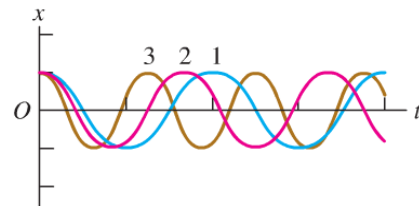
7. (7 pts) A satellite is doing a uniform circular motion around a planet. Which one of the following statements is correct?

- A. The satellite's rotational period depends on its mass
 B. The satellite's rotational radius depends on its mass
 C. The satellite's momentum is constant
 D. The satellite's angular momentum is constant
 E. The satellite's total mechanical energy is constant and positive

A

8. (7 pts) The right figure shows the simple harmonic motion of three different spring-driven oscillators that all have the same mass. Which one of the following statements is correct?

- A. Oscillator 1 has the smallest spring constant
 B. Oscillator 1 has the largest frequency
 C. Oscillator 1 has the smallest period
 D. Oscillator 3 has the smallest angular frequency
 E. None of the above is correct



C

9. (7 pts) A torsion spring is displaced an angle of 5° from equilibrium and oscillates with a frequency of f . If instead it is displaced at an angle of 15° from equilibrium and released, what will be its frequency of oscillation?

- A. $1/9 f$ B. $1/3 f$ C. f D. $3f$ E. $9f$

10. (9 pts) An ice skater begins to spin about a vertical axis on the point of one skate. Friction is negligible. She begins with her arms and free leg extended away from her axis of rotation. As she brings her arms and free leg closer to her axis of rotation,

a. What happens to her rotational speed ω ? (Select one.)

- ω increases. ω remains constant. ω decreases.

b. What happens to her angular momentum L ? (Select one.)

- L increases. L remains constant. L decreases.

c. What happens to her rotational kinetic energy K_{rot} ? (Select one.)

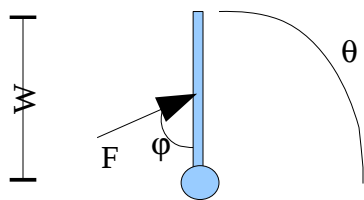
- K_{rot} increases. K_{rot} remains constant. K_{rot} decreases.

A

11. (7 pts) A ball of mass M oscillates with period T when attached to a spring with spring constant k_1 . If another ball having mass $4M$ is to oscillate with the same period T , what must be the spring constant k_2 of the spring to which it is attached?

- A. $k_2 = 4 k_1$ B. $k_2 = 2 k_1$ C. $k_2 = k_1$ D. $k_2 = k_1/2$ E. $k_2 = k_1/4$

12. (25 pts) You try to close a door by pushing with a force F at angle ϕ to the plane of the door directly in the center of the door. The door has mass M , height H , and width W . Give an expression for the time, t , required for the door to close if it is initially open at some angle θ . A top view of the door is shown. Assume the door starts at rest.



$\Sigma \tau = I \alpha$ find α then use angular kinematics to find time needed to move a certain angular distance θ

I for a slab = $\frac{1}{3} M W^2$

$\tau = \vec{r} \times \vec{F} = r F \sin \phi$

$= \frac{1}{2} W F \sin \phi$

$\Sigma \tau = I \alpha$

$\frac{1}{2} W F \sin \phi = \frac{1}{3} M W^2 \alpha$

$\frac{3 F \sin \phi}{2 M W} = \alpha$

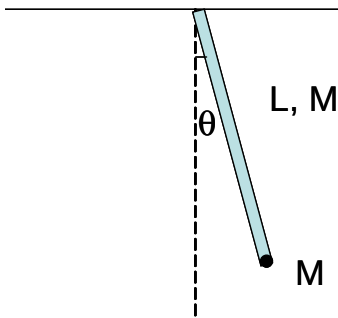
now for kinematics part

$\theta_i = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

$t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2\theta \cdot 2 M W}{3 F \sin \phi}}$

$t = \sqrt{\frac{4 M W \theta}{3 F \sin \phi}}$

13. (25 pts) A uniform rod with length L and mass M is pivoted at one end. point object with the same mass M is attached to the other end of the rod. Calculate the oscillation period of this composite system for a small-amplitude oscillation.



This is a physical pendulum where

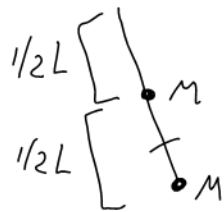
$$\omega = \sqrt{\frac{Mgd}{I}} \quad \text{and} \quad P = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{Mgd}}$$

m here = $2M$, the total mass of the system

I for this two-component system is

$$I = I_{\text{rod}} + I_{\text{point}} \\ = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$$

d is the distance from the center of mass to the pivot point.



so the center of mass is at $\frac{3}{4}L = d$

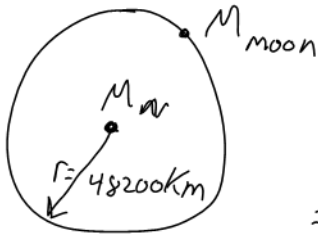
therefore

$$P = 2\pi \sqrt{\frac{\frac{4}{3}ML^2}{2Mg \frac{3}{4}L}} = 2\pi \sqrt{\frac{16L}{9 \cdot 2g}} \\ = \frac{8\pi}{3} \sqrt{\frac{L}{2g}}$$

or

$$\frac{4\pi}{3} \sqrt{\frac{2L}{g}}$$

14. (25 pts) Neptune's moon Naiad orbits Neptune at an average distance of 48.2×10^3 km. Neptune's mass is 1.02×10^{26} kg, and Naiad's mass is 1.4×10^{17} kg. How much additional kinetic energy would Naiad need to obtain for it to escape from Neptune's orbit?



$$E_1 = E_2 \quad \text{conservation of energy}$$

$$K_1 + U_1 + E_A = K_2 + U_2 \quad U_2 \rightarrow 0 \text{ for escape}$$

$$K_2 \rightarrow 0 \text{ for barely escaping.}$$

$$K_1 + U_1 + E_A = 0$$

$$\frac{1}{2} M_m V_{m1}^2 + \frac{-G M_N M_m}{r} + E_A = 0 \quad E_A = \text{additional energy required}$$

$$V_{m1} \approx V_{\text{circular}} = \sqrt{\frac{G M_N}{r}}$$

$$E_A = -\frac{1}{2} M_m \sqrt{\frac{G M_N}{r}}^2 + \frac{G M_N M_m}{r} = \frac{1}{2} \frac{G M_N M_m}{r}$$

$$= \frac{1}{2} \frac{6.67 \times 10^{-11} \cdot 1.02 \times 10^{26} \cdot 1.4 \times 10^{17}}{4.8 \times 10^7}$$

$$\approx 9.7 \times 10^{24} \text{ J}$$